Module: ACDA 6009 Structural Mechanics

Assignment: Deflection of beams

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Abstract

A customer has provided a design of a test rig to measure the deflection of ferrous and non-ferrous cantilever beams. An experiment has been conducted to determine the suitability of the test rig by comparing theoretical calculations of deflection using Macaulay's method against measured results. It was determined the test rig was suitable for the testing of deflection of cantilever beams. The results being within 0.2mm. It is recommended that the cantilever material be mild steel, due to having the least deflection, and the least variance in deflection in the variation of load. Further research is investigating the design of the beam and comparing against an industry standard "I" beam.

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Introduction

Beams and Loads

Beams are used for a variety of applications, to bridges, buildings, and frames. They are supported through different methods and are exposed to different loads (Figure No.1). They can be designed for a variety of roles, and can be adapted to resist mechanical forces, such as bending, shear, and compressive. Overall, their purpose is to support loads. [1]

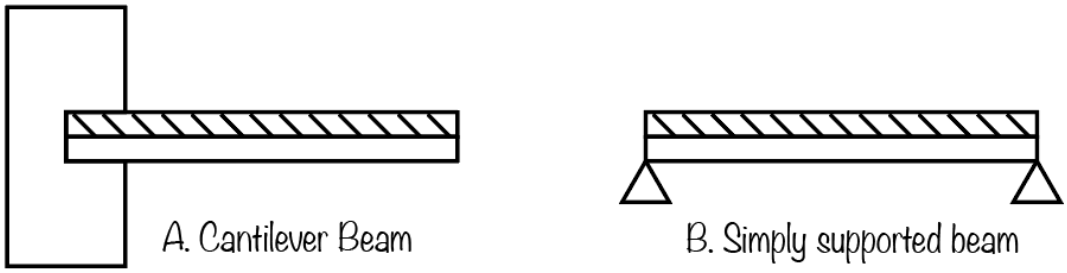

Figure No.1 – Cantilever Beam, and a simply supported beam (SSB)

Problem and Purpose

A client has requested the test of three materials, for the construction of a cantilever beam that will suspend a piece of test equipment from the end.

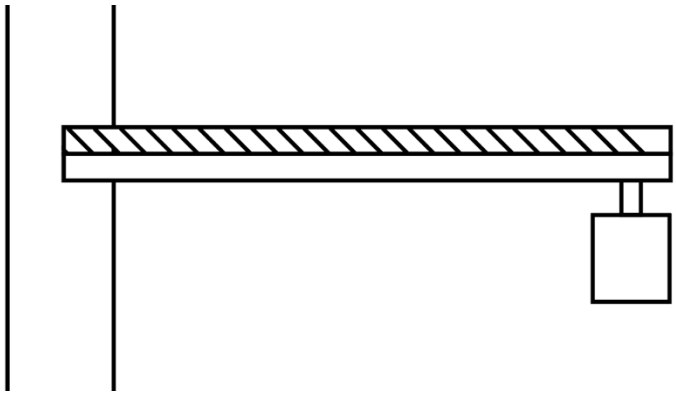

Figure No.2 – Trial equipment set-up. A cantilever beam with a mass on the end.

They have requested this report to identify a suitable material that will be able to suspend the test equipment, while having minimal deflection. This information will then determine the selection of a suitable material, with consideration to cost compared against performance.

Due to the nature of the test equipment, the load applied will vary. Since the cantilever beam will be used in an experiment it has been requested to suggest a beam with minimal variation to deflection. This will then reduce the impact of variation within their testing.

Objective:

The objective is to conduct a small-scale test, replicating the operating conditions of the cantilever beam against a variety of loads. Replicating a variety of test equipment being suspended by the beam.

Additionally, the aim of this report is to provide a suggestion for the material to be selected, based on the results of the small-scale test. Considering factors such as cost of materials, maximum deflection and variation of deflection when a variety of loads are suspended by the beam.

Background

Force

A force is an action applied to a body to change its state. It has three characteristics, magnitude, direction, and point of application. [2] Therefore, it is considered a vector quantity. The equation for a force is:

$$F = ma$$

Equation No.1 [3]

For the applications of beams, a force could be considered a person walking over a simply supported beam (Fixed at two points). The persons weight, along with gravity will apply a force to a beam where the person is standing. Thus, creating a point load.

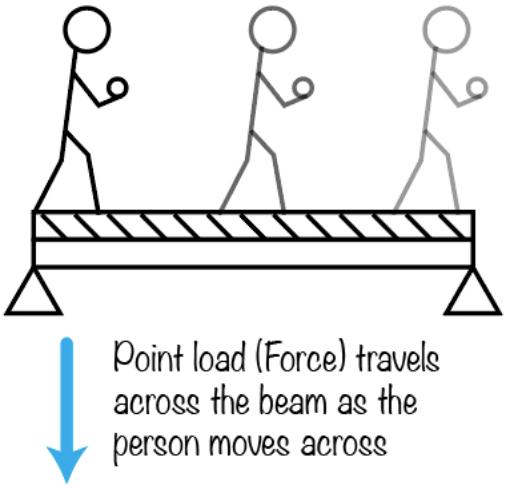

Figure No.3 – Force acting on an SSB

Depending on the deflection of the beam it can cause both compressive and tensile forces (Figure 4). If a beam is split equally in half, the side that undergoes the most bending will experience a tensile force as the material is stretched, while the other side will undergo a compressive force. [4]

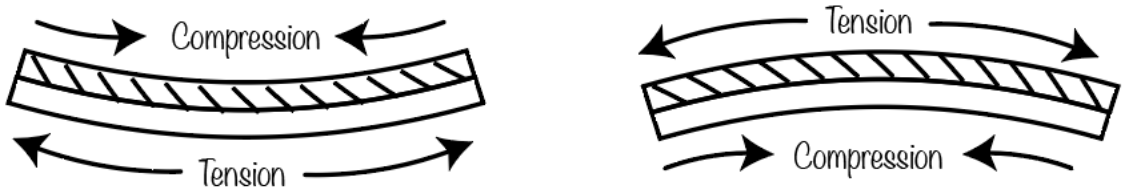

Figure No.4 – How deflection affects a beam.

Equal and Opposite Reaction

For a beam to be static and in equilibrium, it must be in the following conditions:

- The upward forces must equal the downward forces.
- The sum of clockwise moments must be equal to anti-clockwise moments. [4]

These rules closely resemble newtons first law:

"Every action (force) must have an equal and opposite reaction." [2]

If these conditions are not met, then the beam would be caused to rotate, or would slowly levitate or be pushed into the ground. Depending on the forces acted on it. Going back to the earlier example (figure 3), if the person was to walk onto the beam, and there was no opposite force, this would cause the beam to fall, and the person with it. Alternatively, if there is no equal anti-clockwise moment then the force will cause the beam to rotate.

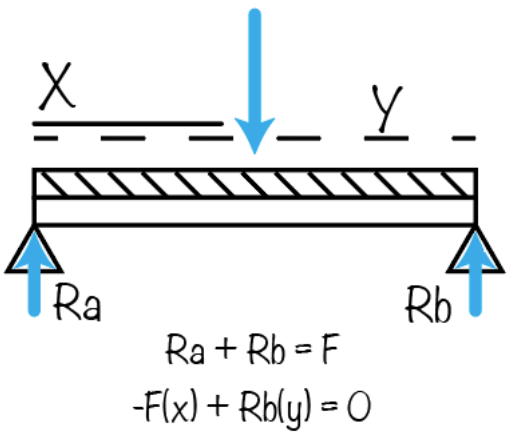

Figure No.5 – An SSB in static equilibrium

Shear and Bending Moments

When a force is applied to a beam, the beam will undergo both shear and bending moments. These forces can be imagined by taking a cross section of a simply supported beam under a point load.

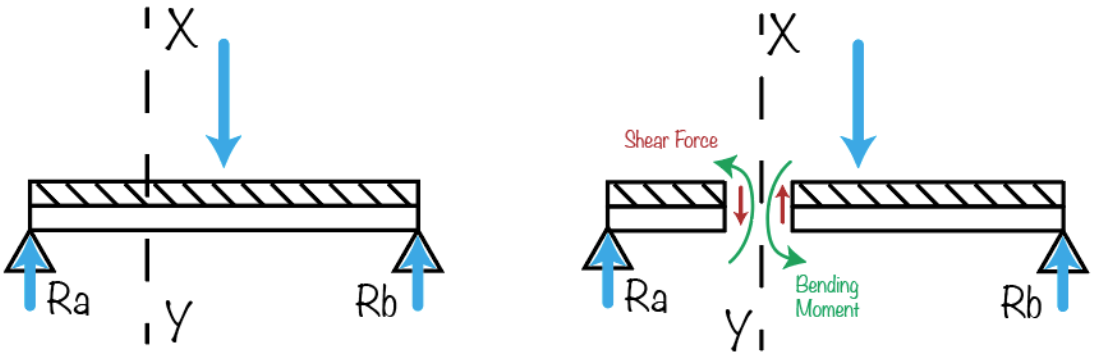

Figure No.6 – How a point load creates shear and bending moments.

The support point R_a is applying a force on the cross-section X-Y - pushing the cross section upwards. While the point load is pushing the other side downwards - but in the same clockwise motion.

Considering the whole beam, to stay in equilibrium, R_b will need to apply an upwards force and bend anti-clockwise. As a result, a shear moment has been created, as well as a bending moment at the cross-section X-Y to keep equilibrium. [2]

Engineers Theory of Bending

When a beam undergoes a bending moment, it can either hog or sag. However, there is a method to calculate the bending moment of any X-Y cross section across a beam, by using Macaulay's method, which uses the engineers theory of bending. [4]

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

Equation Number.2 [5] [6]

This equitation is derived by measuring the strain caused from bending of a beam. [7] Using figure, No.7 a beam cross section has been selected of distance x. Currently, the beam has a neutral axis (cd) which is placed halfway between ab and gh. When the beam is not being bent the distance between ab, cd, ef, and gh are all the same.

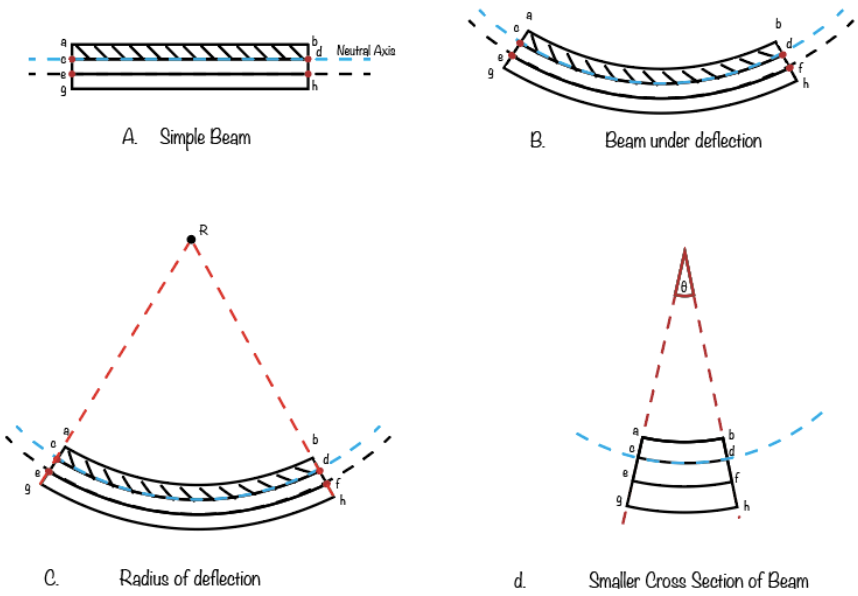

Figure No.7 – Simple Beam under deflection.

When the beam sags (deflection from a load), this bending would cause all the points (a,b,c,d,e,f,g,h) to become part of a circle (Figure No.7 – C). Where the points are distance R from the centre of the circle. During the bending motion, cd on the neutral axis does not change in length. Therefore, no strain is applied. However, ef does change length longitudinally. [2] Therefore, a strain can be calculated via equation 3.

$$\varepsilon = \frac{\Delta L}{L}$$

Equation No.3 [3]

However, to measure the change in length the radius and length of cross section (radians) must be used to calculate the length of arc. Therefore, using equation no.4.

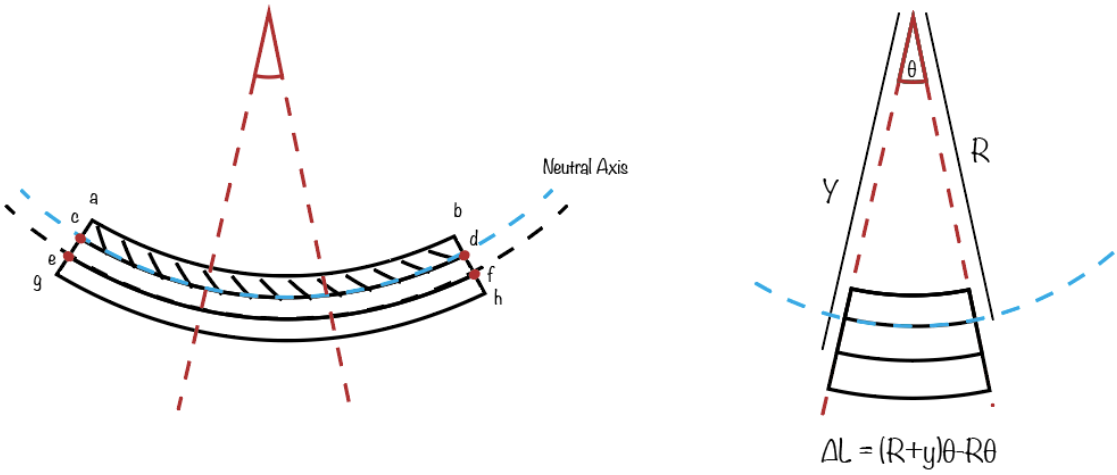

Figure No.8 – How change in length for a cross section is measured

Equation No.4 is used to calculate the new length of arc, in figure 8. This will then allow the change in length to be measured.

$$\varepsilon = \frac{(R + y)\theta - R\theta}{R\theta}$$

Equation No.4 [2]

Expanding the brackets.

$$\varepsilon = \frac{R\theta + Y\theta - R\theta}{R\theta}$$

Equation No.5

Simplifying the equation.

$$\varepsilon = \frac{Y}{R}$$

Equation No.6

Substituting equation No.6 into Equation No.7 will provide equation No.8.

$$\varepsilon = \frac{\sigma}{E}$$

Equation No.7

$$\frac{Y}{R} = \frac{\sigma}{E}$$

Equation No.8

Producing the first half of the engineering theory of bending. [7]

Returning back to the beam, if a section was considered (Figure No.9) with the force applied across the area of the beam, then equation No.9 can be applied. [5] Which is change in force is equal stress multiplied by change of area.

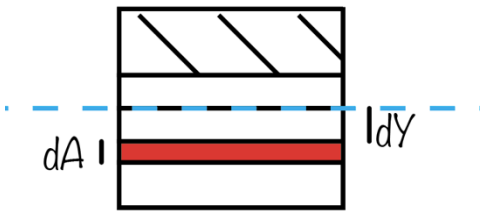

Figure No.9 – Showing how area changes with distance to neutral axis

$$\Delta F = \sigma \Delta A$$

Equation No.9

Where the change of force from the resistance applied from the bending moment changes from the stress over the change in area. To calculate the total longitudinal force equation 10 can be used.

$$F = \int_A \Delta F = \int_A \sigma \Delta A$$

Equation No.10 [5]

The last integral gives the second moment of area, resulting in equation.

$$M = \frac{EI}{R}$$

Equation No.11

Finally, rearranging equation No.11 and substituting into equation no.8 this will provide the engineering theory of bending equation no.12.

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

Equation Number.12

This expression combines the moment of bending with the geometry and deformation of the beam. Using this equation, this will be used to calculate the theoretical beam deflection during the test. [5]

Methodology

Equipment:

- Dial Indicator
- Varying loads (50g, 100g, 150g, and 250g)
- Three beams
    - Aluminium
    - Brass
    - Mild Steel
- Vernier Calipers
- 30cm Rule
- Work Panel and mout
- Pen and paper
- Camera

Independent Variables

Three identical beams in design apart from materials properties were used for the experiment.

Beam	**Material**	Youngs Modulus	Width (mm)	Depth (mm)	Length (mm)
1	**Aluminium**	69 GPa	9.5	3.1	240
2	**Brass**	105 GPa	9.5	3.1	240
3	**Mild Steel**	200 GPa	9.5	3.1	240

Table No.1 – Material and Physical Properties of beams [1]

Additionally, four masses (50g, 100g, 150g, and 200g) were applied at the end of each beam.

Control Variable

Each beam was installed into the test rig with 24cm overhang. The same dial clock was used throughout the experiment. Additionally, the same masses were used throughout.

Dependent Variable

Using the dial indicator, the displacement of the beam was measured (Figure 10, 12).

Experiment Method

For the experiment, the equipment was set up as per figure 10, and a beam was installed into the test rig; and clamped down to fix it in place. The beam length was determined to be 240mm from the edge of the clamp to the end of the beam. This was measured using a rule to check the set-up of each test.

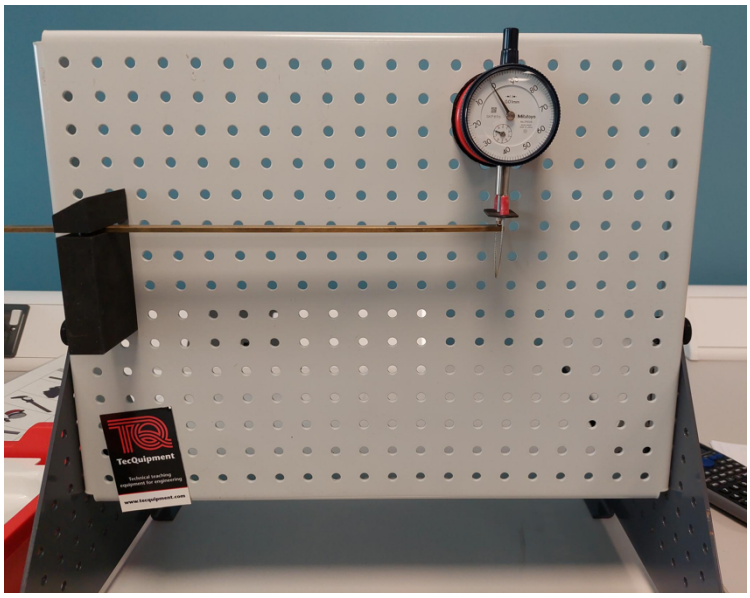

Figure No. 10 – Set up of equipment

A mechanical clock was used to measure the deflection of the beam. Before adding weight to the end of the beam, the clock was calibrated to zero. The clock was placed a few millimetres from the edge of the beam would rest on the surface of the beam. To allow the clock to measure the change in deflection, the clock was slightly pushed against the beam to allow the recording of deflection.

Figure No.11 – Masses used in 50g Stacks

Four test samples were prepared. Each of the samples were in increments of 50g, starting from 50g and finishing at 200g.

For the execution of the experiment, the load was added to the end of the beam. This would deflect the beam, and the mechanical clock that rested on the beam would measure the deflection in mm. This would be recorded. After each load, more mass would be added continually.

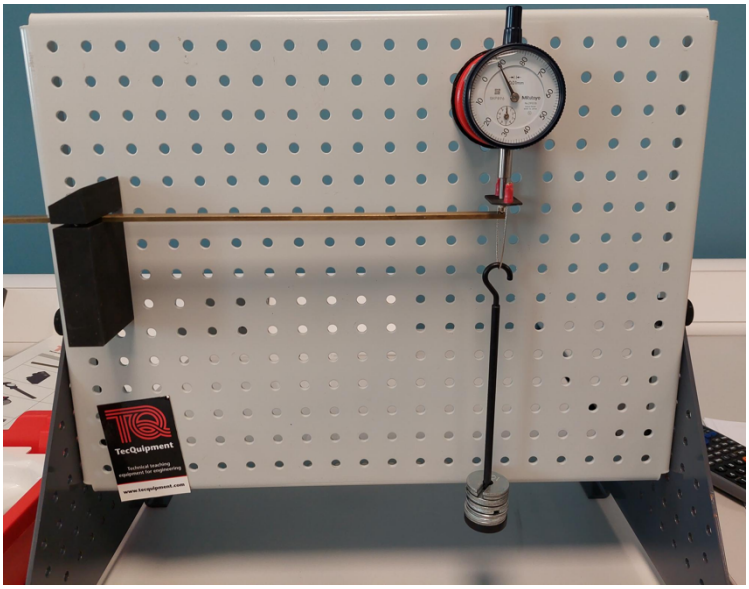

Figure 12 – Load added onto beam and measurement captured

Theoretical Method:

Below is the theoretical method used to calculate the deflection of the beam using Macaulay's method.

The bending moment can be calculated at any point using equation No.13

$$-M = EI\frac{d^2y}{dx^2}$$

Equation Number.13 [4]

The moments of the beam were added to the equation. The equation was then rearranged to make y the subject of the equation.

$$y = \frac{1}{EI}(-R_aX - F_1[L - X])$$

Equation No.14

The first integration of Equation No.14

$$y = \frac{1}{EI}(-\frac{R_a X^2}{2} - \frac{F_1[L-X]^2}{2} + C)$$

Equation Number.15

Second Integration of Equation No.15

$$y = \frac{1}{EI}(-\frac{R_a X^3}{6} - \frac{F_1[X-L]^3}{6} + Cx + D)$$

Equation Number.16

Applying first boundary conditions

- When X = 0, there is no deflection at the cantilever position. As it is fixed into the wall. Therefore, Y = 0, and therefore D = 0.
- Assuming the rate of change in deflection is zero at the maximum point, therefore when X = L, Y = 0.

Using this assumption, the Equation 17, can be solved for C.

$$y = \frac{1}{EI}(-\frac{R_a X^3}{6} - \frac{F_1[X-L]^3}{6} + Cx)$$

Equation Number.17

First moved EI to other side of the equation to remove.

$$yEI = (-\frac{R_a X^3}{6} - \frac{F_1[X-L]^3}{6} + Cx)$$

Equation Number.18

Added maximum length of beam.

$$0\ EI = (-\frac{R_a 0.24^3}{6} - \frac{F_1[0.24-0.24]^3}{6} + 0.24C)$$

Equation Number.19

Removed values that equalled zero.

$$0 = (-\frac{R_a 0.24^3}{6} - 0 + 0.24C)$$

Equation Number.20

Re-arranged to determine C.

$$-\frac{R_a 0.24^3}{6} = 0.24C$$

Equation Number.21

Substituted Equation No.21 into equation No.18, while removing values that would equal to zero.

$$Y = \frac{1}{EI}(-\frac{R_a 0.24^3}{6} + -\frac{R_a 0.24^3}{6})$$

Equation Number.22

Simplified equation 22.

$$Y = \frac{1}{EI}(-\frac{2R_a 0.24^3}{6})$$

Equation Number.23

Simplified equation 23, into common textbook equation 24, for the deflection of a cantilever beam.

$$Y = -\frac{FL^3}{3EI}$$

Equation No.24

Results:

	Mass	Aluminium	Brass	Mild Steel
	(Kg)	(mm)	(mm)	(mm)
1	0.05	1.31	0.81	0.45
2	0.1	2.72	1.8	0.88
3	0.15	4.38	2.71	1.46
4	0.2	5.5	3.67	1.87

Table No.2 – Measured Deflection

Mass	**Force**
(Kg)	(N)
0.05	0.49
0.1	0.98
0.15	1.47
0.2	1.96

Table No.3 – Mass and the force applied

	Aluminium	Brass	Mild Steel
	(Pa)	(Pa)	(Pa)
E	6.9E+10	1.05E+11	2.00E+11

Table No.4 – Mechanical properties of each beam

Length	Width	Depth	**I**
(m)	(m)	(m)	(m^4)
0.24	0.0095	0.0031	2.36E-11

Table No.5 – Beam dimensions and the second moment of inertia

$$Y = -\frac{FL^3}{3EI}$$

Equation No.24 – Deflection of beam under a load

	Mass	Aluminium	Brass	Mild Steel
	(Kg)	(mm)	(mm)	(mm)
1	0.05	1.39	0.91	0.48
2	0.1	2.78	1.83	0.96
3	0.15	4.17	2.74	1.44
4	0.2	5.56	3.65	1.92

Table No.6 – Calculated Deflection of Beams

	Mass	Aluminium		Brass		Mild Steel	
	(Kg)	(mm)	(mm)	(mm)	(mm)	(mm)	(mm)
1	0.05	1.31	1.39	0.81	0.91	0.45	0.48
2	0.1	2.72	2.78	1.80	1.83	0.88	0.96
3	0.15	4.38	4.17	2.71	2.74	1.46	1.44
4	0.2	5.50	5.56	3.67	3.65	1.87	1.92

Table No.7 – Comparison of measured and calculated deflection

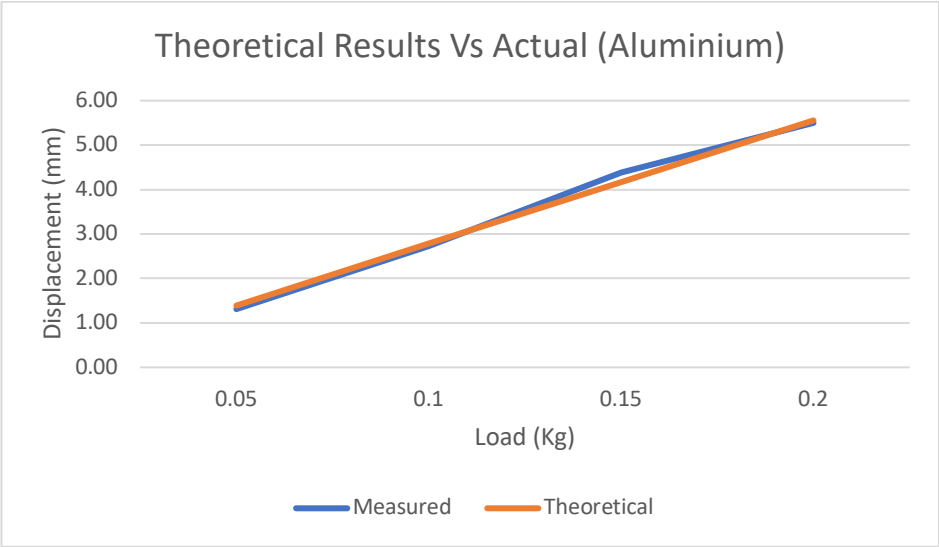

Figure No.13 – Theoretical results vs actual for aluminium

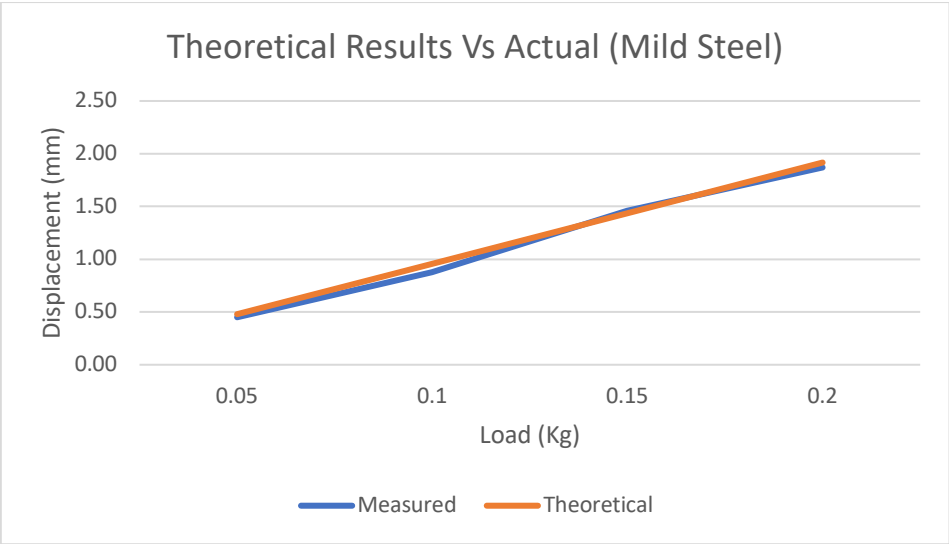

Figure No.14 – Theoretical results vs actual for Mild Steel

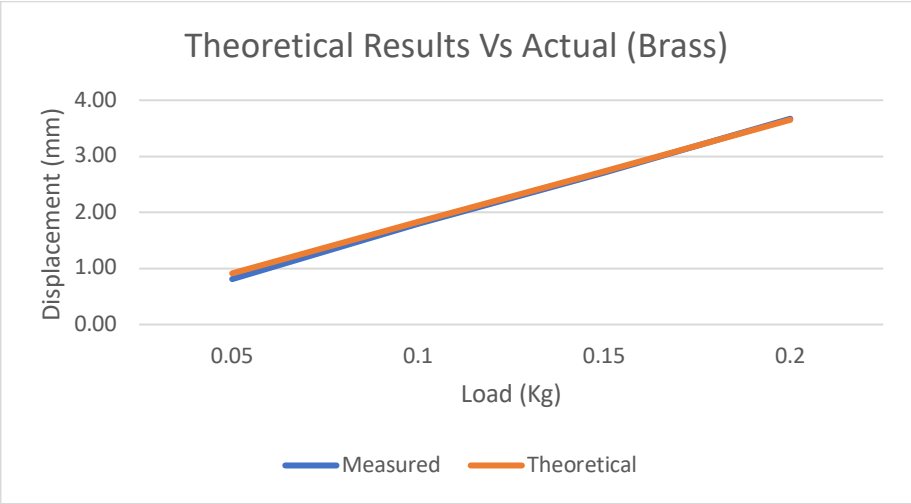

Figure No.15 – Theoretical results vs actual for Brass

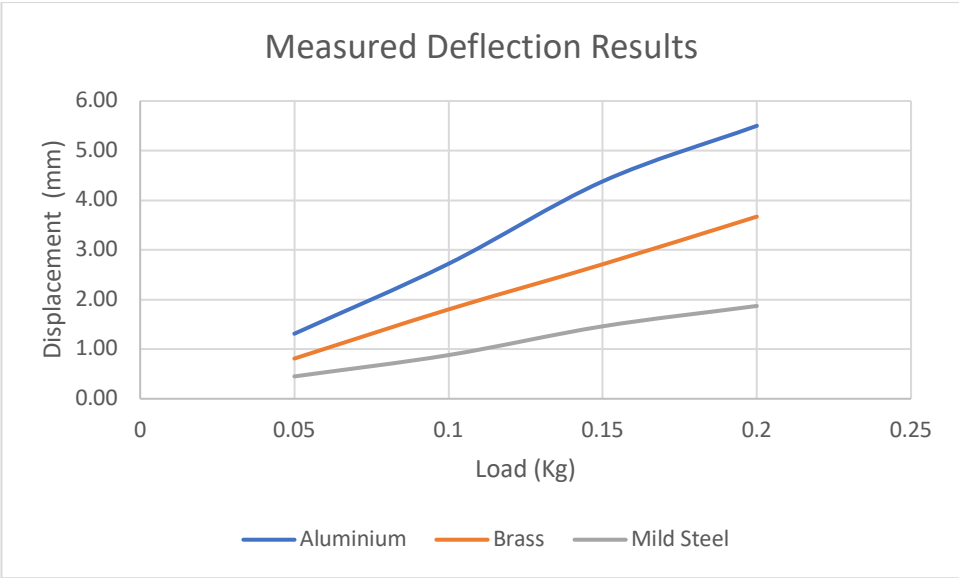

Figure No.16 – Measured deflection results for aluminium, Brass, Mild Steel

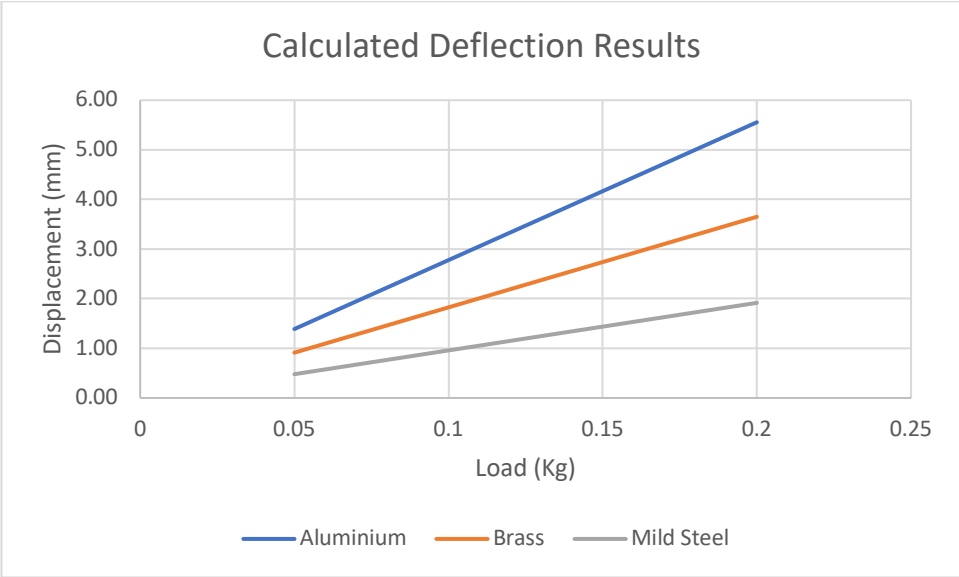

Figure No.17 – Calculated deflection results for aluminium, Brass, Mild Steel

Discussion

From the experiment it can be seen the measured results for aluminium, brass, and mild steel closely resemble the theoretical results. With the largest error being 0.2mm (Aluminium, 0.1Kg, Table No.5). This could be due to the equipment. When a load was removed for the experiment, the clock did not return to zero. This could indicate a human error from recording the data.

All materials behaved linearly. As the load increased so did the deflection. However, aluminium showed the greatest gradient. This could be due to the aluminium having the lowest young's modulus of the three materials (Table No.3). As such this material has the least stiffness, and resistance to load compared to Mild Steel and brass.

Comparing figure 17 to figure 16 the linear trend was slightly broken around 0.15Kg for aluminium and mild steel. However, this did not cause failure or great deflection. This could indicate an error with the load added. Ultimately, this has been put down to human error during the recording.

Table 5, and Graph 5 shows, mild steel has the least variance in deflection with loads applied when compared to Brass and aluminium. This is a result of having the greatest young's modulus. Therefore, it has the most resistance to deflection. It should be noted for the theoretical results as young's modulus increase the deflection of the beam decreased proportionally. For example, using table 4, and 7. Mild steel has roughly twice the young's modulus of brass. Table 7 shows that the deflection of mild steel is half of that compared to brass. This indicates that young's modulus has a linear affect to the deflection of beams.

Overall, the measured results are similar to the calculated results. Therefore, indicating the small-scale test is accurate up to a tolerance of 0.2mm. However, this tolerance error could be any of the following reasons:

- Human error, this could be recording result.
- Rounding errors, due to the mathematics and round of results this can increase the tolerance, therefore reducing the accuracy of the theoretical compared to measured.

Conclusion

The requirements for the client were to determine an appropriate material, with minimal deflection and variance in deflection when under varying load, in align with cost.

It is therefore, recommended to manufacture the equipment using mild steel. This is due to a low cost per Kilo for Mild steel, and best performance for the experiment. Showing small variance in deflection when under varying loads, and least amount of deflection.

The experiment results were confirmed to be accurate from the theoretical calculated results with the greatest variance of 0.2mm. However, this was potential due to human error when collecting a measurement. All materials showed similar behaviour, showing a linear relationship between deflection and load. As the load increased so did the deflection of each material, however at different rates. This was suspected due to the material's young modulus.

Further investigation should be made to repeat the load at 0.15 to determine the cause of the break in trend and to improve accuracy.

Additionally, recommendations for the client to further reduce deflection are:

- Beam design should be considered to minimise deflection. By increasing the depth of the beam, this will increase the 2nd moment of inertia, and therefore reducing the deflection.

- Select a material with a greater young's modulus, which has shown to correlate with the resistance to deflection.

- Support Joist: To further reduce the deflection of the beam, a support joist could be added. This would then add a support point, which will be able to reduce the acting moments, and therefore reduce the deflection.

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