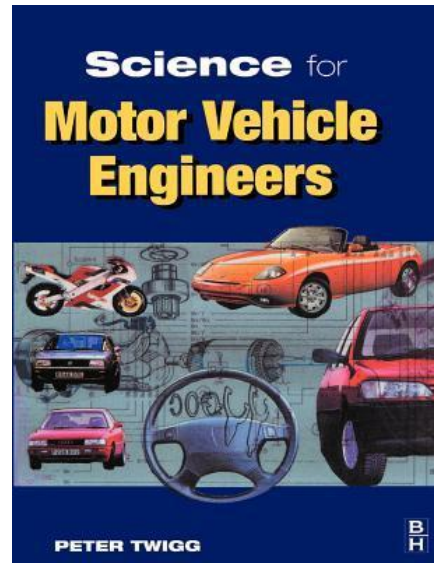
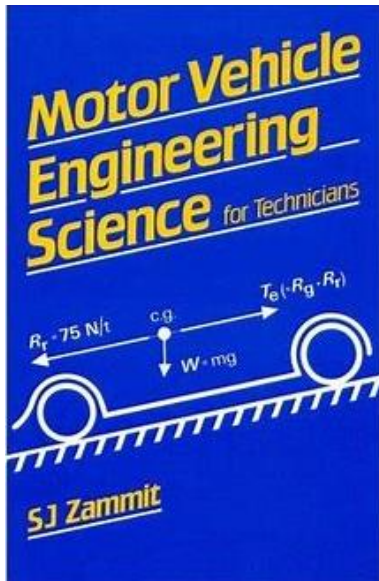


Engineering Science



Assessment

70% Exam 30% Coursework

Topics

Equations of Motion

Equations of Angular Motion

Work

Energy

Power

Friction

SHM

Heat Transfer

Stress & Strain

Bending

Torsion

Linear Velocity and Acceleration

'Equations of Motion'

Linear Velocity maybe defined as 'the rate of change of distance (or displacement) with time'. Velocity differs from the more general term 'speed' in that velocity accounts for a direction of motion as well as having magnitude hence it is a vector quantity. 'Speed' defines only magnitude hence it is a scalar quantity.

Linear Acceleration may similarly be defined as 'the rate of change of velocity with time'. Like velocity acceleration is a vector quantity.

The SI 'base units' of velocity and acceleration are respectively:

Velocity – m/s or ms^{-1} , commonly represented by symbols u and v .

Acceleration – m/s^2 or ms^{-2} , commonly represented by symbol a .

Velocity/ Time Graphs with Uniform (Constant) Acceleration

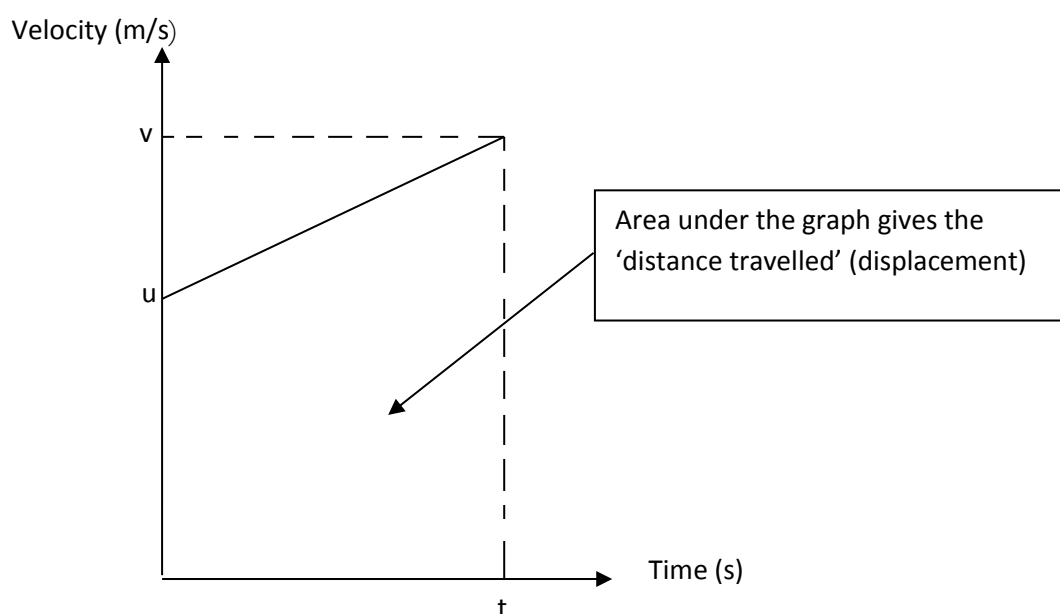


Figure 1.

Provided that the acceleration is uniform (constant), the velocity increases linearly as a straight line graph as in Figure 1. Given the definition 'acceleration is the change in velocity with respect to time', and that for motions of this kind, u represents the velocity at time $t=0$ ('the beginning'), and v represents the velocity after a time lapse t , referring to the graph we get the following formula:

$$a = \frac{v - u}{t}$$

However with the equation rearranged to the customary layout of $y = mx + c$ then the equation of motion would be:

$$v = u + at \dots\dots\dots(1)$$

Where a is the gradient of the line, representing the uniform acceleration.

'Distance Travelled' ('Displacement'), denoted by s

Because the velocity increases uniformly, the average or mean velocity can be accurately determined from the area under the velocity –time plotted graph which has the shape of a trapezium.

$$s = \frac{(u + v)}{2} t \dots\dots\dots(2)$$

If we substitute for v from equation (1), we could get:

$$s = \frac{(u + (u + at))t}{2}$$

$$s = (2u + at) \frac{t}{2}$$

$$s = ut + \frac{at^2}{2} \dots\dots\dots(3)$$

Finally to complete the set of the Equations of Motion (for uniform acceleration), we may combine equation (1) and equation (2) to eliminate 't' from the formula:

From (1): $t = \frac{v - u}{a}$, therefore substituting for t into equation (2):

$$s = \frac{u(v - u)}{a} + \frac{1}{2} \frac{a(v - u)^2}{a^2}$$

Simplifies to:

$$2as = 2vu - 2u^2 + (v^2 - 2uv + u^2)$$

Giving

$$2as = v^2 - u^2$$

Or:

$$v^2 - u^2 = 2as \dots\dots\dots(4)$$

Giving the equations of motion:

$$v = u + at$$

$$s = \frac{(u + v)}{2} t$$

$$s = ut + \frac{at^2}{2}$$

$$v^2 - u^2 = 2as$$

Rotary (Angular) Motion: Angular Velocity and Angular Acceleration

‘Equations of Motion’

When considering rotary (angular) motion with uniform (constant) angular acceleration, the analysis is very similar to that with linear motion. Such considerations apply when considering ‘bodies’ rotating about a centre, as with a wheel rotating in Figure 2:

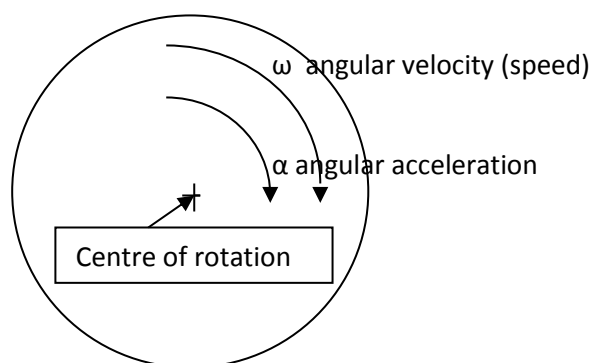


Figure 2

An angular velocity/ time graph maybe similarly shown, with uniform (constant) angular acceleration.

The following graph and analysis is very similar to that for linear motion, as considered.

Angular Velocity/Time Graph with Uniform (Constant) Angular Acceleration

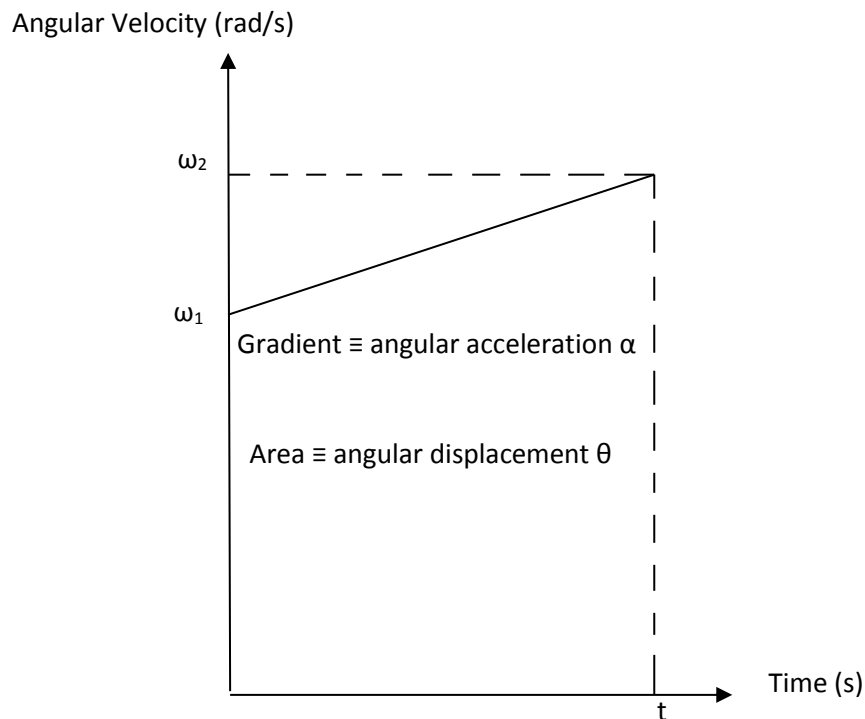


Figure 3.

Where:

ω_1 = initial angular velocity, with units radian per second (rad/s),

ω_2 = final angular velocity, with units rad/s,

α = uniform or constant angular acceleration, with unit rad/s^2 ,

t = time, with units seconds (s)

Applying the same principles as those applied to the linear conditions an alternative and corresponding set of equations can be determined:

$$\omega_2 = \omega_1 + \alpha t \dots\dots\dots(1)$$

$$\theta = \frac{(\omega_1 + \omega_2)t}{2} \dots\dots\dots(2)$$

$$\theta = \omega_1 t + \frac{\alpha t^2}{2} \dots\dots\dots(3)$$

$$\omega_2^2 = \omega_1^2 + \alpha \theta \dots\dots\dots(4)$$

Relationship between Angular Speed and Tangential Velocity

Given the following scenario:

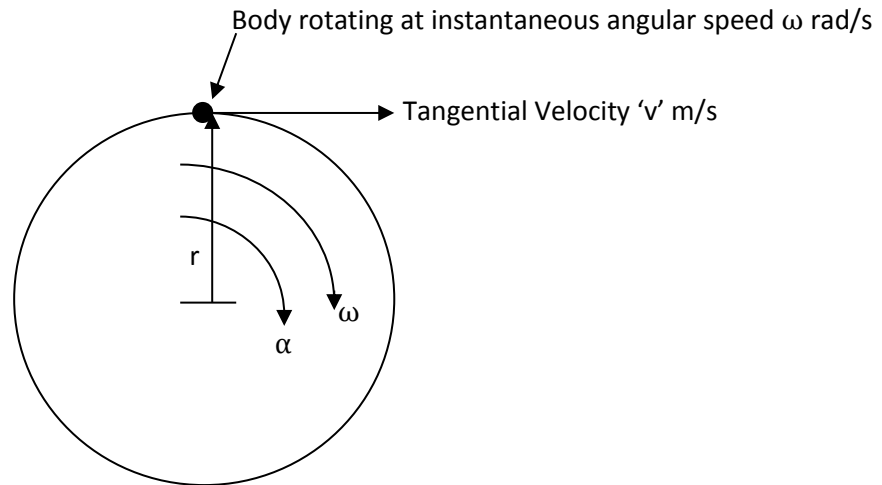


Figure 4.

For above:

$$v = r\omega$$

If the angular speed changes i.e there is an angular acceleration (or deceleration) α rad/s², then proportionally so there will be a tangential acceleration given by:

$$a = r\alpha$$

Newton's Laws of Motion

1. 'Everybody continues in its state of rest or uniform motion in a straight line, unless compelled to change that state by the action of an external force'.
2. 'The rate of change of momentum of a body is directly proportional to the resultant external force which is producing the change, and takes place in the direction of the force'

Momentum = mv

i.e Force $F \propto \frac{d(mv)}{dt}$ Rate of change of momentum

$$F = \frac{d(mv)}{dt}$$

Or

$$F = v \frac{dm}{dt} + m \frac{dv}{dt}$$

Differentiation of a product via the product rule.

But if the mass remains constant then $\frac{dm}{dt} = 0$

$$\text{Hence } F = m \frac{dv}{dt}$$

Or more commonly $F = ma$

3. 'To every action there is an equal and opposite reaction'.

Torque and Angular Acceleration

Consider the situation:

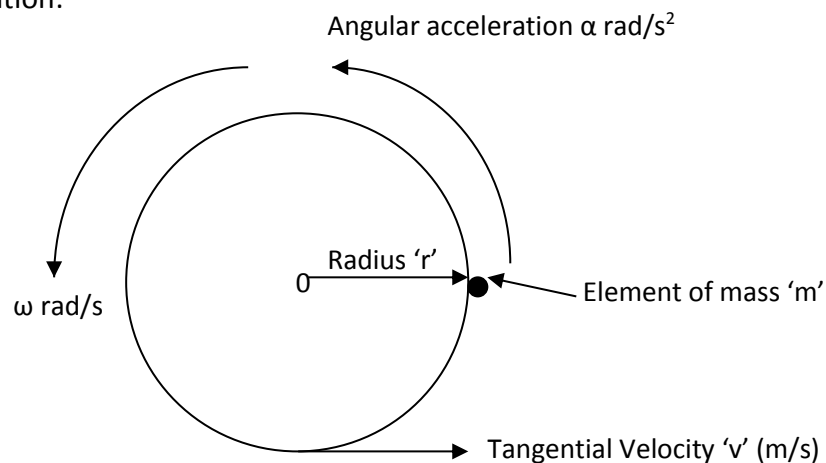


Figure 5.

Figure 5 represents a solid object rotating about a fixed axis 'O' perpendicular to the page. Considering a single element of mass 'm' rotating about O at a distance 'r', should the speed of the element of mass change then there must be some tangential force being applied to it in accordance with Newton's Second Law.

$$F = ma$$

But since $a = r\alpha$, page 5 then,

$$F = mr\alpha$$

The torque acting on the element of mass can be found since torque is produced by a force acting at a perpendicular distance of the line of action of the force from the point of application.

$$T = Fxr = m\alpha r^2 = m\alpha r^2$$

But when considering the whole of the solid object, the sum of all of the elements of mass of which the solid object consists, acting at many radii need to be considered. The sum of all these elements is known as the Mass Moment of Inertia or simply Moment of Inertia, 'I'.

$$\text{Since } I = \sum mr^2$$

$$\text{Giving } T = I\alpha \text{ compare with } F = ma$$

Calculating values of I is difficult except for simple cases:

$$I = mk^2$$

Where 'k' is the radius of gyration, and is the distance from the centre of rotation to the point at which the mass of the solid object is considered to be concentrated from the perspective of its 'rotary inertia'.

$$\text{So for a uniform disc rotating about its geometric centre, } k^2 = \frac{d^2}{8}$$

Work, Energy & Power

When a force is displaced ('moves through a distance'), then 'work is done', or energy is expended.

$$\text{Work Done} = \text{Force} \times \text{Distance}$$

$$J = N \times m$$

$$J = Nm$$

Potential Energy

Suppose a mass is moved through a vertical distance 'h':

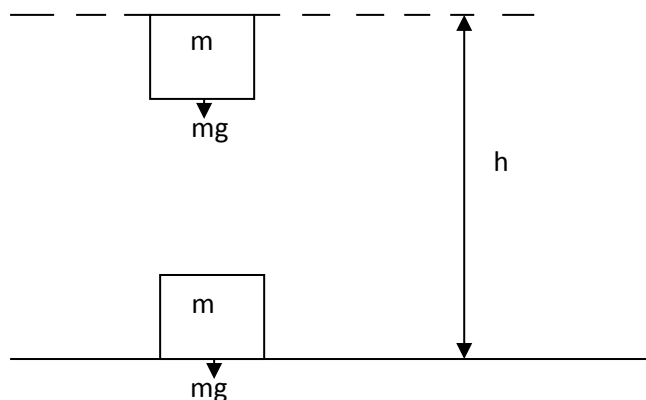


Figure 6.

Constant force acting on body = Gravitational force = weight = mg

$\therefore$ work done = Force x distance

$$= mgh$$

This is the work done (i.e. energy expended) can be reclaimed by allowing the body to move back to its datum. It would accelerate under gravity and have kinetic energy when it falls through the datum. Thus we have potential energy at its elevated position equal to the work done.

$$PE = mgh$$

Kinetic Energy

Kinetic Energy is the energy possessed by a body by virtue of its velocity (motion). Consider a 'body' moving from rest to attain a velocity v in time ' t ' whilst moving through a distance ' s '.

Then:

Work done = Force x Distance = $F.s$

$\therefore$ Kinetic Energy = $F.s$ (Principle of conservation of energy assuming no losses due to friction etc)

$$\text{i.e. } K.E. = (ma) \times \frac{(v^2)}{2a}$$

$$K.E. = \frac{1}{2}mv^2$$

Similarly the K.E. of a 'rotating body' can be shown to be:

$$K.E. = \frac{1}{2}I\omega^2$$

Where I = mass moment of inertia (kgm^2)

ω = angular speed (rad/s)

Power

Defined as 'the rate of doing work'.

Average Power = Work done/time taken

Power = $\text{J/s} = \text{W}$ (watt)

If a force F is moving at a velocity v , then:

$$P = F.v$$

For angular motion:

$$P = T\omega$$

Where T = Torque and ω = angular velocity

$$\omega = \frac{2\pi n}{60} \text{ where } n = \text{rev/min}$$

Giving

$$P = \frac{2\pi nT}{60} \text{ W}$$

Friction

Friction is ever present where there is 'contact' between two surfaces which are moving relative to each other. Consider the principles associated with Dry friction, when two dry and clean surfaces have contact and motion as described.

Laws of Dry Friction

1. If a force is applied tending to move a body over another, the opposing friction that arises is tangential to the surfaces in contact and is just sufficient to balance the applied force.
2. As the applied force increases, there will be a 'limit' reached when the frictional resistance cannot resist the applied force tending to move the body, and there will be a limiting value of frictional resistance when at the 'point of motion'.
3. The 'limiting force' is 'proportional' to the normal (perpendicular) force between the surfaces, and is independent of the area of contact.
4. The ratio of the limiting frictional force (F) and the normal reaction (R_n) is a constant which depends on the nature (type/roughness) of the surfaces in contact.

Thus

$$\frac{F}{R_n} = \mu \text{ where } \mu \text{ is a constant called the Coefficient of Static or Limiting Friction}$$

$$\text{Or } F = \mu R_n$$

Note $0 < \mu < 1$

5. When sliding starts, the direction of the resistive frictional force is always opposite to that of the motion.

The frictional force is again given by $F = \mu R_n$, but μ now assumes a value generally slightly lower than the static limiting values. Otherwise, sliding friction obeys the same laws of static friction.

Linear Velocity and Acceleration, Example Questions.

1. A motor vehicle starting from rest travels with uniform acceleration of 1.2 m/s^2 for 15s. It then travels with uniform velocity for 2 minutes and is finally brought to rest with a uniform deceleration of 1.8 m/s^2 . Determine:

- a) the maximum velocity in km/h,
- b) the time taken for the deceleration period and,
- c) the total distance travelled.

[64.8km/h, 10s, 2.385km]

2. The ram of a horizontal forging machine moves from rest with a uniform acceleration of 160 m/s^2 until it strikes the work piece. Assuming that the ram is brought to rest with a uniform deceleration of 500 m/s^2 in compressing the work piece and that the time taken for the whole operation is 33ms determine:

- a) the velocity at impact,
- b) the amount that the work is compressed, and
- c) the total distance moved by the ram during the operation.

[4m/s, 16mm, 66mm]

3. A cage travels down a mine shaft 800m deep. It accelerates uniformly from rest for the first 200m, travels at constant velocity for the next 500m and then comes to rest again at the bottom of the shaft by retarding uniformly for the last 100m. If the total time for descent is 1 minute calculate the maximum velocity.

[18.33m/s]

4. A train which is being accelerated uniformly passes a station with a velocity of 60km/h and at the next station its velocity is 96km/h. If the distance between the two stations is 1.3 km determine the acceleration in m/s^2 .

[0.167 m/s^2]

5. In a particular type of hardness test a steel hammer is allowed to fall on to a material being tested. If the hammer bounces to a height of 400mm determine the velocity after bouncing on the specimen, take $g = 9.8\text{m/s}^2$. [2.8m/s]

6. A motor vehicle, travelling initially with a velocity of 72km/h, is given a uniform retardation of 0.5m/s^2 for a period of 30s. Sketch the velocity time graph and determine:

a) the final velocity in km/h, and

b) the distance travelled (in meters) during the specified period of time.

[18km/h, 375m]

7. During a cutting operation on a shaping machine the motion of the ram on its forward stroke of length 126mm is divided into three phases as follows: accelerating from rest to a velocity of 500mm/s for the first 20mm, decelerating from 500mm/s to 300mm/s while taking a cut 100mm long and decelerating to rest for the last 6mm. Assuming that all the changes of velocity are uniform sketch the velocity time graph and determine the time taken for the forward stroke. [0.37s]

8. A special motor car, designed to travel at high speeds, is able to accelerate as 2m/s^2 while increasing velocity from 540 to 576 km/h and at 1m/s^2 while increasing velocity from 576 to 612 km/h. Working on the assumption that the acceleration during each of the two periods is uniform determine:

a) the minimum time taken to increase speed from 540 to 612 km/h, and

b) the distance travelled during this time.

[15s, 2.425km]

9. A goods lift in a multi storey factory is able to make its complete ascent of 24m in 30 s. For the first quarter of the distance the velocity is uniformly accelerated, and during the last quarter is uniformly retarded, the acceleration being numerically equal to the retardation. Determine the uniform velocity of the lift during the centre portion of its ascent.

[1.2m/s]

10. A cage in a mine shaft performs the first part of its decent with a uniform acceleration of 1m/s^2 and the remainder of its decent to the bottom of the shaft with uniform retardation of 2m/s^2 . If the total time for decent is 1 minute calculate the depth of the shaft.

[1200m]

11. In a particular laboratory experiment a steel ball was allowed to fall from rest under the influence of gravity. The time taken for it to descend through a distance of 1m was measured accurately.

a) If the time taken was 0.452s calculate the value of gravitational acceleration.

b) Using the value of 'g' determine the velocity of the ball at the bottom of its descent.

[9.79m/s^2 , 4.425m/s]

12. A ball is thrown vertically upwards with a velocity of 9.8m/s . Determine:

a) its height when its velocity is 4.9m/s , and

b) the times when it will be at this height.

[3.675m, 0.5s and 1.5s]

Engineering Science.

Angular Velocity and Acceleration Examples.

1. The speed of a flywheel is increased uniformly from 210rpm to 250rpm in 5 seconds. Determine the angular acceleration and the number of revolutions made by the wheel in the 5 second period. [0.84 rad/s², 19.2 revs]

2. A grinding wheel is rotating at 2800 rpm when the power is switched off. If it takes 4 minutes for the wheel to come to rest calculate the angular retardation, assuming it to be uniform. [1.22 rad/s²]

3. A lift-winding drum accelerates uniformly from rest for 1.5 seconds until it is rotating at 80 rpm. At this speed the drum makes 2.6 revolutions before coming uniformly to rest in 1.2 seconds. Determine
 - a. The angular acceleration and retardation,
 - b. The total time the drum is rotating,
 - c. The total number of revolutions made by the drum.[5.6 rad/s²; 7 rad/s²; 4.64s; 4.4revs]

4. The flywheel of an internal-combustion engine is accelerated uniformly at 15 rad/s² for 3 seconds, until the engine speed is 3500rpm. What is the initial speed? If the rotating parts of the engine have a moment of inertia of 1.3 kgm², determine the accelerating torque. [3070rpm; 19.5 Nm]

5. The radius of gyration of a flywheel can be assumed to be 215mm. If the torque required to give the flywheel an angular acceleration of 6 rad/s² is 11 Nm, estimated the mass of the flywheel. [39.59kg]

6. The belt pulley on a motor-driven shaft is 160mm diameter and is accelerated uniformly from rest to its maximum speed of 1470 rpm in a time of 3 seconds. Calculate,

- the maximum angular velocity in rad/s,
- the maximum linear velocity of the belt in m/s,
- the angular acceleration in rad/s^2 , and
- the linear acceleration of the belt in m/s^2 .

[154 rad/s, 12.32 m/s, 51.3 rad/s^2 , 4.104 m/s^2]

7. In a laboratory test a flywheel is accelerated uniformly from rest for a brief period during which time the flywheel turned through 4.2 rev. The flywheel was then allowed to come to rest against friction with uniform retardation during which time it is turned through 42 rev. If the total time of the motion was 88 seconds determine,

- The maximum angular velocity in rad/s,
- the angular acceleration in rad/s^2 , and
- the angular retardation in rad/s^2 .

[6.6 rad/s; 0.825 rad/s^2 ; 0.082 rad/s^2]

8. A cyclist accelerates from a speed of 12 km/h to a speed of 20 km/h in a time of 20 seconds while descending an incline. Determine,

- the angular acceleration of the road wheels if the diameter of the wheel is 0.66 m;
- the distance travelled in this time.

[0.34 rad/s^2 ; 89 m]

9. A shaft of diameter 0.3 m is rotating at 2000 rpm. Determine:

- its angular velocity;
- the linear velocity of a point on the circumference of the shaft;
- the angular acceleration required to increase the shaft speed to 3000 rpm in a period of 50 s.
- the linear acceleration of a point on the circumference of the shaft.

[209.4 rad/s; 31.4 m/s; 2.094 rad/s^2 ; 0.314 m/s^2]

Engineering Science.

Work, Energy & Power Examples.

1. The energy expended in lifting an engine of mass 159kg is 6kJ. Calculate the height through which it has been lifted. [3.85m]

2. A motor vehicle of mass 1.6kg is travelling on the level at a speed of 72 km/h. What is its Kinetic Energy? If the engine is switched off, how far will the vehicle travel before coming to rest if the average resistance to motion is 1kN. [320J, 0.32m]

3. The inlet valve of a small car engine has a mass of 50g and reaches a maximum acceleration of 8000ms^{-2} during the operation period in 0.001 s from rest. Calculate the kinetic energy of the valve at the end of the acceleration. [1.6J]

4. The speed of a vehicle of total mass 1.5 tonnes is reduced from 54 km/h to 18km/h by the action of a retarding force of 3kN.
 - a. Calculate the change in kinetic energy,
 - b. Show that the change in kinetic energy is equal to the work done against the retarding force. [150kJ]

5. An engine has a mass of 150kg and is suspended from a crane by a sling 4m above the ground.
 - a. Determine the potential energy of the crane. [5.8kJ]
 - b. If the sling were to break and the engine to fall to the ground what would the velocity and the kinetic energy of the engine before just at the point of impact with the ground. [8.86m/s. 5.8kJ]

6. The flywheel of a shearing machine has a diameter of 500mm and a mass of 800kg:.
- Calculate the energy stored in the flywheel when it rotates at 210rev/min. [6045.1J]
 - Calculate the energy given up when the speed of the flywheel is reduced to 105 rev/min. [4533.8J]
 - If the energy given up is used to shear material of thickness 11mm, calculate the average shearing force in kN. [412.165kN]
7. The road wheel brake drum and rotating parts of the wheel assembly of a motor car have a total mass of 20kg and their radius of gyration is 0.23 m. The wheel accelerates uniformly from rest to a 840rev/min in 22s. Determine:
- The angular acceleration of the wheel assembly,
 - The number of revolutions made by the wheel assembly,
 - The moment of inertia of the rotating assembly,
 - The kinetic energy stored in the assembly at the end of 22s.
- [4 rad/s², 154 rev, 1.06kg m², 4.1 kJ]

Engineering Science.

Work, Energy & Power More Examples.

1. A train and Locomotive have a total mass of 400 tonne. Calculate the power needed when the train is travelling at a constant speed of 72km/h against a constant resistance of 100N/tonne. [800kW]

2. A motor car of mass 1000kg is being driven up an incline of 1 in 10 measured along the slope. If the engine develops an output power of 35kW, 40 % of which is lost through the transmission to the wheels, and there is a constant frictional resistance of 420N. Calculate :
 - a. The constant speed which the car can maintain up the incline.
 - b. The speed with which the car would travel along a level road if friction effects and the power available at the road wheels remains unchanged. [54km/h, 180 km/h]

3. The fuel conveyor system at a power station consists of a number of trucks, each of which has a mass of 4240kg when fully laden. The fuel is conveyed in the trucks up an inclined railway with a gradient of 1 in 2.5 measured along the slope. There is a constant friction resistance to motion of 250N per truck, and the maximum haulage power available is 136kW. Calculate:
 - a. The maximum number of fully laden trucks that can be hauled up the incline at a constant speed of 2 m/s.
 - b. The power needed to haul this number of trucks along a level part of the track at a constant speed of 2m/s. [4, 2kW]

4. A road vehicle has a mass of 1600kg and is uniformly accelerated from rest to a speed of 60km/h in a distance of 48m on a level surface against a tractive resistance of 0.18 N/kg. Determine:
 - a. The propulsive force of the vehicle.

- b. The change in momentum. [4917.63N, 26720kgm/s]
5. Crates of machined components are allowed to slide down a delivery chute 10m long inclined at 1 in 5 measured along the slope. Each full crate has a mass of 25kg and the velocity at the bottom of the chute is 6 m/s. Calculate the energy absorbed in overcoming friction. [40.5J]
6. A flywheel of the titanic of mass 8 tonnes has a radius of gyration of 1.6m. Determine the kinetic energy in the flywheel when it is rotating at 90 rev/min. If the speed of the flywheel is reduced to 80 rev/min, what will be the change in kinetic energy. [904.8kJ, 182.3kJ]
7. The rotating parts of an engine may be considered to have a mass of 3.5kg acting at a radius of gyration of 0.1m. If the mass of the engine itself is 80kg, determine the total kinetic energy of the engine when it is propelling the vehicle at 40km/h while rotating at 2100rev/min. [5.78kJ]

Engineering Science.

Friction Examples.

1. It is found that a vehicle having a mass of 900kg can be moved slowly and steadily along a horizontal surface by a force of 5.8kN with the wheels locked. Calculate the coefficient of friction. [0.66]

2. A metal block having a high friction lining and having a mass of 4.5kg requires a horizontal pull of 15.74N to move it at a steady speed along the horizontal steel surface. What is the coefficient of friction for the high friction lining and Steel. [0.36]

3. A train wagon has a coefficient of friction of 0.35 and requires a force of 1500N to overcome friction and keep it moving at a steady speed.
 - a) What is the mass of the wagon? [436.87kg]
 - b) If the number of wagons being pulled by a train is 8, what will the force exerted by the train need to be in order to overcome friction? [11999.74N]

4. A 50 kg weight being pulled at a constant velocity up a plain angled at 30 degrees from the horizontal has a coefficient of friction of 0.25. Calculate the total force acting down the plain. [351.45N]

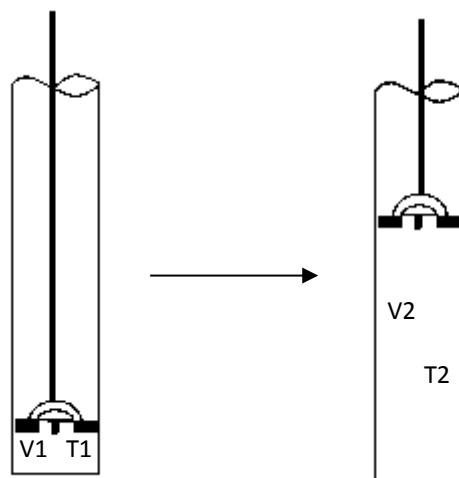
Gas Laws Lesson
Engineering Science

Charles Law.

'At constant pressure, the volume of a given mass of an ideal gas increases or decreases by the same factor as its temperature on the [absolute temperature scale](#).'

E.g. If, P, the pressure remains constant for every °C rise the temp the volume expands. Charles' Law describes the direct relationship of temperature and volume of a gas.

So imagine if you double the temperature of the air in a plastic bottle of a certain mass of gas, in simple terms, the particles become excited and move about more, effectively taking up more double volume.



Therefore: $V_1/T_1 = V_2/T_2$ where the pressure of the gas is constant, and

V_1 = Initial Volume (m^3)

V_2 = Final Volume (m^3)

T_1 = Initial Temperature (K)

T_2 = Final Temperature (K)

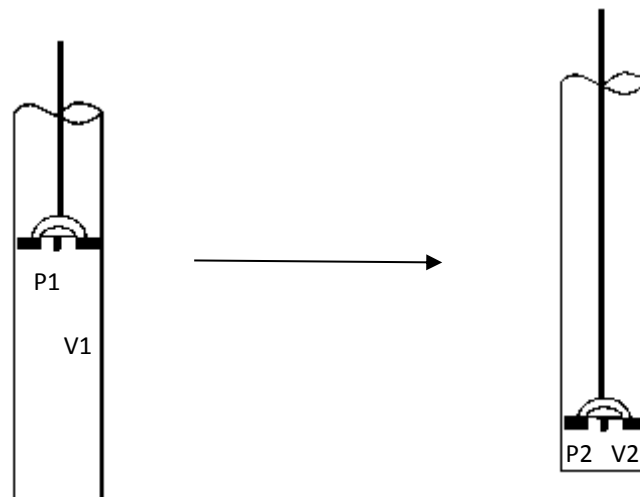
To convert °C to K add 273.

Boyle's Law.

'The Volume of a gas of fixed mass, is inversely proportional to the pressure, provided the temperature remains constant.'

E.g. If, T , the temperature remains constant and P , the pressure increases then V the volume filled by the gas decreases.

So imagine a volume of gas in a plastic bottle, if a pressure is applied to the bottle then the volume taken up by the gas gets less and the bottle will contract.



Therefore: $P_1V_1 = P_2V_2$ where the temperature of the gas is constant, and

V_1 = Initial Volume (m^3)

V_2 = Final Volume (m^3)

P_1 = Initial Pressure (Pa)

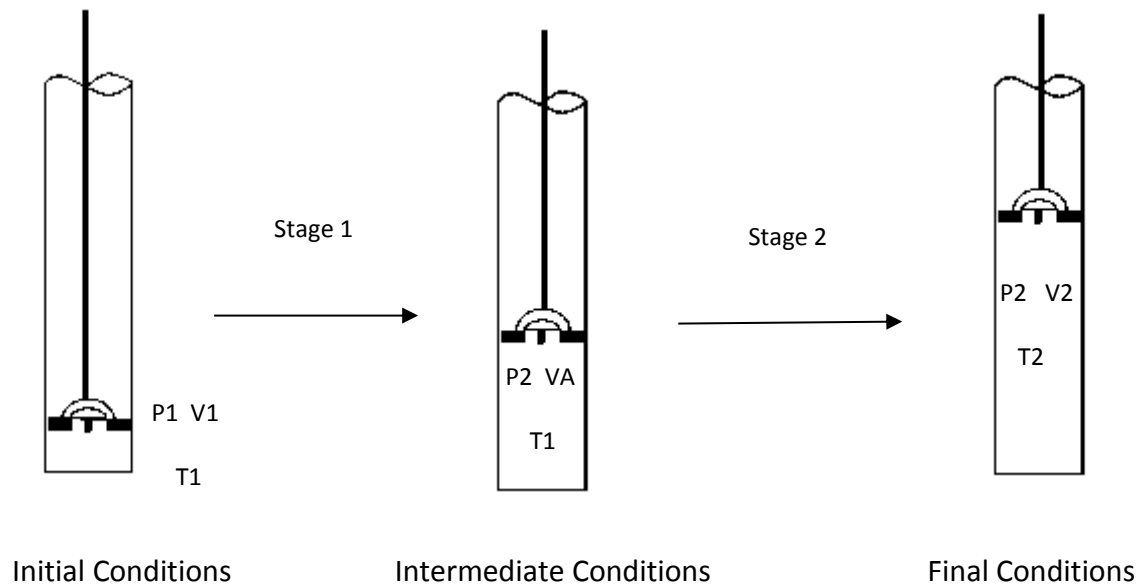
P_2 = Final Pressure (Pa)

But we can combine Boyle's and Charles's Laws to get the Gas equation.

Imagine changes according to both Laws in two stages...

Stage 1 where T is constant.

Stage 2 where P is constant.



So from Stage 1: $P_1V_1 = P_2V_A$

$$V_A = (P_1V_1)/P_2 \quad \text{Equation 1.}$$

And from Stage 2: $V_A/T_1 = V_2/T_2$

$$V_A = (V_2T_1)/T_2 \quad \text{Equation 2.}$$

Equate 1 and 2.

$$(P_1V_1)/P_2 = (V_2T_1)/T_2$$

OR
$$(P_1V_1)/T_1 = (P_2V_2)/T_2$$

Gas Law Examples

1. A given mass of gas occupies a volume of 200 litres at a temperature of 27°C. What would the volume of this mass of gas be at:

- a. 90°C,
- b. -30°C, under the same pressure conditions?

N.B: 1 Liter is 0.001m³

[0.242m³, 0.162m³]

2. A quantity of gas at a temperature of 15°C is heated at constant pressure until the volume is doubled. What will be the final temperature of the gas? [576K]

3. The compression ratio of an engine is 9 to 1, the pressure at the commencement of the compression stroke is 110kN/m² absolute and the temperature 115°C. Calculate the absolute pressure at the end of the compression stroke if the temperature has then risen to 180°C. [1155.9kN/m²]

4. At the commencement of the compression a cylinder contains 480cm³ of gas at a pressure of 96kN/m² absolute and temperature 100°C. At the end of the compression the volume is 80cm³ and the pressure 725kN/m² absolute. What will be the temperature at the end of compression? N.B 1cm³=1x10⁻⁶m³ [469.5K]

5. When the pressure in a car tyre was checked at a temperature of 12°C, the tyre gauge showed a reading of 1.75bar. What would be the tyre gauge reading when the temperature had increased to 38°C, assuming the volume of air in the tyre to be constant? (take atmospheric pressure to be 1.013 bar. 1bar=1x10⁵N/m²) [2.003bar]

6.

a) A compression ignition engine has a bore of 14cm and a stroke 12.2cm. The clearance volume is 128cm³. Calculate the compression ratio of the engine.

b) In the above engine, the air pressure at the commencement of the compression stroke is 100kN/m² and the temperature is 35°C. Calculate the temperature at the end of the compression stroke when the pressure is 4000kN/m². [15.7:1, 786.1K]

7. A compression ignition engine has a compression ratio of 14:1. The pressure of the air at the intake is 1.013bar, temperature 18°C and 2525cm³ per cylinder. If the pressure at the end of the compression is 38.5bar, calculate the temperature of the air at the end of compression. [789.98K]

8. A quantity of gas occupies a volume of 0.056m³ at and pressure of 14 bars and a temperature of 27°C. It is allowed to expand until the pressure has fallen to 8.4bars and the temperature to 15°C. What will the new volume be? If the temperature then increases to 38°C without a change in volume, what will be the new pressure? [0.0896 m³, 9.07bar]

9. A quantity of air in a cylinder is compressed to one fifth of its volume. The pressure is initially 105kN/m² and the temperature rises during compression from 15°C to 120°C. What is the pressure of the compressed air? If this air is allowed to expand at constant temperature to its original volume, what will be the new pressure? [716.4kN/m², 143.28kN/m²]

10. In the compression stroke a diesel engine, 3 litres of air at pressure 93kN/m² and a temperature of 45°C are compressed to a pressure of 2900kN/m² and a temperature of 690°C. The compression follows the combine's laws of Boyle and Charles. Calculate the volume of air, in millilitres at the end of the compression. [291.3ml]

The Concept of an Ideal Gas and the Ideal Gas Equation

No gas obeys the gas laws exactly, however they do provide a fairly accurate description of the way gases behave when they are at low pressures and are at temperatures which are well above those at which they liquefy. A useful concept is that an ideal or perfect gas which obeys the laws exactly.

The behaviour of such a gas can be accounted for by:

The ideal gas Law as $pV = nR_uT$,

Where p = pressure (N/m^2 or Pa)

V = volume (m^3)

n = number of moles

R_u = the universal gas constant (J/molK)

T = temperature (K)

Number of moles, n

To find n , the number of moles divide the mass in grams by the mass of one mole in grams.

Use the ideal gas equation by first making the substitution to give:

$$PV = \frac{\text{mass}(g)}{(\text{mass_of_1mole}(g))} RT$$

A value for R_u will be given you if you need it, or you can look it up in a data source. The SI value for R_u is 8.31441 J/mol K .

The Individual Gas Constant, R

The Individual Gas Constant depends on the particular gas and is related to the molecular weight of the gas. The value is independent of temperature. In the SI system the units are J/kg K .

The Universal Gas Constant - R_u

The Universal Gas Constant, R_u , as it appears in the ideal gas law and can be expressed as the product between the Individual Gas Constant, R , for the particular gas - and the *Molecular mass*- M_{gas} - for the gas, and is the same for all ideal or perfect gases:

$$R_u = M_{gas} R$$

where

R_u = universal gas constant

M_{gas} = molecular mass of the ideal gas or mixture of gases

This equation is known as the equation of state of an ideal gas or the ideal gas equation, and it embodies the 3 gas laws.

In summary:

- (i) An ideal gas obeys the gas laws and $pV = nR_uT$ exactly. No such gas exists.
- (ii) The internal energy of the gas is entirely kinetic and depends on only its temperature.
- (iii) The behaviour of a real gas can be described by $pV = nR_uT$ if they are at low pressures and are at temperatures which are well above those at which they liquefy.

Example

1. Nitrous Oxide (N_2O) gas of mass 20g is compressed into a cylinder of volume $0.02m^3$ at the temperature of $28^\circ C$. Calculate the pressure of the gas. (Molecular mass 44.013g/mol, Individual gas constant 0.28kJ/molK.) [83.46MPa]
2. Propane Gas (C_3H_8) is compressed into a cylinder of diameter 0.3m and height 1.6m at $22^\circ C$. Given that the Propane had a mass of 1.5kg and has an individual gas constant of 0.24kJ/molK calculate the pressure of the gas. (Molecular mass 44.10g/mol) [939.7MPa]

Fluid Flow

Bernoulli's theorem is a statement of the principle of the conservation of energy applied to a flowing liquid, and states that:

'For a flowing liquid provided there is no loss of energy by friction or leakage, the total energy at any part of the liquid is constant.'

The principle is a simplification of Bernoulli's equation, which states that the sum of all forms of energy in a fluid flowing along an enclosed path. It is named after a Dutch/Swiss mathematician and scientist Daniel Bernoulli, though it was previously understood by Leonhard Euler and others. In fluid flow with no viscosity, and therefore, one in which the pressure difference is only the accelerating force; it is the equivalent to Newton's laws of motion. It is important to note that the only cause of the change in fluid velocity is the difference in pressure on either side of it. It may be misunderstood to be that a change in velocity simply causes a change in pressure; the Bernoulli principle does not make any such statement.

In other words:

Potential Energy + Pressure Energy + Kinetic Energy = Constant

For a unit mass:

$$gh + (p/\rho) + (v^2/2) = \text{Constant}$$

For a unit weight (divide by g):

$$h + (p/g\rho) + (v^2/2g) = \text{Constant}$$

Thus between any two points (1) and (2) in a connected liquid, Bernoulli's equation may be written as follows:

$$gh_1 + (p_1/\rho) + (v_1^2/2) = gh_2 + (p_2/\rho) + (v_2^2/2)$$

Where:

g = gravity (ms^{-2}),

h = height (m),

ρ = density of the fluid (kg/m^3),

v = flow velocity (ms^{-1}),

p = pressure (N/m^2).

Quantity of Flow of Liquid

The quantity of liquid flowing is measured in cubic meters per second (m^3/s) or 'cumecs'. It is also measured in litres per minute and is denoted by the letter 'Q'

Consider a uniform pipe running full of liquid. Let the cross sectional area of a pipe be 'A' in square meters and the velocity of the liquid be 'v' meters per second. Then:

Quantity of liquid flowing

= cross-sectional area of the pipe x velocity of the liquid.

i.e. $Q = Av \text{ m}^3/\text{s}$

Also if ρ the density of the liquid in kg/m^3 , then:

Mass of liquid flowing per second = ρQ

$$= \rho Av$$

Continuity of Flow

Consider a liquid flowing through a tapered pipe (Figure 1). Let the cross sectional area of the pipe at A and B respectively be A_1 and A_2 and let the velocity of the liquid at these two sections be v_1 and v_2 . Then:

Quantity of liquid flowing through A = $A_1 v_1$

Quantity of liquid flowing through B = $A_2 v_2$

For the continuity of flow, the quantity of liquid flowing through the section A must be equal to the quantity of liquid flowing through section B. Thus:

$$A_1 v_1 = A_2 v_2$$

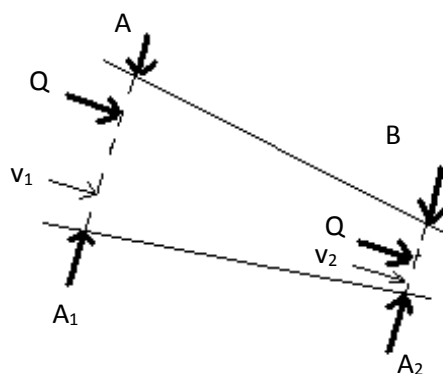


Figure 7

This is known as the equation of continuity. From this equation we can obtain the ratio of the velocities of the liquid at any two sections in the pipe.

If d_1 and d_2 are the pipe diameters at A and B respectively, then:

$$(\pi/4) \times d_1^2 \times v_1 = (\pi/4) \times d_2^2 \times v_2$$

$$d_1^2 \times v_1 = d_2^2 \times v_2$$

Or:

$$v_1 / v_2 = d_2^2 / d_1^2$$

This last equation shows that the velocity of the flow is inversely proportional to the square of the diameter of the pipe, so that if the diameter of the pipe is reduced to half then the velocity of flow will be increased by four times the original value.

Examples

1. The diameters for a tapered pipe at two sections A and B are 0.5m and 0.4m respectively. The flow is $0.55\text{m}^3/\text{s}$ and the pipe is running full. Find velocities at A and B and the pressure difference between the two points given that A is 1 m above B and that the density of the fluid is $1000\text{kg}/\text{m}^3$.
[2.81m/s, 4.38m/s, 4165.85N/m²]
2. At a section of horizontal pipe running full of oil, the diameter of the pipe is 0.6m and the velocity of the flow is 1m/s. Calculate the velocity of the oil at another section of the pipe, B, where the diameter is 0.45m. What is the flow rate in litres per minute? ($1\text{m}^3 = 1000\text{liters}$) [16980l / m]
3. A pipe has three cross sections A, B, and C of diameters 0.5m, 0.45m and 0.25m respectively calculate the velocities at each of the cross sections given the flow rate is $2\text{m}^3/\text{s}$. Find the velocities at each cross section.
[10.2m/s, 12.5m/s, 40.8m/s]

Applications of Bernoulli's Equation

Jets and Nozzles

Bernoulli's equations suggests that for a fluid flow where the potential energy change is $h\rho g$ is very small or zero, as in a horizontal pipe, **the pressure falls when the velocity rises**. The velocity increases at a restriction a slow stream of water from a tap can be converted into a fast jet by narrowing the exit. And the greater the change in cross sectional area the greater the increase in velocity and so the greater the pressure drop. Examples are a Bunsen burner (figure 8) and a paint spray gun (figure 9).



Figure 8: Bunsen Burner



Figure 9: Spray Gun

A Spinning Ball

If a tennis ball is 'cut' or a golf ball is 'sliced' it spins as it travels through the air and experiences a sideways force which causes it to curve in flight. This is due to air being dragged around the spinning ball, thereby increasing the air flow on one side and decreasing it on the other, thus creating a pressure difference. See figure 10.

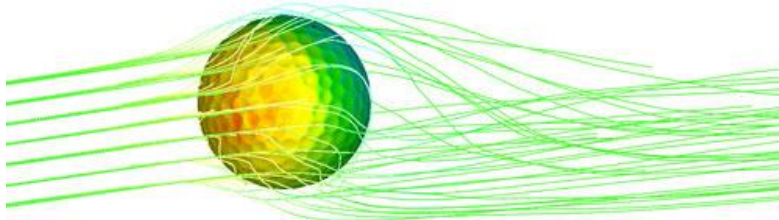


Figure 10: Spinning golf ball

An Aerofoil

This is a device which is shaped so that the relative motion between it and a fluid produces a force perpendicular to the flow. Examples are aircraft wings, turbine blades and propellers. Figure 11 shows the shape of an aerofoil section such that the fluid flows faster over the top surface than over the bottom. By Bernoulli's it should follow that the pressure underneath is increased and that above decreased. A resultant upward force is created, normal to the flow and it is this force which provides the 'lift'.

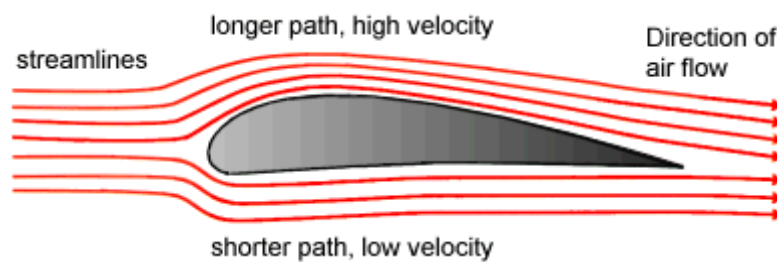


Figure 11: Aerofoil.

More Examples

1. A horizontal pipe of diameter 36cm has a gas travelling through it with velocity of 30m/s. The pipe then tapers to a diameter of 18cm, and has a pressure of $1.8 \times 10^5 \text{ N/m}^2$. Given the density of the gas is 1.16 kg/m^3 , Find:

- the velocity of the gas in the taper. [120m/s]
- the initial pressure of the gas. [187830N/m²]

2. Water, density 1000 kg/m^3 , flows down a pipe which tapers from 150mm diameter to 100mm diameter at the lower end. The difference in height between the two ends is 3m and the rate of flow is $0.8 \text{ m}^3/\text{min}$. Calculate the pressure difference between the upper and lower ends of the pipe.

[28423 N/m²]

3. A horizontal pipe carries water of density 1000 kg/m^3 . The pipe tapers from 80mm diameter to 50mm diameter, the pressure at the wider end being 100kPa and the other end being 70kPa. Calculate the velocity of the water at each end of the pipe and the volume passing through the pipe per second.

[0.016m³/s]

4. Oil of density 850kg/m^3 flows along a tapered pipe at the rate of $6\text{m}^3/\text{s}$. The cross sectional area of the two pipes at A and B respectively are 0.4m^2 and 1m^2 , and the level of B is 6m below that of A. Neglecting frictional losses find the difference in pressure in kN/m^2 . [130.4 kN/m^2]

5. At a section A a pipe running full of water, the diameter of the pipe is 0.9m and the velocity of water is 1.2m/s . At another section B, which is 10m below the level of A, the diameter of the pipe is 0.6m . Determine the difference in pressure between the water at A and B. Express this in kN/m^2 . Neglect frictional losses and take the density of water to be 1000kg/m^3 . [95.175 kN/m^2]

The Venturi Meter

The *Venturi meter* is an instrument which applies Bernoulli's theorem for measuring the rate of flow of a liquid in a pipe-line (figure 1). It consists of a tube which converges from the entrance to a cylindrical portion called the throat, and then diverges gradually to the normal diameter of the pipe. The meter is usually fitted in a horizontal position in the pipe line, and piezometer tubes are inserted in the entrance at A and at the throat B. A loss of pressure head H will occur at the throat but the velocity of the liquid in this narrow section will be greater than at the entrance, so that the same quantity of liquid will flow through a small area.

A piezometer is a small tube or manometer used to measure the pressure of a fluid at a specific location in a column.

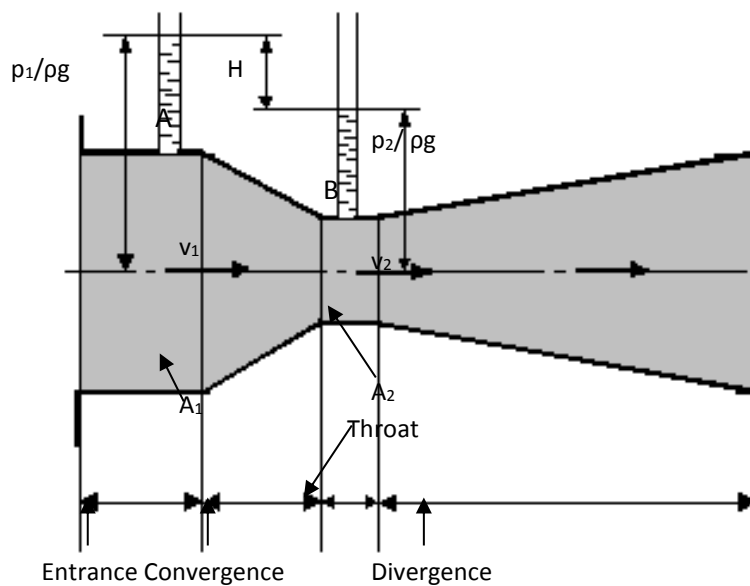


Figure 12

By applying Bernoulli's equation to sections A and B, it can be shown that the rate of flow of liquid in the pipe is proportional to the square root of H i.e. $Q = C \sqrt{H}$ where C is a constant for any given Venturi meter.

So neglecting all losses:

Since the venturi is horizontal, there is no potential energy:

$$p_1/\rho g + v_1^2/2g = p_2/\rho g + v_2^2/2g$$

Or

$$(p_1 - p_2)/\rho g = (v_2^2 - v_1^2)/2g$$

But $(p_1 - p_2)/\rho g = H$ the difference in pressure head between A and B.

So:

$$(v_2^2 - v_1^2)/2g = H$$

Or:

$$(v_2^2 - v_1^2) = 2gH \dots\dots (1)$$

From Continuity of flow:

$$A_1 v_1 = A_2 v_2$$

Or

$$v_2 = (A_1/A_2)v_1 \dots\dots (2)$$

Substituting (2) into (1):

$$[(A_1/A_2)v_1]^2 - v_1^2 = 2gH$$

$$v_1^2 [(A_1/A_2)^2 - 1] = 2gH$$

$$v_1^2 = 2gH/[(A_1/A_2)^2 - 1]$$

$$v_1 = \sqrt{2gH/[(A_1/A_2)^2 - 1]}$$

Rate of flow:

$$Q = A_1 v_1$$

Substituting:

$$Q = A_1 \sqrt{2gH/[(A_1/A_2)^2 - 1]}$$

Examples

1. A venturi meter is 160mm diameter at the entrance and 80mm diameter at the throat. If the water pressures at the entrance and at the throat are 7 bar and 6.3 bar respectively, given the density of water is 1000kg/m^3 , determine the discharge of water assuming no frictional losses. (1bar = $1 \times 10^5\text{N/m}^2$) [0.06m³/s]
2. Water of density 1000kg/m^3 passes through a horizontal venturi meter which has an inlet diameter of 140mm and a throat diameter of 30mm. If the pressure difference between the inlet and the throat is 2kPa, what is the velocity of the water entering the meter? Neglect any losses. [0.09m/s]
3. A venturi meter is used to measure the flow of oil, of density 820kg/m^3 , in a horizontal pipe of 120mm diameter. If the throat of the meter is 40mm diameter, what will the flow rate of oil be if the pressure difference between the manometers is 800Pa. Assume there are no losses. [0.0018m³/s]