

LOCATION: SWANSEA

LLEOLIAD: ABERTAWE

LEVEL 6

LEFEL 6

EXAMINATION PERIOD
MAY 2020

CYFNOD ARHOLIADAU
MAI 2020

MODULE CODE: M6X00420
ADVANCED STRESS ANALYSIS AND FEA

TIME: 2 HOURS

ANSWER THREE OUT OF FOUR QUESTIONS

Equipment or materials required – None

Adnoddau neu ddefnyddiau sydd eu hangen – Dim

A formula sheet is provided for reference.

Calculators are permitted.

Assume 'g' = 9.81 m/s².

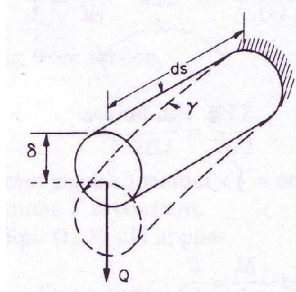
ONE

Your choice of elements will greatly influence your results in any analysis that you carry out in FEA software, as such;

- a) Discuss how you would choose the element type and what physical property information would be needed for each type. [6 marks]
- b) Discuss 4 ways that you could use to help you decide what element size you should use. [6 marks]
- c) Discuss how you would choose element order. [4 marks]
- d) Discuss what 'h-adaptive' and 'p-adaptive' meshes are. [4 marks]

TWO

- (a) (i) Consider the bar subjected to shear loading as shown below. Prove from first principles that the shear strain γ , due to the load, is given by; $\gamma = \frac{Q}{AG}$, where Q is the shear load, ds is the length of the bar, A is the cross sectional area and G is the Shear Modulus.



[2 Marks]

- (ii) Hence, prove that the strain energy, U , for a bar in Shear is given by;

$$U = \frac{Q^2 L}{2AG}$$

[4 Marks]

- (b) (i) A hollow rectangular stainless steel box section has external dimensions 30 mm by 20 mm and wall thickness 2.5 mm. If the box section has a length of 2.4 m, determine the strain energy when a stress of 480 MPa is produced in the box section by tensile loading. $E = 190$ GPa.

[3 Marks]

- (ii) The box section then has a metal collar rigidly fitted at the lower end when fixed vertically, and a mass, m , is dropped onto the collar. Show, by considering the energy transferred from the falling weight to the box section, that the relationship between stress, σ , experienced in the section and the drop height, h , is given by;

$$h = \frac{\sigma^2 V}{2EW}$$

where, V is the volume of the box section. Assume that the extension of the box section is *NOT* significant in the energy transfer.

[3 Marks]

- (iii) If a mass of 20 kg fell from a height of 450 mm onto the collar fitted to the bottom of the box section, what is the value of stress experienced in the material?

[3 Marks]

- (iv) Rather than using a hollow rectangular box section a hollow stainless steel circular pipe of external diameter 20 mm and length 2.4 m was fixed vertically with a collar rigidly fitted to the bottom end. In order that the stress was no greater than the value found in part (iii) above, what is the required wall thickness of the pipe? The same mass of 20 kg falls through a height of 450 mm.

[5 marks]

THREE

Consider a hollow cylindrical shaft that is made from low carbon steel with an external diameter of 75 mm and wall thickness of 5 mm. The shaft is only subjected to torsion as a result of its operation in power transmission, and no axial loading is present. If the elastic limit of steel in tension is found to be 325 MN/m^2 , and the value of Poisson's ratio for the material is 0.3, determine the maximum torque that can be transmitted by the hollow shaft if a safety factor of 6 is to be used in the design process, and the applied criterion of elastic failure is;

- (a) The maximum shear stress theory (*Guest-Tresca*), and
- (b) The maximum shear strain energy per unit volume theory (*Von Mises*).

[12 marks]

Note: For an element in a complex stress system that is subjected to shear stress only, as in the case of pure torsion, the maximum and minimum principal stresses are known to be τ_{xy} and $-\tau_{xy}$ respectively.

Due to the assumption of a 2-dimensional stress plane, $\sigma_2 = 0$, and there are no direct stresses as a result of torsional loading only.

- (c) In order to undertake a weight-saving design analysis, it is decided to change the material and dimensions of the hollow power transmission shaft. Applying the *Von Mises* elastic failure criterion, determine the maximum torque that could be transmitted by a hollow shaft manufactured from titanium alloy Ti6AL4V, if the external diameter was 50 mm and wall thickness 3 mm, and the factor of safety is 2.75. For the titanium alloy being considered, the elastic limit is given as 880 MN/m^2 .

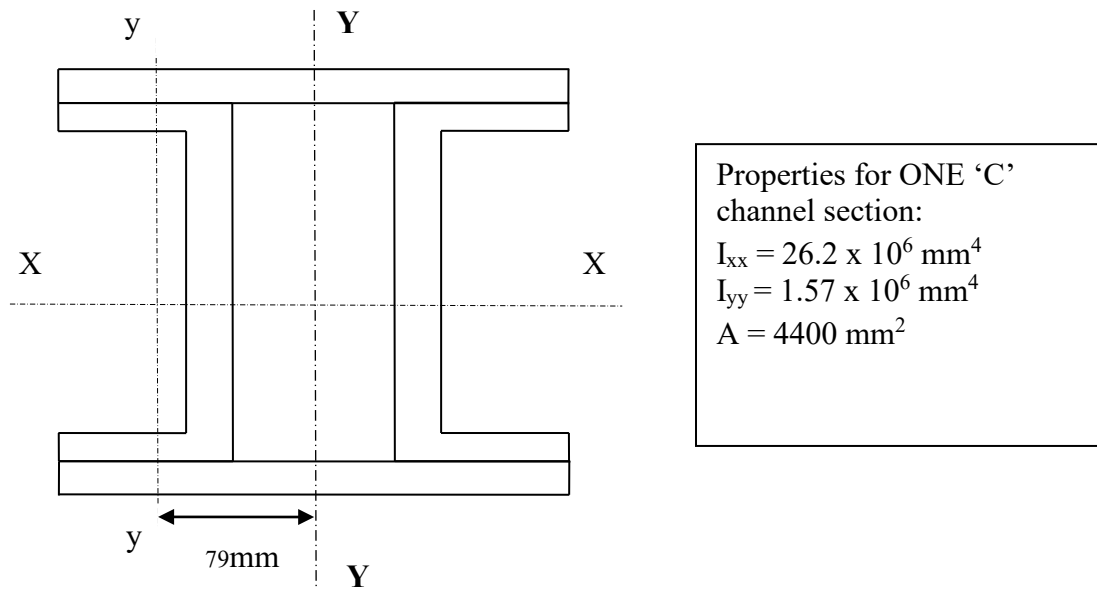
[6 marks]

- (d) What is the percentage difference in torque that could be transmitted when comparing the two hollow shafts of different materials and dimensions?

[2 marks]

FOUR

A fabricated strut manufactured from mild steel (low carbon steel), is formed from two horizontal flat steel plates, each measuring 300 mm wide and 12 mm thick, and two vertical 'C' channel sections, 100 mm wide by 200 mm deep as shown below;



The fabricated strut is 8 m long and is to be pinned at both ends, Young's Modulus for mild steel is 200 GN/m^2 for the plates and channels, the end condition constant 'a' is $1/7500$ for both ends pinned, and the compressive yield stress is 320 MN/m^2 .

- (a) Determine about which axis the fabricated strut will tend to buckle. Show your complete calculations and values for both the second moment of area about the X and Y axes. [7 marks]
- (b) What is the radius of gyration of the strut about the buckling axis? [1 mark]
- (c) What is the value of the slenderness ratio for the strut? [1 mark]
- (d) According to Euler theory, what load will this strut buckle at? [3 marks]
- (e) According to Rankine-Gordon theory, what load will this strut buckle at? [3 marks]
- (f) If it was subsequently decided to rigidly fix two vertical steel plates of dimensions 224 mm by 6 mm to the fabricated strut so that these plates formed an enclosed box section, determine the increase in predicted Euler buckling load. All other parameters and properties remain unchanged. [5 marks]

FORMULA SHEET

Energy

$$U = \frac{P^2 L}{2AE} \quad \text{Strain energy in tension}$$

$$U = \frac{Q^2 L}{2AG} \quad \text{Strain energy in shear}$$

$$U = \frac{M^2 L}{2EI} \quad \text{Strain energy in bending}$$

$$U = \frac{T^2 L}{2GJ} \quad \text{Strain energy in torsion}$$

$$\text{P.E.} = mgh \quad \text{Potential energy}$$

$$\text{K.E.} = \frac{1}{2} mv^2 \quad \text{Kinetic energy}$$

Buckling of Struts

$$I = \frac{bd^3}{12} \quad \text{Second moment of area about neutral axis (rectangle)}$$

$$I = \frac{\pi d^4}{64} \quad \text{Second moment of area about neutral axis (circle)}$$

$$J = \frac{\pi d^4}{32} \quad \text{Polar moment of inertia (circle)}$$

$$\text{slenderness ratio} = \frac{L}{k}$$

$$k = \sqrt{\frac{I}{A}} \quad \text{Radius of gyration}$$

$$I_{NA} = I + Ah^2 \quad \text{Parallel Axis Theorem}$$

$$P_e = \frac{\pi^2 EI}{L^2} \quad \text{Euler Buckling: pinned-pinned}$$

$$P_e = \frac{\pi^2 EI}{4L^2} \quad \text{Euler Buckling: fixed-free}$$

$$P_e = \frac{4\pi^2 EI}{L^2} \quad \text{Euler Buckling: fixed-fixed}$$

$$P_e = \frac{2\pi^2 EI}{L^2} \quad \text{Euler Buckling: fixed-pinned}$$

$$P_R = \frac{\sigma_y A}{1 + a(L/k)^2} \quad \text{Rankine Buckling}$$

Theories of Elastic Failure – criterion for failure

Maximum Principal Stress (*Rankine*) $\sigma_1 = \sigma_y$

Maximum Shear Stress (*Guest-Tresca*) $\sigma_1 - \sigma_3 = \sigma_y$

Maximum Principal Strain (*Saint-Venant*) $\sigma_1 - \nu\sigma_2 - \nu\sigma_3 = \sigma_y$

Total Strain Energy/Unit Volume (*Haigh*) $\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1) = \sigma_y^2$

Shear Strain Energy/Unit Volume (*Von Mises*) $[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2] = 2\sigma_y^2$

Elastic Loading

Bending: $\frac{\sigma}{y} = \frac{M}{I} = \frac{E}{R}$ Torsion: $\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L}$

Factor of Safety $n = \frac{\text{Yield stress}}{\text{Allowable working stress}}$

Contact Mechanics

Maximum pressure p_0 and deflection δ of two bodies in point of contact.

Spheres: $p_0 = 1.5 \left(\frac{F}{\pi a^2} \right)$ Cylinders: $p_0 = \frac{2}{\pi} \frac{F}{aL}$

(A) Sphere on a flat surface:

Cylinder on a flat surface:

$$a = 0.880 \sqrt[3]{F r_1 \Delta}$$

$$a = 1.076 \sqrt{\frac{F}{L} r_1 \Delta}$$

For $E_1 = E_2 = E$

$$\delta = 0.775 \sqrt[3]{F^2 \frac{\Delta^2}{r_1}}$$

$$\delta = \frac{0.579 F}{E L} \left(\frac{1}{3} + \ln \frac{2 r_1}{a} \right)$$

(B) Two spherical balls:

Two cylindrical rollers:

$$a = 0.880 \sqrt[3]{F \frac{\Delta}{m}}$$

$$a = 1.076 \sqrt{\frac{F \Delta}{L m}}$$

$$\delta = 0.775 \sqrt[3]{F^2 \Delta^2 m}$$

$$\text{Where: } \Delta = \frac{1}{E_1} + \frac{1}{E_2} \quad \text{and, } m = \frac{1}{r_1} + \frac{1}{r_2}$$