

3.

Let,

$$f(x, y) = x^2 \sin(2x + 3y)$$

$$a) \frac{\partial f(x, y)}{\partial x}$$

$$\frac{\partial f}{\partial x} = 2x \sin(2 + 3y)$$

$$b) \frac{\partial^2 f(x, y)}{\partial y \partial x}$$

$$\frac{\partial f}{\partial x} = 2x \sin(2 + 3y)$$

$$\frac{\partial f}{\partial y} = x^2 \sin(2x + 3)$$

$$\frac{\partial^2 f}{\partial y \partial x} = \sqrt{2x \sin(2+3)}$$

$$= \sqrt{2x \sin 5}$$

$$2. \quad y = \frac{(3x+1) \cdot \sin^3(x)}{e^{2x}}$$

$$\ln y = \left[\frac{\ln(3x+1) \cdot \sin^3(x)}{e^{2x}} \right]$$

$$\ln y = \ln[(3x+1) \cdot \sin^3(x)] - \ln e^{2x}$$

$$\ln y = \ln(3x+1) + \ln[\sin^3(x)] - \ln e^{2x}$$

$$\ln y = \ln(3x+1) + 3\ln[\sin(x)] - (2x)\ln e$$

~~$$\ln y = \ln(3x+1) + 3\ln[\sin(x)] - 2x$$~~

$$\frac{y'}{y} = \frac{3}{3x+1} + 3 \cdot \frac{\cos(x)}{\sin(x)} - 2$$

$$y' = y \left(\frac{3}{3x+1} + \frac{3\cos(x)}{\sin(x)} - 2 \right)$$

$$y' = \left(\frac{(3x+1) \cdot \sin^3(x)}{e^{2x}} \right) \cdot \left(\frac{3}{3x+1} + \frac{3\cos(x)}{\sin(x)} - 2 \right)$$

$$b) \ g(x) = \frac{5x+3}{2x+1}$$

$$g(x) = \frac{\cancel{(5x+3)} \cdot 2x \cdot 5 \cdot (2x+1) - 2 \cdot (5x+3)}{(2x+1)^2}$$

$$g'(x) = \frac{10x+1 - 10x-6}{4x^2+4x+1}$$

$$g'(x) = \frac{-5}{4x^2+4x+1}$$

$$g''(x) = \frac{-5}{8x+4}$$

$$g''(x) = \frac{-5}{4(2x+1)}$$

$$4.a. f(x) = -2x^3 + x^2 + 8x - 10$$

$$f'(x) = -6x^2 + 2x + 8$$

$$4b. \frac{-2 \pm \sqrt{4 - (4 \times (-6) \times 8)}}{2 \times (-6)}$$

$$= \frac{-2 \pm \sqrt{196}}{-12}$$

$$= \frac{-2 \pm 14}{-12}$$

$$x = \frac{-2 + 14}{-12}$$

$$x = \frac{-2 - 14}{-12}$$

$$x = -1$$

$$x = \frac{4}{3}$$

$$= -2 \times (-1)^3 + (-1)^2 + 8 \times (-1) - 10$$

$$= 2 + 1 - 8 - 10$$

$$= -15$$

$$(-1, -15)$$

$$= (-2 \times (1.33)^3) + (1.33)^2 + (8 \times 1.33) - 10$$

$$= -3.84$$

$$-4.7 + 1.77 + 10.64 - 10$$

$$= -2.29$$

$$(1.33, -2.29)$$

$$4c) (-1, -15) = f'x < 0 \quad (1.33, -2.29) = f'x < 0$$

This means the functions between the 2 critical point is increasing

$$4d) f'(x) = -6x^2 + 2x + 8$$

$$f''(x) = -12x + 2$$

$$4e) \text{ solve } f''(x) = 0$$

$$x = \frac{-2}{-12}$$

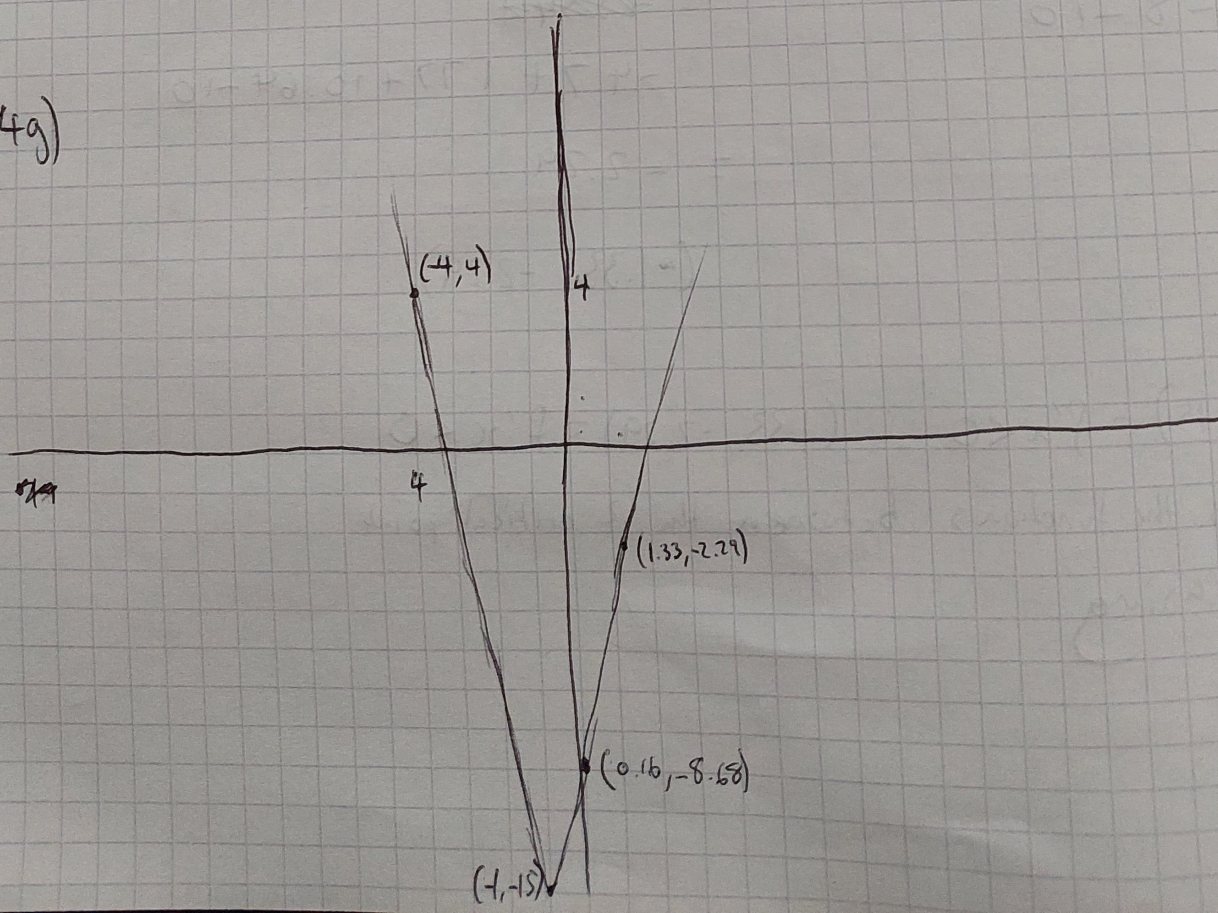
$$x = \frac{1}{6}$$

$$\begin{aligned} \text{4f) } (0.16, f(0.16)) &= -2(0.16)^3 + (0.16)^2 + (8 \times 0.16) - 10 \\ &= -8.68 \end{aligned}$$

$$\therefore (0.16, -8.68)$$

4f) As $f''(x) < 0$ $(0.16, -8.68)$ this shows that f is still increasing

4g)



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Analytical Methods

ACHA4000

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3pm

Question 1:

$$a) \quad p(x) = \sqrt{\frac{1}{2x^3} + (2x)^3 - \frac{6}{7}}$$

$$p(x) = \left(\frac{1}{2x^3} + \frac{(2x)^3}{\cancel{(2x)^3}} - \frac{6}{7} \right)^{\frac{1}{2}}$$

$$p(x) = \left(\frac{x^{-3}}{2} + 8x^3 - \frac{6}{7} \right)^{\frac{1}{2}}$$

$$p'(x) = \frac{1}{2} \left(\frac{x^{-3}}{2} + 8x^3 - \frac{6}{7} \right)^{-\frac{1}{2}} \cdot \left(-\frac{3x^{-4}}{2} + 24x^2 - \frac{6}{7} \right)$$