

Question 1.

a. Length of total area = $20 + 2w$ Area = Length $\times$ Width
 Width of total area = $10 + 2w$ (1)

$$(20 + 2w)(10 + 2w) = 600 \quad (2)$$

$$200 + 40w + 20w + 4w^2 = 600$$

$$4w^2 + 60w + 200 = 600 \quad (1)$$

$$4w^2 + 60w - 400 = 0 \quad (1)$$

$$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$w = \frac{-60 \pm \sqrt{60^2 - 4 \times 4 \times -400}}{2 \times 4}$$

$$w = \frac{-60 \pm \sqrt{3600 - 4 \times 4 \times -400}}{2 \times 4}$$

$$w = \frac{-60 \pm \sqrt{3600 - -6400}}{8} \quad (6)$$

$$w = \frac{-60 \pm \sqrt{10000}}{8}$$

$$w = \frac{-60 + 100}{8} = \underline{\underline{5m}} \quad \text{CORRECT ANSWER} \quad (2)$$

$$w = \frac{-60 - 100}{8} = \underline{\underline{-20m}} \quad \text{IMPOSSIBLE}$$

b. Area of decking = Total area - Internal area
 $= 600 - (20 \times 10)$
 $= 400 \quad (1)$

$$\text{Cost} = \text{Area of decking} \times \text{price per } m^2$$

$$\begin{aligned}
 \text{Cost} &= \text{Area of decking} \times \text{price per m}^2 \\
 &= 400 \times 65 \\
 &= \underline{\underline{\pounds 26,000}}
 \end{aligned}$$

(1)

$$\begin{aligned}
 \text{c. Volume of pool} &= \text{Area} \times \text{depth} \\
 &= 200 \times 1.5 \\
 &= 300 \text{ m}^3
 \end{aligned}$$

(2)

$$\begin{aligned}
 \text{Time} &= \text{Volume of pool} \div \text{Flowrate} \\
 &= 300 \div 0.02 \\
 &= 15000 \text{ s} \\
 &= 4 \text{ hours } 10 \text{ minutes}
 \end{aligned}$$

(2)

(1)

Question 2.

$$\begin{aligned}
 \text{a. } P &= \int_0^6 800 + \frac{1}{2} x^3 \cdot dx \\
 &= \left[800x + \left(\frac{1}{(3+1)} \frac{x^{(3+1)}}{2} \right) \right]_0^6 = \left[800x + \frac{1}{4} \frac{x^4}{2} \right]_0^6 = \left[800x + \frac{x^4}{8} \right]_0^6 \quad (5) \\
 P &= \left[(800 \times 6) + \frac{6^4}{8} \right] - \left[800 \times 0 + \frac{0^4}{8} \right] \\
 P &= 4800 + 162 - 0 \\
 &= \underline{\underline{4962 \text{ N}}} \quad (4)
 \end{aligned}$$

$$\begin{aligned}
 \text{b. } R &= \int_0^6 800x + \frac{x^4}{8} \cdot dx \\
 &= \left[\frac{1}{1+1} \times 800x^{1+1} + \frac{1}{4+1} \times \frac{x^{4+1}}{8} \right]_0^6 \\
 &= \left[\frac{1}{2} \times 800x^2 + \frac{1}{5} \times \frac{x^5}{8} \right]_0^6 = \left[400x^2 + \frac{x^5}{40} \right]_0^6 \quad (6)
 \end{aligned}$$

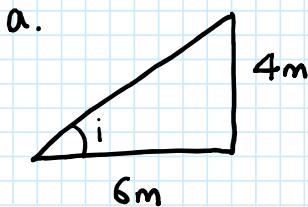
$$= \left[\frac{1}{2} \times 800 x^2 + \frac{1}{5} \times \frac{x^5}{8} \right]_0^6 = \left[400 x^2 + \frac{x^5}{40} \right]_0^6 \quad (6)$$

$$R = \left[400 \times 6^2 + \frac{6^5}{40} \right] - \left[400 \times 0^2 + \frac{0^5}{40} \right]$$

$$= 14400 + 194.4 - 0$$

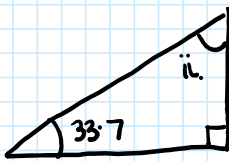
$$= 14594.4 = \underline{\underline{14.6 \text{ kNm}}} \quad (5)$$

Question 3.

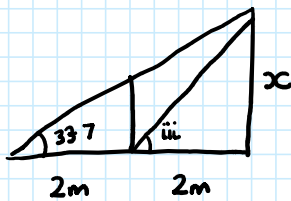


$$\tan i = \frac{4}{6}$$

$$i = \tan^{-1} \left(\frac{4}{6} \right) = 33.7^\circ$$



$$\begin{aligned} ii &= 180 - 90 - 33.7 \\ &= 56.3^\circ \end{aligned}$$

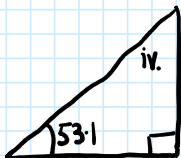


$$\tan 33.7 = \frac{x}{4}$$

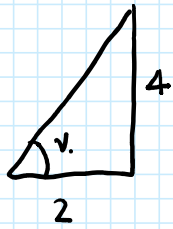
$$\begin{aligned} x &= 4 \times \tan 33.7 \\ &= 2.67 \text{ m} \end{aligned}$$

$$\tan iii = \frac{2.67}{2}$$

$$iii = \tan^{-1} \left(\frac{2.67}{2} \right) = 53.1^\circ$$

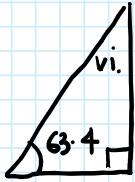


$$iv = 180 - 90 - 53.1 = 36.9^\circ$$



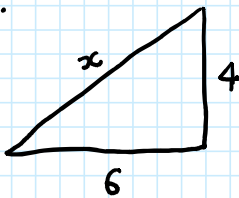
$$\tan v = \frac{4}{2}$$

$$v = \tan^{-1} \left(\frac{4}{2} \right) = 63.4^\circ$$



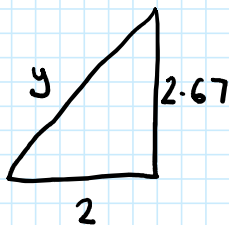
$$\begin{aligned} v_i &= 180 - 90 - 63.4 \\ &= 26.6^\circ \end{aligned}$$

b.



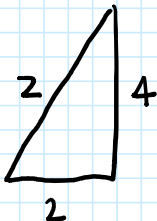
$$\begin{aligned} x &= \sqrt{6^2 + 4^2} \\ &= \sqrt{36 + 16} \\ &= \sqrt{52} = 7.21\text{m} \end{aligned}$$

(2)



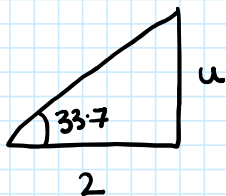
$$\begin{aligned} y &= \sqrt{2^2 + 2.67^2} \\ &= \sqrt{4 + 7.13} \\ &= \sqrt{11.13} = 3.34\text{m} \end{aligned}$$

(2)



$$\begin{aligned} z &= \sqrt{4^2 + 2^2} \\ &= \sqrt{16 + 4} \\ &= \sqrt{20} = 4.47\text{m} \end{aligned}$$

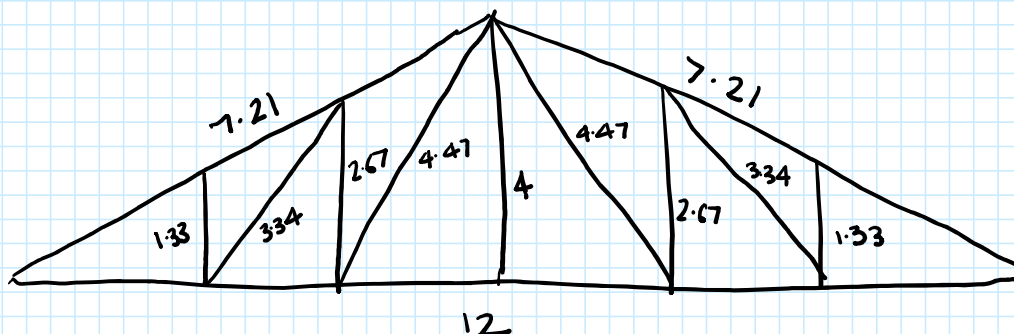
(2)



$$\tan 33.7 = \frac{u}{2}$$

$$u = 2 \times \tan 33.7 = 1.33 \text{ m}$$

(2)



$$\begin{aligned} \text{Total Length} &= 12 + 4 + (2 \times 7.21) + (2 \times 1.33) + (2 \times 3.34) + (2 \times 2.67) + (2 \times 4.47) \\ &= 12 + 4 + 14.42 + 2.66 + 6.68 + 5.34 + 8.94 \\ &= \underline{\underline{54.04 \text{ m}}} \end{aligned}$$

(2)

Question 4.

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & -1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ T \end{bmatrix} = \begin{bmatrix} -9 \\ -29 \\ 0 \end{bmatrix}$$

(3)

$$\begin{aligned} |A| &= 1(2 \times 0 - 1 \times -1) - 0(0 \times 0 - 1 \times -1) + -1(0 \times 1 - 2 \times 1) \\ &= 1(0 + 1) - 0(0 + 1) - 1(0 - 2) \\ &= 1(1) - 1(-2) \\ &= 1 + 2 = 3 \end{aligned}$$

(2)

$$\begin{bmatrix} -9 & 0 & -1 \\ -29 & 2 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

(1)

$$\begin{aligned}
 |A_a| &= -g(2 \times 0 - 1 \times -1) - 0(-2g \times 0 - -1 \times 0) + -1(-2g \times 1 - 2 \times 0) \\
 &= -g(0 + 1) - 0(0 + 0) - 1(-2g - 0) \\
 &= -g(1) - 1(-2g) \\
 &= -g + 2g = g \quad (2)
 \end{aligned}$$

$$a = \frac{|A_a|}{|A|} = \frac{g}{3} \quad (2)$$

$$\begin{matrix} A_b \\ \begin{bmatrix} 1 & -g & -1 \\ 0 & -2g & -1 \\ 1 & 0 & 0 \end{bmatrix} \end{matrix} \quad (1)$$

$$\begin{aligned}
 |A_b| &= 1(-2g \times 0 - -1 \times 0) - -g(0 \times 0 - 1 \times -1) + -1(0 \times 0 - 1 \times -2g) \\
 &= 1(0 + 0) + g(0 + 1) - 1(0 + 2g) \\
 &= g(1) - 1(2g) \\
 &= g - 2g = -g \quad (2)
 \end{aligned}$$

$$b = \frac{|A_b|}{|A|} = \frac{-g}{3} \quad (2)$$

$$\begin{matrix} A_T \\ \begin{bmatrix} 1 & 0 & -g \\ 0 & 2 & -2g \\ 1 & 1 & 0 \end{bmatrix} \end{matrix} \quad (1)$$

$$\begin{aligned}
 |A_T| &= 1(2 \times 0 - 1 \times -2g) - 0(0 \times 0 - 1 \times -g) + -g(0 \times 1 - 2 \times 1) \\
 &= 1(0 + 2g) - 0(0 + 2g) - g(0 - 2) \\
 &= 1(2g) - g(-2) \\
 &= 2g + 2g = 4g \quad (2)
 \end{aligned}$$

$$\begin{aligned}
 &= 1(2g) - g(-2) \\
 &= 2g + 2g = 4g \quad (2)
 \end{aligned}$$

$$T = \frac{|A_T|}{|A|} = \frac{4g}{\underline{\underline{3}}} \quad (2)$$

Question 5.

a. Hemispherical Section

$$\begin{aligned}
 A_{\text{hem}} &= 2\pi r^2 = 2 \times \pi \times 1.5^2 \\
 &= 14.1 \text{ m}^2 \quad (2)
 \end{aligned}$$

Cylindrical Section

$$\begin{aligned}
 A_{\text{gl}} &= 2\pi rh = 2 \times \pi \times 1.5 \times 4 \\
 &= 37.7 \text{ m}^2 \quad (2)
 \end{aligned}$$

Frustum

$$A_{\text{frus}} = \pi (r_1 + r_2) l \quad \text{where}$$

$$\begin{aligned}
 l &= \sqrt{(r_1 - r_2)^2 + h^2} = \sqrt{(2.5 - 1.5)^2 + 20^2} \\
 &= \sqrt{1^2 + 20^2} \\
 &= \sqrt{401} = 20.02 \text{ m} \quad (2)
 \end{aligned}$$

$$\begin{aligned}
 A_{\text{frus}} &= \pi (2.5 + 1.5) \times 20.02 \\
 &= \pi \times 4 \times 20.02 \\
 &= 251.6 \text{ m}^2 \quad (2)
 \end{aligned}$$

$$\begin{aligned}
 \text{Total Area} &= A_{\text{hem}} + A_{\text{gl}} + A_{\text{frus}} \\
 &= 14.1 + 37.7 + 251.6 \\
 &= 303.4 \text{ m}^2 \quad (2)
 \end{aligned}$$

b. Hemispherical Section

$$\begin{aligned} V_{\text{hem}} &= \frac{2}{3} \pi r^3 \\ &= \frac{2}{3} \times \pi \times 1.5^3 \\ &= 7.07 \text{ m}^3 \end{aligned} \quad (2)$$

Cylindrical Section

$$\begin{aligned} V_{\text{cyl}} &= \pi r^2 h \\ &= \pi \times 1.5^2 \times 4 \\ &= 28.27 \text{ m}^3 \end{aligned} \quad (2)$$

Frustum Section

$$\begin{aligned} V_{\text{frus}} &= \frac{1}{3} \pi h (r_1^2 + r_1 r_2 + r_2^2) \\ &= \frac{1}{3} \times \pi \times 20 \times (2.5^2 + 2.5 \times 1.5 + 1.5^2) \\ &= \frac{1}{3} \times \pi \times 20 \times (6.25 + 3.75 + 2.25) \\ &= \frac{1}{3} \times \pi \times 20 \times 12.25 \\ &= 256.56 \text{ m}^3 \end{aligned} \quad (4)$$

$$\begin{aligned} \text{Total Volume} &= V_{\text{hem}} + V_{\text{cyl}} + V_{\text{frus}} \\ &= 7.07 + 28.27 + 256.56 \\ &= \underline{\underline{291.9 \text{ m}^3}} \end{aligned} \quad (2)$$