

06/01/2021
Dovydas Paulavicius
1904197

Analytical Methods (ACHA4000)

Question 1

a) i)
$$\begin{pmatrix} 8 & -1 & 7 \\ -5 & 2 & 5 \\ 3 & 0 & 2 \end{pmatrix} \times \begin{pmatrix} -2 & 8 & 0.5 \\ 2 & 5 & 2 \\ 0 & -9 & 1 \end{pmatrix} = \begin{pmatrix} -18 & -4 & 9 \\ 14 & -75 & 6.5 \\ -6 & 6 & 3.5 \end{pmatrix}$$

ii)
$$\begin{pmatrix} -2 & 8 & 0.5 \\ 2 & 5 & 2 \\ 0 & -9 & 1 \end{pmatrix} \times \begin{pmatrix} 8 & -1 & 7 \\ -5 & 2 & 5 \\ 3 & 0 & 2 \end{pmatrix} = \begin{pmatrix} -54.5 & 18 & 27 \\ -3 & 8 & 43 \\ 48 & -18 & -43 \end{pmatrix}$$

iii)
$$\begin{pmatrix} 8 & -1 & 7 \\ -5 & 2 & 5 \\ 3 & 0 & 2 \end{pmatrix} \times \begin{pmatrix} 2 & 5 \\ -6 & 0 \\ 1 & 12 \end{pmatrix} = \begin{pmatrix} 29 & 124 \\ -17 & 35 \\ 8 & 39 \end{pmatrix}$$

iv)
$$\begin{pmatrix} 2 & 5 \\ -6 & 0 \\ 1 & 12 \end{pmatrix} \times \begin{pmatrix} 8 & -1 & 7 \\ -5 & 2 & 5 \\ 3 & 0 & 2 \end{pmatrix} \neq \text{IMPOSSIBLE!}$$

v)
$$A = \begin{pmatrix} 8 & -1 & 7 \\ -5 & 2 & 5 \\ 3 & 0 & 2 \end{pmatrix} \quad |A| = 8(2 \times 2 - 0 \times 5) - 1(-5 \times 2 - 5 \times 3) + 7(-5 \times 0 - 2 \times 3)$$
$$= 32 - 25 - 42$$
$$= -35$$

vi)
$$B = \begin{pmatrix} -2 & 8 & 0.5 \\ 2 & 5 & 2 \\ 0 & -9 & 1 \end{pmatrix} \quad |B| = -2(5 \times 1 - -9 \times 2) - 8(2 \times 1 - 2 \times 0) + 0.5(2 \times -9 - 5 \times 0)$$
$$= -46 - 16 - 9$$
$$= -71$$

06/01/2021

Dovydas Paulavicius
1904197

Analytical Methods (ACHA4000)

vii)
$$\begin{pmatrix} 8 & -1 & 7 \\ -5 & 2 & 5 \\ 3 & 0 & 2 \end{pmatrix} + \begin{pmatrix} -2 & 8 & 0.5 \\ 2 & 5 & 2 \\ 0 & -9 & 1 \end{pmatrix} = \begin{pmatrix} 6 & 7 & 7.5 \\ -3 & 7 & 7 \\ 3 & -9 & 3 \end{pmatrix}$$

viii)
$$\begin{pmatrix} 8 & -1 & 7 \\ -5 & 2 & 5 \\ 3 & 0 & 2 \end{pmatrix} \times \begin{pmatrix} -2 & 8 & 0.5 \\ 2 & 5 & 2 \\ 0 & -9 & 1 \end{pmatrix} = \begin{pmatrix} -18 & -4 & 9 \\ 14 & -75 & 6.5 \\ -6 & 6 & 3.5 \end{pmatrix}$$

$$\begin{pmatrix} -18 & -4 & 9 \\ 14 & -75 & 6.5 \\ -6 & 6 & 3.5 \end{pmatrix} + \begin{pmatrix} 8 & 14 \\ 0.25 & 21 \end{pmatrix} \neq \text{IMPOSSIBLE!}$$

06/01/2021

Dovydas Paulavicius

1904197

Analytical Methods (ACHA 4000)

$$d) A = \begin{pmatrix} 2 & 3 & 2 \\ 1 & 2 & 5 \\ 3 & 1 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 14 \\ 4 \\ 19 \end{pmatrix} \quad |A| = 2(-9) - 3(-17) + 2(-5) = 23$$

$$A_1 = \begin{pmatrix} 14 & 3 & 2 \\ 4 & 2 & 5 \\ 19 & 1 & -2 \end{pmatrix} \quad |A_1| = 14(-9) - 3(-103) + 2(-34) = 115$$

$$A_2 = \begin{pmatrix} 2 & 14 & 2 \\ 1 & 4 & 5 \\ 3 & 19 & -2 \end{pmatrix} \quad |A_2| = 2(-103) - 14(-17) + 2 \times 7 = 46$$

$$A_3 = \begin{pmatrix} 2 & 3 & 14 \\ 1 & 2 & 4 \\ 3 & 1 & 19 \end{pmatrix} \quad |A_3| = 2 \times 34 - 3 \times 7 + 14(-5) = -23$$

$$x = \frac{|A_1|}{|A|} = 5$$

$$y = \frac{|A_2|}{|A|} = 2$$

$$z = \frac{|A_3|}{|A|} = -1$$

106/01/2021
Dovydas Paulavicius
11904197

Analytical Methods (ACHA4000)

Question 3

$$\begin{aligned} \text{a) i)} \quad & \int (x^5 - 2x)(5x^3 + 6x) dx \\ &= \int 5x^8 + 6x^6 - 10x^4 - 12x^2 dx \\ &= \int 5x^8 dx + \int 6x^6 dx - \int 10x^4 dx - \int 12x^2 dx \\ &= \frac{5x^9}{9} + \frac{6x^7}{7} - 2x^5 - 4x^3 + C \end{aligned}$$

$$\begin{aligned} \text{ii)} \quad & (e^{5x} + 1)(e^{3x} + 2) dx \\ &= dx(e^{8x} + 2e^{5x} + e^{3x} + 2) \\ &= e^{8x} dx + 2e^{5x} dx + e^{3x} dx + 2dx \end{aligned}$$

$$\begin{aligned} \text{iii)} \quad & \int_0^2 (3x^3 + 2x)^3 + 4x + 3 dx \\ &= \int_0^2 \left(\frac{9x^6 - 4x^2}{3x^3 - 2x} \right)^3 + 4x + 3 dx \\ &= \int_0^{\sqrt{\frac{2}{3}}} \left(\frac{9x^6 - 4x^2}{3x^3 - 2x} \right)^3 + 4x + 3 dx = \sqrt{6} + \frac{256}{45} \\ &= \sqrt{6} + \frac{256}{45} - \sqrt{6} + \frac{44254}{9} \\ &= \frac{24614}{5} = 4922.8 \end{aligned}$$

06/01/2021
Dovydas Paulavicius
1904197

Analytical Methods (ACHA4000)

iv) $\int_2^4 (2x+3)(x+5) dx$

$$= \int_2^4 2x^2 + 13x + 15 dx$$

$$= \int_2^4 2x^2 dx + \int_2^4 13x dx + \int_2^4 15 dx$$

$$= \frac{112}{3} + 78 + 30$$

$$= \frac{436}{3} = 145.33$$

b) i) $\int 4x \cos 3x dx$

$$= 4x \int x \cos(3x) dx$$

$$= 4 \times \int \frac{u \cos(u)}{9} du$$

$$= 4 \times \frac{1}{9} \times \int u \cos(u) du$$

$$= 4 \times \frac{1}{9} (u \sin(u) - (-\cos(u)))$$

$$= \frac{4}{9} (3x \sin(3x) + \cos(3x))$$

$$= \frac{4}{9} (3x \sin(3x) + \cos(3x)) + C$$

06/01/2021
Dovydas Paulavicius
1904197

Analytical Methods (ACHA 4000)

$$\begin{aligned} \text{b) ii) } & \int_1^2 3x e^{2x} dx \\ &= 3 \times \int_1^2 x e^{2x} dx \\ &= 3 \times \int_2^4 \frac{eu}{4} du \\ &= 3 \times \frac{1}{4} \times \int_2^4 e^u u du \\ &= 3 \times \frac{1}{4} [e^u u - \int e^u du]_2^4 \\ &= 3 \times \frac{1}{4} [e^u u - e^u]_2^4 \\ &= \frac{3}{4} [e^u u - e^u]_2^4 \\ &= \frac{3}{4} (3e^4 - e^2) \end{aligned}$$

$$\begin{aligned} \text{c) i) } & \int \frac{21x^2 + 10x - 3}{7x^3 + 5x^2 - 3x - 1} dx \\ &= \int \frac{1}{u} du \\ &= \ln |7x^3 + 5x^2 - 3x - 1| + C \end{aligned}$$

06/01/2021

Dovydas Paulavicius

1904197

Analytical Methods (ACHA4000)

ii) $\int \frac{8}{3x+10} dx$

$$= 8 \times \int \frac{1}{3x+10} dx$$

$$= 8 \times \int \frac{1}{3u} du$$

$$= 8 \times \frac{1}{3} \times \int \frac{1}{u} du$$

$$= 8 \times \frac{1}{3} \ln|u|$$

$$= \frac{8}{3} \ln|3x+10| + C$$

iii) $\int \frac{1}{x^2+9} dx$

$$= \int \frac{1}{3(u^2+1)} du$$

$$= \frac{1}{3} \times \int \frac{1}{u^2+1} du$$

$$= \frac{1}{3} \arctan(u)$$

$$= \frac{1}{3} \arctan\left(\frac{x}{3}\right) + C$$
