

Maths Paper 2

1)

a) i) $y = \frac{x^2}{2x+1}$

$$\therefore \frac{dy}{dx} = \frac{(2x) \cdot (2x+1) - (x^2) \cdot (2)}{(2x+1)^2}$$

$$= \frac{4x^2 + 2x - 2x^2}{4x^2 + 4x + 1}$$

$$= \frac{2x^2 + 2x}{4x^2 + 4x + 1}$$

b) ii) $y = \sqrt{x^2+1}$

$$\therefore \frac{dy}{dx} = \frac{1}{2} \cdot 2x \cdot 1$$

$$\therefore \frac{dy}{dx} = x(x^2+1)^{-1/2}$$

$y = u^{1/2}, u = x^2+1 \quad \frac{du}{dx} = 2x$

$$\therefore \frac{dy}{dx} = \frac{1}{2} u^{-1/2}$$

(iii) $y = 3x^{3/2} + \frac{3}{x^2} - 6x$

$$\therefore \frac{dy}{dx} = \frac{9}{2} x^{1/2} - \frac{3}{x^3} - 6$$

(iv) $y = x + \sin 2x$

$$\therefore \frac{dy}{dx} = 1 + 2\cos 2x$$

$y = \sec^3 4x$

$\therefore y = u^3, u = \sec 4x$
 $\therefore \frac{dy}{du} = 3u^2$

$$\therefore \frac{dy}{dx} = (3\sec^2 4x)(4\sec 4x \tan 4x)$$

$$= 12\sec^3 4x \tan 4x$$

$$\frac{du}{dx} = 4\sec 4x \tan 4x$$

$$(v) \quad y = x + \sin 2x$$

$$\therefore \frac{dy}{dx} = 1 + 2\cos 2x$$

$$(vi) \quad y = \frac{1-10x}{(2x-1)^2}$$

$$(2x-1)^2 = 4x^2 - 4x + 1$$

$$\therefore \frac{dy}{dx} = \frac{(-10)(2x-1)^2 - (1-10x)(8x-4)}{(2x-1)^4}$$

$$\therefore \frac{dy}{dx} = \frac{-40x^2 + 40x - 10 - (-80x^2 + 48x - 4)}{(2x-1)^4}$$

$$= \frac{40x^2 - 8x - 6}{(2x-1)^4}$$

	$1-10x$
$8x$	$8x - 80x^2$
-4	$-4 + 40x$

$$b) \quad y = \frac{1}{x} + \cos 2x$$

$$\therefore y' = -\frac{1}{x^2} - 2\sin 2x$$

$$\therefore y'' = \frac{1}{x^3} - 4\cos 2x$$

$$\therefore y''' = -\frac{1}{x^4} + 8\sin 2x$$

$$\therefore y'''' = \frac{1}{24x^5} + 16\cos 2x$$

$$\therefore \frac{d^5 y}{dx^5} = \frac{-1}{120x^6} - 32\sin 2x$$

$$c) f(x) = \ln(\operatorname{cosec} x - \cot x)$$

it is known if $f(x) = \ln(g(x))$, $f'(x) = \frac{g'(x)}{g(x)}$
Hence:

$$f'(x) = \frac{(-\operatorname{cosec} x \cot x - (-\operatorname{cosec}^2 x))}{\operatorname{cosec} x - \cot x}$$

$$\therefore f'(x) = \frac{\operatorname{cosec}^2 x - \operatorname{cosec} x \cot x}{\operatorname{cosec} x - \cot x}$$

$$\therefore f'(x) = \frac{\operatorname{cosec} x (\operatorname{cosec} x - \cot x)}{\operatorname{cosec} x - \cot x}$$

$$= \operatorname{cosec} x \quad \text{QED}$$

2)

$$a) \quad (i) \int \frac{2x}{4+3x^2} dx = \int \left(\frac{1}{3} \cdot \frac{6x}{4+3x^2} \right) dx$$

$$= \frac{1}{3} \int \frac{6x}{3x^2+4} dx = \frac{1}{3} \ln |3x^2+4| + c$$

$$(ii) \int_0^4 e^{1/2x} dx = \left[2e^{1/2x} \right]_0^4 = 2e^2 - 2e^0 = 2e^2 - 2$$

$$(iii) I = \int_0^4 \frac{5x+13}{(2x+1)(x+4)} dx$$

$$(2x+1)(x+4) = 2x^2 + 9x + 4$$

$$\text{let } \frac{5x+13}{(2x+1)(x+4)} = \frac{A}{(2x+1)} + \frac{B}{(x+4)}$$

$$\therefore 5x+13 = A(x+4) + B(2x+1)$$

$$\text{let } x = -4$$

$$\therefore -20+13 = 0A - 7B$$

$$\therefore -7 = -7B \therefore B = 1$$

$$\text{let } x = -\frac{1}{2}$$

$$\therefore -\frac{5}{2} + 13 = \frac{7}{2}A + 0B$$

$$\therefore A = 3$$

$$\therefore I = \int_0^4 \left(\frac{3}{2x+1} + \frac{1}{x+4} \right) dx$$

$$= \frac{3}{2} \int_0^4 \frac{2}{2x+1} dx + \int_0^4 \frac{1}{x+4} dx$$

$$= \left[\frac{3}{2} \ln |2x+1| \right]_0^4 + \left[\ln |x+4| \right]_0^4$$

$$= \left(\frac{3}{2} \ln 9 - \frac{3}{2} \ln 1 \right) + (\ln 8 - \ln 4) \quad (\ln 1 = 0)$$

$$= \frac{3}{2} \ln 9 + \ln 2$$

$$b) f(x) = \frac{4}{x(1+4\ln x)^2} dx = \cancel{uv'} \quad uv'$$

$$\therefore \int f(x) \quad \text{let } u = \frac{4}{(1+4\ln x)^2} \quad \text{and } v' = \frac{4}{x}$$

$$\therefore u = 4(1+4\ln x)^{-2}$$

$$\therefore \text{let } a = 1+4\ln x$$

$$\therefore u = 4a^{-2} \quad \therefore a' = \frac{4}{x}$$

$$\therefore u' = \frac{4}{x \times 8(1+4\ln x)^3}$$

$$\therefore u' = \frac{4}{-8x(1+4\ln x)^3}$$

By reverse chain rule:

$$\text{if } y = \frac{4}{(1+4\ln x)}$$

$$y' = \frac{4}{x} \times (4 \times -1 \times (1+4\ln x)^{-2})$$

$$\therefore y' = \frac{4}{-4x(1+4\ln x)^2}$$

$$\therefore -4y' = f(x)$$

$$\therefore -4 \int y' dx = \int f(x) dx$$

$$\therefore \int f(x) dx = -4y + c$$

$$\therefore \int f(x) dx = \cancel{-16} \frac{-16}{1+4\ln x} + c$$

c)

(i) let $u = 3e^{2x}$ $u = (e^x - 1)^{-1/2}$ $v' = 3e^{2x}$
 $\therefore u' = 6e^{2x}$ $u' = -1/2 e^x (e^x - 1)^{-3/2}$ $\therefore v = \frac{3}{2} e^{2x}$

$$\therefore \int_{\ln 2}^{\ln 5} \frac{3e^{2x}}{\sqrt{e^x - 1}} dx = \left[\frac{3}{2} e^{2x} \cdot (e^x - 1)^{-1/2} \right]_{\ln 2}^{\ln 5} - \int_{\ln 2}^{\ln 5} \frac{3}{4} e^x \cdot e^{2x} \cdot (e^x - 1)^{-3/2} dx$$

$$= (18.75 - 6) + \frac{3}{4} \int_{\ln 2}^{\ln 5} e^{3x} \cdot (e^x - 1)^{-3/2} dx$$

~~18.75~~
 $\therefore I = 12.75 + \frac{3}{4} \int_2^5 u^2 (u-1)^{-3/2} du$

(let $a = u^2$ $b' = (u-1)^{-3/2}$
 $\therefore a' = u$ $b = -\frac{2}{1/2} (u-1)^{-1/2}$)

~~SO $I = 12.75 + \frac{3}{4} \left(\left[-\frac{2}{1/2} u^2 (u-1)^{-1/2} \right]_2^5 + \int_2^5 \frac{2}{1/2} u (u-1)^{-1/2} du \right)$~~

~~$= -13.25 - 9.525 + \frac{3}{4} \int_2^5 u (u-1)^{-1/2} du$~~

~~let $c = u$ $d' = (u-1)^{-1/2}$
 $\therefore c' = 1$~~

$$I = 12.75 + \frac{3}{4} \left(\left[-\frac{2}{1/2} u^2 (u-1)^{-1/2} \right]_2^5 \right) - \int_2^5 \frac{2}{1/2} u (u-1)^{-1/2} du$$

let $c = u$ $d' = (u-1)^{-1/2}$
 $\therefore c' = 1$ $d = -2(u-1)^{1/2}$

let $u = e^x$
 $\frac{du}{dx} = e^x$
 $\therefore dx = \frac{du}{e^x}$
 $\therefore \int e^{3x} \cdot (e^x - 1)^{-3/2} dx$
 $= \int u^3 \cdot (u-1)^{-3/2} \cdot \frac{du}{u}$
 $= \int u^2 (u-1)^{-3/2} du$

$$\therefore I = 8.5 - \left(-\frac{1}{2} \int_2^5 u \right)$$

$$I = 8.5 - \left(-\frac{1}{2} \left[-2u(u-1)^{1/2} \right]_2^5 + 2 \int_2^5 -\frac{1}{2}(u-1)^{1/2} du \right)$$

$$= 8.5 + \frac{1}{2} \left(-16 - \left[2(u-1)^{3/2} \right]_2^5 \right)$$

$$= 8.5 + \frac{1}{2} (-16 - 14)$$

$$= -6.5$$

$$(ii) \int_1^e \ln x dx = I \quad \text{let } u = \ln x \quad u' = 1$$

$$\therefore u = \frac{1}{x} \quad \therefore v = x$$

$$\therefore \int \ln x dx = x \ln x - \int \frac{x}{x} dx + c$$

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$$= x \ln x - x + c$$

$$\therefore I = [x \ln x - x]_1^e$$

$$= (e \ln e - e) - (1 \ln 1 - 1)$$

$$= (e - e) - (0 - 1)$$

$$= 1$$

3)

a)

$$(i) \left(\frac{1+i}{1-i} \right)^3 = i$$

$$(ii) \frac{3}{(1+i)(2i)} = \frac{3}{i^2 + 3i + 2} = \frac{3}{3i+1} = \frac{3}{10} - \frac{9i}{10}$$

$$(iii) \left(\frac{1+i}{1-i} \right) - \frac{(1+2i)(2+2i) + (3-i)}{(1+i)}$$

$$= \frac{(1+i)^2}{2} - \frac{(-2+6i) + \frac{2-4i}{2}}{2}$$

$$= 3-7i$$

$$(iv) 2i(i-1) + \sqrt{3} - i + (1+i)(1+i)$$

$$= 2i^2 - 2i + \sqrt{3} - i + 2$$

$$= -2 - 2i + \sqrt{3} - i + 2$$

$$= \sqrt{3} - 3i$$

$$b) Z = (4+7i)(3-2i)$$

$$\therefore Z = 104 + 52i$$

$$|Z| = \sqrt{104^2 + 52^2} = 52\sqrt{5}$$

$$\arg z = \arctan \frac{52}{104} = 0.4636 \text{ rad}$$

$$\therefore z = 52\sqrt{5} \angle 0.4636$$

c) let $z = \frac{1+3i}{1+i}$, $z \in \mathbb{C}$

$$\therefore z = \frac{(1+3i)(1-i)}{(1+i)(1-i)}$$

$$= \frac{4+2i}{2} = 2+i$$