

HEAT TRANSFER

In Physics, heat transfer is the process by which energy in the form of heat is exchanged between bodies or parts of the same body at different temperatures.

Heat flows from high to low temperature regions.

This heat transfer generally occurs following one or several of the 3 processes that are:

- i. Conduction,
- ii. Convection,
- iii. Radiation.

I. CONDUCTION

Conduction is the mode of heat transfer in which energy exchange takes place in solids or fluids at rest (i.e. no convective motion resulting from the displacement of the macroscopic portion of the medium) due to an internal temperature gradient. Direct physical contact between interacting systems is required for conduction to happen. The transfer of thermal energy occurs at molecular and atomic levels.

In solids, conduction of heat proceeds by both lattice vibration and migration of free electrons.

Heat conduction in metals

The electrons in piece of metal can leave their atoms and move about in the metal as free electrons. The parts of the metal atoms left behind are now charged metal ions. The ions are packed closely together and they vibrate continually. The hotter the metal, the more *kinetic energy* these vibrations have. This kinetic energy is transferred from hot parts of the metal to cooler parts by the free electrons. These move through the structure of the metal, colliding with ions as they go.

In fluids, energy is transferred by molecular interactions.

Although the phenomenon of heat conduction had been observed for centuries, it was not until 1882 that the French mathematician Jean Baptiste Joseph Fourier gave it precise mathematical expression in what is now regarded as Fourier's law of heat conduction.

The physical law states that the rate at which heat is conducted through a body per unit cross sectional area is proportional to the negative of the temperature gradient existing in the body:

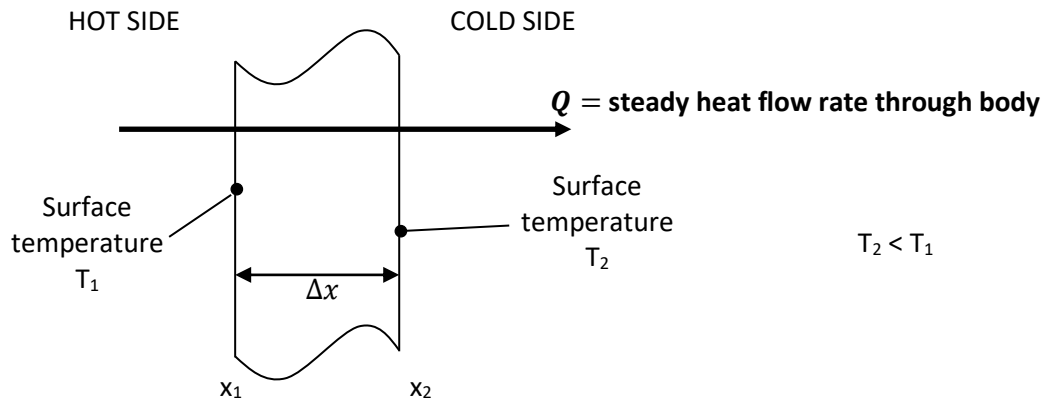
$$Q = -k \cdot A \cdot \frac{dT}{dx}$$

Fourier's law of heat conduction
(One-dimensional, steady-state)

- The derivative term dT/dx is called the temperature gradient.
- k is called the thermal conductivity which is a material property.
A material with a large thermal conductivity is known as thermal conductor, and one with a small value of k as a thermal insulator.
- A is the surface area of the material considered.
- Q is the steady heat flow rate through the material, i.e. the amount of heat transferred, i.e. heat leakage, i.e. heat loss.

1. Plane wall with fixed surface temperatures.

The simplest heat transfer problem is that of one-dimensional steady-state conduction in a plane wall of homogeneous material having constant thermal conductivity and with each face held a constant uniform temperature as shown in the following figure:



Separation of variables and integration of the Fourier's law of heat conduction where the gradient direction is x results in

$$Q \int_{x_1}^{x_2} dx = -k \cdot A \cdot \int_{T_1}^{T_2} dT$$

$$\therefore Q = -k \cdot A \cdot \frac{(T_2 - T_1)}{(x_2 - x_1)}$$

$$\therefore Q = \frac{-k \cdot A \cdot (T_2 - T_1)}{\Delta x}$$

or $Q = -U \cdot A \cdot (T_2 - T_1)$ where $U = \frac{k}{x}$

U is called the heat transfer coefficient.

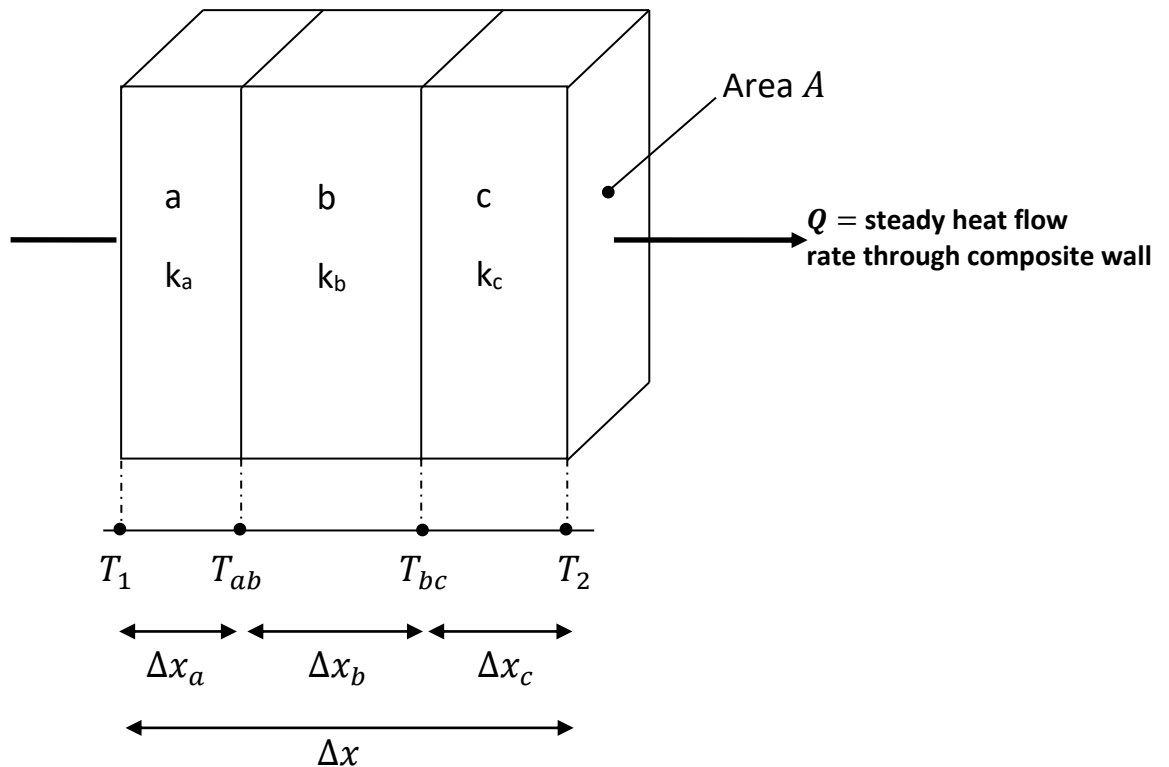
- T as for units $^{\circ}\text{C}$ or Kelvin.
- Δx as for units m .
- A as for units m^2 .
- k as for units $\text{W}/(\text{m} \cdot ^{\circ}\text{C})$ or $\text{W}/(\text{m} \cdot \text{K})$.
- Q as for units W .

$^{\circ}\text{C} = \text{K} - 273.15$

$\text{K} = ^{\circ}\text{C} + 273.15$

2. Conduction through a composite wall.

Consider a composite wall, consisting of 3 materials a, b, c as shown:



Important points:

- The temperature at the material interfaces is common to the surface temperatures of both materials concerned.
- The steady heat flow rate through each layer of the composite wall will be the same, i.e.:

$$Q = Q_a = Q_b = Q_c$$

Applying the basic equation we have:

$$Q_a = -U_a \cdot A \cdot (T_{ab} - T_1) \quad \text{where: } U_a = \frac{k_a}{\Delta x_a}$$

Transposing to make the temperature the subject we get:

$$T_1 - T_{ab} = \frac{Q_a}{U_a \cdot A} \quad (1)$$

Similarly for layers b and c we get:

$$T_{ab} - T_{bc} = \frac{Q_b}{U_b \cdot A} \quad (2)$$

And:

$$T_{bc} - T_2 = \frac{Q_c}{U_c \cdot A} \quad (3)$$

Adding equations (1), (2) and (3) we get:

$$T_1 - T_{ab} + T_{ab} - T_{bc} + T_{bc} - T_2 = \frac{Q_a}{U_a \cdot A} + \frac{Q_b}{U_b \cdot A} + \frac{Q_c}{U_c \cdot A}$$

$$\therefore T_1 - T_2 = \frac{Q}{A} \cdot \left(\frac{1}{U_a} + \frac{1}{U_b} + \frac{1}{U_c} \right) \quad \text{where: } Q = Q_a = Q_b = Q_c$$

If we define the value U such that:

$$\boxed{\frac{1}{U} = \frac{1}{U_a} + \frac{1}{U_b} + \frac{1}{U_c}}$$

where U is called the **overall heat transfer coefficient**

Then we get:

$$T_1 - T_2 = \frac{Q}{A} \times \frac{1}{U}$$

Which gives:

$$Q = UA(T_1 - T_2)$$

$$\therefore \boxed{Q = -UA(T_2 - T_1)}$$

The latest equation takes the same form as the “basic equation” derived in previous section but with U being the **overall heat transfer coefficient**.