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Engineering Maths Exam
ACMA4006

(a)(i) AB

$$A = \begin{pmatrix} 2 & -5 & 3 \\ -5 & 0 & 5 \\ 3 & 0 & 2 \end{pmatrix} \quad B = \begin{pmatrix} -5 & 8 & 0 \\ 2 & 6 & 2 \\ 0 & -2 & 3 \end{pmatrix}$$

$3 \times 3 \quad \quad \quad 3 \times 3$

$$\begin{pmatrix} -10 + -10 + 0 & 16 - 30 - 6 & 0 - 10 + 9 \\ 25 + 0 + 0 & -40 + 0 - 10 & 0 + 0 + 15 \\ -15 + 0 + 0 & 24 + 0 - 4 & 0 + 0 + 6 \end{pmatrix}$$

$$AB = \begin{pmatrix} -20 & -20 & -1 \\ 25 & -50 & 15 \\ -15 & 20 & 6 \end{pmatrix}$$

(ii) BA

$$B = \begin{pmatrix} -5 & 8 & 0 \\ 2 & 6 & 2 \\ 0 & -2 & 3 \end{pmatrix} \quad A = \begin{pmatrix} 2 & -5 & 3 \\ -5 & 0 & 5 \\ 3 & 0 & 2 \end{pmatrix}$$

$$\begin{array}{ccc} -10 - 40 + 0 & 25 + 0 + 0 & -15 + 40 + 0 \\ 4 - 30 + 6 & -10 + 0 + 0 & 6 + 30 + 4 \\ 0 + 10 + 9 & 0 + 0 + 0 & 0 - 10 + 6 \end{array}$$

$$BA = \begin{pmatrix} -50 & 25 & 25 \\ -20 & -10 & 40 \\ 9 & 0 & -4 \end{pmatrix}$$

(iii) $3 \times \boxed{3} \times 2 \quad \checkmark \quad AC$

$$A = \begin{pmatrix} 2 & -5 & 3 \\ -5 & 0 & 5 \\ 3 & 0 & 2 \end{pmatrix} \quad C = \begin{pmatrix} 12 & 5 \\ -3 & 2 \\ 2 & 5 \end{pmatrix}$$

$$24 + 15 + 6$$

$$10 - 10 + 15$$

$$-60 + 0 + 10$$

$$-10 + 0 + 25$$

$$36 + 0 + 4$$

$$15 + 0 + 10$$

$$AC = \begin{pmatrix} 45 & 15 \\ -50 & 15 \\ 40 & 25 \end{pmatrix}$$

(iv) $CA \quad 3 \times \boxed{2} \times 3 \quad \text{can not be multiplied.}$

(v) $|A| \quad A = \begin{pmatrix} 2 & -5 & 3 \\ -5 & 0 & 5 \\ 3 & 0 & 2 \end{pmatrix}$

$$|A| = 2(0 \times 2 - 0 \times 5) + 5(-5 \times 2 - 3 \times 5) + 3(-5 \times 0 - 3 \times 0)$$

$$= 2(0) + 5(-25) + 3(0)$$

$$= 0 - 20 + 0$$

$$= -20$$

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(vi) $|B|$ $B = \begin{pmatrix} -5 & 8 & 0 \\ 2 & 6 & 2 \\ 0 & -2 & 3 \end{pmatrix}$

$$|B| = -5(6 \times 3 - 2 \times 2) - 8(2 \times 3 - 0 \times 2) + 0$$

$$-5(14) - 8(6)$$

$$= -70 - 48$$

$$|B| = -118$$

(vii) $A + B$

$$A = \begin{pmatrix} 2 & -5 & 3 \\ -5 & 0 & 5 \\ 3 & 0 & 2 \end{pmatrix}$$

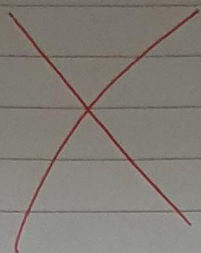
$$B = \begin{pmatrix} -5 & 8 & 0 \\ 2 & 6 & 2 \\ 0 & -2 & 3 \end{pmatrix}$$

$$A + B = \begin{pmatrix} -3 & 3 & 3 \\ -3 & 6 & 7 \\ 3 & -2 & 5 \end{pmatrix}$$

(viii) Can not be done as matrix are not the
 6 11 11 5

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6)



$$c) \begin{pmatrix} 3 & 1 & 4 \\ 5 & 3 & 1 \\ 2 & 4 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 21 \\ 15 \\ 10 \end{pmatrix}$$

$$\begin{aligned} \text{Find } |A| &= 3(3 \times 3 - 4 \times 1) - 1(5 \times 3 - 2 \times 1) + 4(5 \times 4 - 2 \times 3) \\ &= 9 - 4 \quad 15 - 2 \quad 20 - 6 \\ &= 3(5) - 1(13) + 4(14) \\ &= 15 - 13 + 56 \end{aligned}$$

$$|A| = 58$$

$$|A_1| = \begin{vmatrix} 21 & 1 & 4 \\ 15 & 3 & 1 \\ 10 & 4 & 3 \end{vmatrix} \quad |A_1| = 21(3 \times 3 - 4 \times 1) - 1(15 \times 3 - 10 \times 1) + 4(15 \times 4 - 10 \times 1)$$

$$= 21(5) - 1(35) + 4(50)$$

$$= 105 - 35 + 200$$

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$$A_2 = \begin{pmatrix} 3 & 21 & 4 \\ 5 & 15 & 1 \\ 2 & 10 & 3 \end{pmatrix}$$

$$\begin{aligned} |A_2| &= 3 \begin{pmatrix} 15 \times 3 - 10 \times 1 \\ 45 - 10 \end{pmatrix} - 21 \begin{pmatrix} 5 \times 3 - 2 \times 1 \\ 15 - 2 \end{pmatrix} + 4 \begin{pmatrix} 5 \times 10 - 2 \times 15 \\ 50 - 30 \end{pmatrix} \\ &= 3(35) - 21(13) + 4(20) \\ &\quad 105 - 273 + 80 \end{aligned}$$

$$|A_2| = -88$$

$$A_3 = \begin{pmatrix} 3 & 1 & 21 \\ 5 & 3 & 15 \\ 2 & 4 & 10 \end{pmatrix}$$

$$\begin{aligned} |A_3| &= 3 \begin{pmatrix} 3 \times 10 - 4 \times 15 \\ 30 - 60 \end{pmatrix} - 1 \begin{pmatrix} 5 \times 10 - 2 \times 15 \\ 50 - 30 \end{pmatrix} + 21 \begin{pmatrix} 5 \times 4 - 2 \times 3 \\ 20 - 6 \end{pmatrix} \\ &\quad 3(-30) - 1(20) + 21(14) \\ &\quad -90 - 20 + 280 \end{aligned}$$

$$|A_3| = 170$$

$$x = \frac{|A_1|}{|A|}$$

$$y = \frac{|A_2|}{|A|}$$

$$z = \frac{|A_3|}{|A|}$$

$$x = \frac{270}{58}$$

$$y = \frac{-88}{58}$$

$$z = \frac{170}{58}$$

$$x = 4.65$$

$$y = -1.51$$

$$z = 2.93$$

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Question 2

$$a) i) Z_1 + Z_3 = (2 + 5j) + (1 - 3j) = \boxed{3 + 2j}$$

$$ii) Z_1 - Z_3 = (2 + 5j) - (-5 - 2j) = \boxed{-3 + 7j}$$

$$iii) \frac{2 + 5j}{1 - 3j} \times \frac{1 + 3j}{1 + 3j}$$

$$\frac{(2 + 5j)(1 + 3j)}{(1 - 3j)(1 + 3j)}$$

$$\frac{2 + 5j + 6j + 8j^2}{1 + 3j - 3j - 6j^2}$$

$$\frac{2 + 11j - 8}{1 + 6}$$

$$\frac{-6 + 11j}{7} = \boxed{\frac{-6}{7} + \frac{11j}{7}}$$

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$$(iv) (-5 - 2j)(-5 - 2j) - (2 - 5j)$$

$$(25 + 10j + 10j + 4j^2) - (2 - 5j)$$

$$(25 + 20j + 4j^2) - (2 - 5j)$$

$$(25 + 20j - 4) - (2 - 5j)$$

$$(21 + 20j) - (2 - 5j)$$

$$21 + 20j - 2 + 5j$$

$$= \boxed{19 + 25j}$$

$$6) \underset{a}{5}x^2 = \underset{b}{-3}x \underset{c}{-12} = 5x^2 + 3x + 9 = 0$$

$$x = \frac{b \pm \sqrt{b^2 - 4(a)(c)}}{2(a)}$$

$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(5)(12)}}{2(5)}$$

$$= \frac{-3 \pm \sqrt{9 - 240}}{10} = \frac{-3 \pm 15.19j}{10}$$

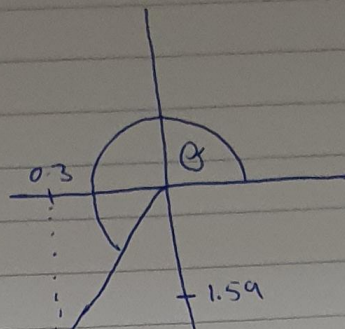
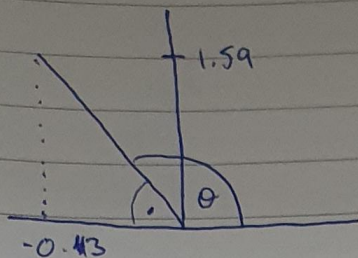
$$x = \frac{-3 \pm 15.19j}{10}$$

$$x = \frac{-3}{10} + \frac{15.19j}{10}$$

$$x = \frac{-3}{10} - \frac{15.19j}{10}$$

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b) continued



$$r^2 = x^2 + y^2$$

$$= 0.3^2 + 1.59^2$$

$$= 0.09 + 2.5281$$

$$= \sqrt{2.6181}$$

$$r = \boxed{1.618}$$

$$\tan \phi = \frac{1.59}{0.3}$$

$$\phi = \tan^{-1} \frac{1.59}{0.3}$$

$$\phi = 79.3$$

$$\theta = 180 - 79.3 = \boxed{100.68}$$

$$\boxed{1.618 (\cos 100.68 + j \sin 100.68)}$$

$$r^2 = x^2 + y^2$$

$$r = 1.618$$

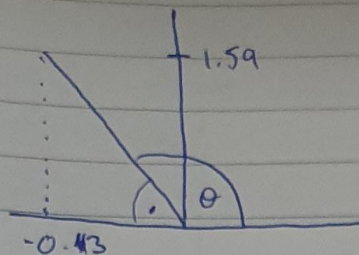
$$\phi = 79.3$$

$$\theta = 79.3 + 180 = 259.3$$

$$\boxed{1.618 (\cos 259.3 + j \sin 259.3)}$$

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b) continued



$$r^2 = x^2 + y^2$$

$$= 0.3^2 + 1.59^2$$

$$= 0.09 + 2.5281$$

$$= \sqrt{2.6181}$$

$$r = \boxed{1.618}$$

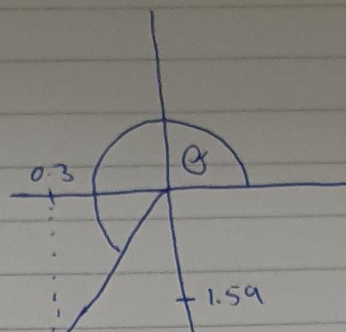
$$\tan \phi = \frac{1.59}{0.3}$$

$$\phi = \tan^{-1} \frac{1.59}{0.3}$$

$$\phi = 79.3$$

$$\theta = 180 - 79.3 = \boxed{100.68}$$

$$\boxed{1.618 (\cos 100.68 + j \sin 100.68)}$$



$$r^2 = x^2 + y^2$$

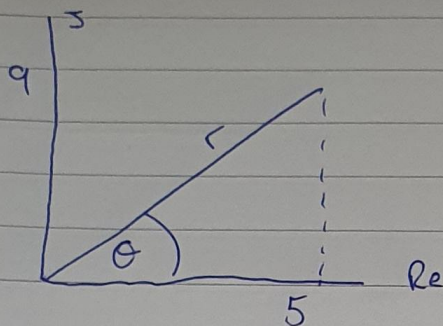
$$r = 1.618$$

$$\phi = 79.3$$

$$\theta = 79.3 + 180 = 259.3$$

$$\boxed{1.618 (\cos 259.3 + j \sin 259.3)}$$

$$c) i) (5 - 9j)^5$$



$$r^2 = 9^2 + 5^2$$

$$81 + 25$$

$$r = \sqrt{106}$$

$$\tan \theta = \frac{9}{3}$$

$$\theta = \tan^{-1} \frac{9}{3}$$

$$\theta = 71.56$$

$$r^n (\cos n \theta + j \sin \theta)$$

$$(106)^4 (\cos 4 \times 71.56 + j \sin 4 \times 71.56)$$

$$126247696 (\cos 286.24 + j \sin 286.24)$$

$$126247696 (0.28 - 0.96j)$$

$$= \boxed{35349354.88 - 121197788.2j}$$

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Question 3

i) $\int (x^8 + 2x^2)(3x^2 + x) dx$

$$3x^{16} + 2x^8 + 6x^4 + 2x^2$$

$$\frac{3x^{17}}{17} + \frac{2x^9}{9} + \frac{6x^5}{5} + \frac{2x^3}{3} + C$$