Prifysgol Cymru
University of Wales

Examination Answer Books

[15 Pages]

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SURNAME: Hare

FORENAME(S): Tomos

1	Student Number	1410129
2	Title of Examination Paper	Maths
3	Module Code	ACDA4008
4	Date and Time of Examination	14/05/20
5	Examination Venue	
6	Seat Number	
7	Signature of Person Supervising	

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Total	

Dogfennau yn cael eu hefyd ar gael yn Gymraeg

1. i) $f(x) = 6x^{-3} + 2$

$f'(x) = -18x^{-4}$

7.5

ii) $f(x) = -4xe^{2x} - 9x$

PR

$f'(x) = u\frac{dv}{dx} + v\frac{du}{dx} - 9$

$u = -4x$

$\frac{du}{dx} = -4$

$v = e^{2x}$

$\frac{dv}{dx} = 2e^{2x}$

$= (-4x)(2e^{2x}) + (e^{2x})(-4) - 9$

$= -8xe^{2x} - 4e^{2x} - 9$

7.5

iv) $f(x) = \sin(5x^3 + x^2)$

CR

$f'(x) = \frac{dy}{du} \times \frac{du}{dx}$

$u = 5x^3 + x^2$

$\frac{du}{dx} = 15x^2 + 2x$

$y = \sin u$

$\frac{dy}{du} = \cos u$

$= \cos u \times (15x^2 + 2x)$

$= 15x^2 \cos(5x^3 + x^2) + 2x \cos(5x^3 + x^2)$

7.5

21.5/30

v) $f(x) = \frac{(5x+3)^2}{3x^2+2x}$

QR

$f'(x) = \frac{v\frac{du}{dx} - u\frac{dV}{dx}}{v^2}$

$= \frac{(3x^2+2x)(50x+30) - (5x+3)^2(6x+2)}{(3x^2+2x)^2}$

$= \frac{(50x+30)(3x^2+2x) - (6x+2)(5x+3)^2}{(3x^2+2x)^2}$

$u = (5x+3)^2$

~~QR~~

~~$u_2 = 5_1$~~

$\frac{du}{dx} = 2(5x+3) \times 5$

$= 10(5x+3)$

$= 50x + 30$

$V = 3x^2 + 2x$

$\frac{dV}{dx} = 6x + 2$

4.5/8

2. $y = 2x^3 - 9x^2 + 12x$

$\frac{dy}{dx} = 6x^2 - 18x + 12$

When $\frac{dy}{dx} = 0$, it will be a turning point.

$6x^2 - 18x + 12 = 0$

~~$(x-6)(x-12)$~~

$6x^2 - 6x : -12x + 12$

~~$6(x^2-3x+2)$~~ $6x(x-2) - 12(x-1)$

$(6x-12)(x-1)$

P 72

S -18

F -6, -12

$\frac{dy}{dx} = 0$ when $x = 2, 1$

$\frac{d^2y}{dx^2} = 12x - 18$

when $x = 2$, $\frac{d^2y}{dx^2} = 6$

$x = 1$ $\frac{d^2y}{dx^2} = -6$

09/10

$y = 2x^3 - 9x^2 + 12x$

is minimum at $x = 1$ and maximum at $x = 2$.

3. i) $\int (22x^3 - 4x^{-2})\,dx$

$= \frac{22}{4}x^4 \frac{-4}{-2}x^{-1} + C$

$= 3x^4 + 4x^{-1} + C$

10

3. (ii) $\int \frac{e^x}{1+e^x}\,dx$

$\int \frac{u}{1+u}\,dx$

$\int \frac{u}{1+u} \times \frac{du}{e^x}$

$\int \frac{1}{1+u}\,du$

$= \ln(1+u) + C$

$= \ln(1+e^x) + C$

###

let $u = e^x$

$\frac{du}{dx} = e^x$

$dx = \frac{du}{e^x}$

Re-done at the end

4

$\frac{3^1}{3^0}$

(iii) $\int \left(\frac{2}{x} + 4 + \ln 3\right) dx$

$= \int (2x^{-1} + 4 + \ln 3)\,dx$

$= 2\ln(x) + 4x + x\ln(3) + C$

10

Question No

Examiner Only

4. $I = \int_3^6 (3x^2 + x^3)\, dx$

$= \left[\frac{3}{3}x^3 + \frac{1}{4}x^4 \right]_3^6$

$= \left[x^3 + \frac{1}{4}x^4 \right]_3^6$

$= \left(6^3 + \frac{1}{4}(6)^4 \right) - \left(3^3 + \frac{1}{4}(3)^4 \right)$

$= (216 + 324) - \left(27 + \frac{81}{4} \right)$

$= 540 - \frac{189}{4}$

$= \frac{1971}{4} = 492.75$

$\frac{10}{10}$

New Page →

Rest of Question

Answer Writing

5. i) $z_1 = \frac{3-2j}{2+j}$, $z_2 = \frac{3+2j}{1-2j}$

find $z_1 - z_2$

$$\frac{(3-2j)(1-2j)}{(2+j)(1-2j)} - \frac{(3+2j)(2+j)}{(1-2j)(2+j)}$$

$$= \frac{3-6j-2j+4j^2}{2-4j+j-2j^2} - \frac{6+3j+4j+2j^2}{2+j-4j-2j^2}$$

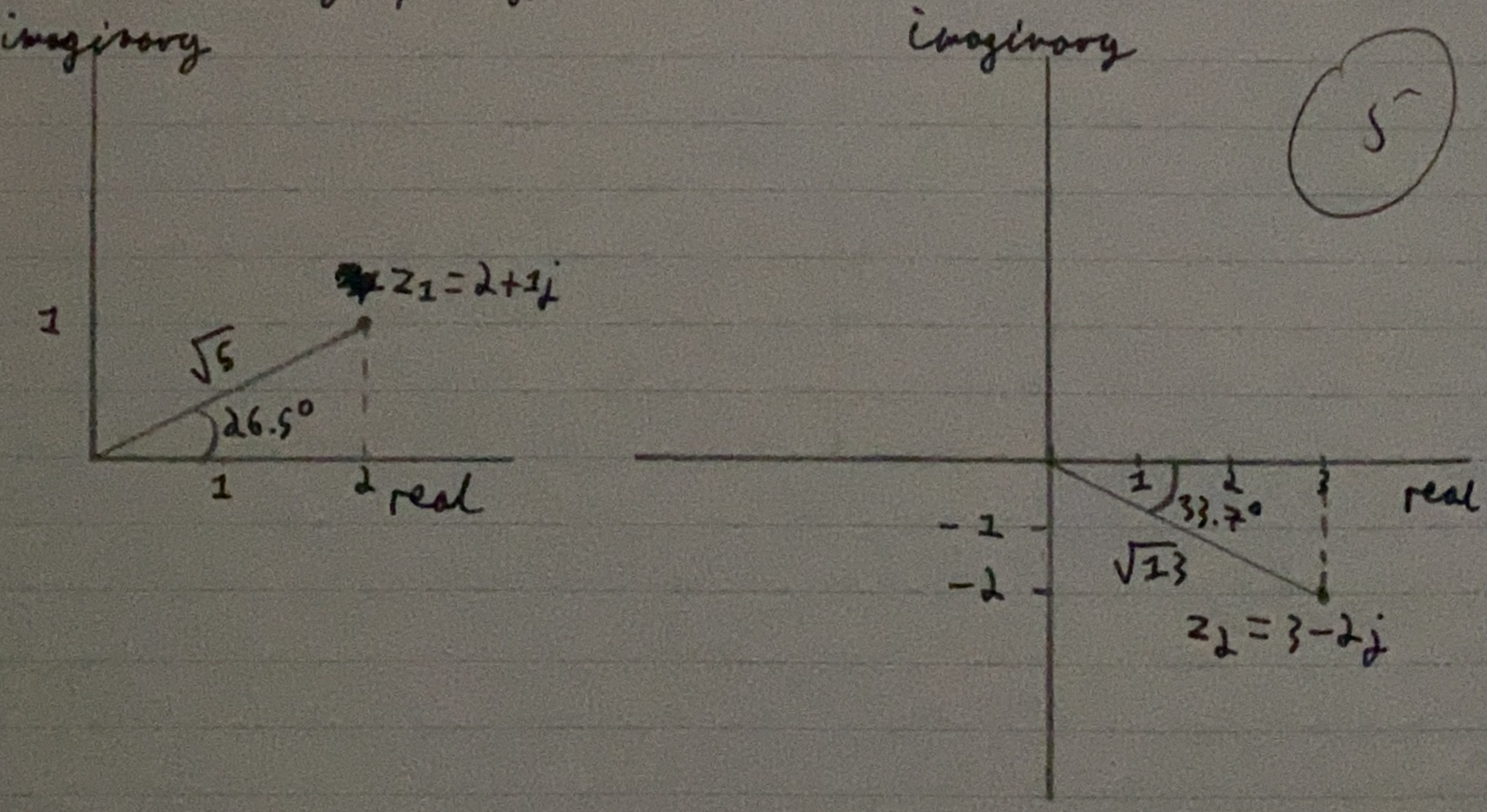
$$= \frac{3-8j-4-(6+7j-2)}{2-3j+2}$$

$$= \frac{-1-8j-(4+7j)}{4-3j}$$

(2)

$$= \frac{-5-15j}{4-3j}$$

ii) $z_1 = 2+1j$, $z_2 = 3-2j$



(5)

$$|z_1| = r_1 = \sqrt{2^2+2^2} = \sqrt{8}$$
$$\theta_1 = \tan^{-1}\left(\frac{2}{2}\right) = 26.5^\circ$$

$$|z_2| = r_2 = \sqrt{3^2+(-2)^2} = \sqrt{9+4} = \sqrt{13}$$

$$\theta_2 = \tan^{-1}\left(\frac{-2}{3}\right) = -33.7^\circ$$

iii) $z = (\sqrt{3}+j)^8 = (3^{\frac{1}{2}}+j)^8$

$$|z| = r = \sqrt{(\sqrt{3})^2+1^2} = \sqrt{4} = 2$$
$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^\circ$$

$$z = \sqrt{2}(\cos 30 + j\sin 30)$$
$$z^8 = (\sqrt{2})^8(\cos(30\times 8) + j\sin(30\times 8))$$
$$= 16(\cos(240) + j\sin(240)$$

or $16 < 240$

Next page →

5. (iv) $y = x^2 - 4x + 13$

$$\frac{dy}{dx} = 2x - 4$$

$$2x - 4 = 0$$
$$x = 2$$

$$(2+3i)(2-3i)$$

$$= 4 - 6i + 6i - 3i^2$$
$$= 4 - 3(-1)$$
$$= 7$$

Polar form

$2+3i$

$$r = \sqrt{2^2+3^2} = \sqrt{13}$$
$$\theta = \tan^{-1}\left(\frac{3}{2}\right) = 56.3^\circ$$

$$\sqrt{13} < 56.3$$

or $\sqrt{13}\,(\cos(56.3) + j\sin(56.3))$

$2-3i$

$$r = \sqrt{2^2+(-3)^2} = \sqrt{13}$$
$$\theta = \tan^{-1}\left(\frac{-3}{2}\right)$$
$$= -56.3^\circ$$

$$\sqrt{13} < -56.3^\circ$$

or $\sqrt{13}\,(\cos(-56.3) + j\sin(-56.3))$

3(a) $\int \frac{e^x}{1+e^x}\,dx$

Let $u = 1+e^x$

$\frac{du}{dx} = e^x$

$du\,dx = \frac{du}{e^x}$

$\int \frac{e^x}{u}\,\frac{du}{e^x}$

$\int \frac{1}{u}\,du$

$= \ln(u) + C$

$= \ln(1+e^x) + C$

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