

VUMAT for the Johnson-Holmquist (JH-2) ceramic model

With the release of Abaqus 6.10, the constitutive model for ceramics and other brittle materials proposed by Johnson and Holmquist (JH-2) is available in the form of a built-in user material for Abaqus/Explicit. The model is intended for the simulation of the mechanical response and failure behavior of brittle materials subjected to large strains, high strain rates, and high pressures.

In order to access this model, the first 7 characters of the material name in the Abaqus input file must be ABQ_JH2; e.g., ABQ_JH2_1.

The model is supported for plane strain, axisymmetric, and 3D solid elements and can be used in both Lagrangian and Eulerian domains.

This document describes the basic equations of the constitutive model and provides detailed information about the user interface to the VUMAT implementation in Abaqus.

1. Description of the JH-2 ceramic model

The JH-2 model consists of three main components: (1) a representation of the deviatoric strength of the intact and fractured material in the form of a pressure-dependent yield surface, (2) a damage model that transitions the material from the intact state to a fractured state, and (3) an equation of state (EOS) for the pressure-density relation that can include dilation (or bulking) effects. Each of these components of the model are described in the following sections.

1.1. Strength

The strength of the material is expressed in terms of the normalized von Mises equivalent stress as:

$$\sigma^* = \sigma_i^* - D(\sigma_i^* - \sigma_f^*)$$

where σ_i^* is the normalized intact equivalent stress, σ_f^* is the normalized fractured equivalent stress, and D is the damage variable (defined in the next section). The normalized equivalent stresses (σ^* , σ_i^* and σ_f^*) have the general form $\sigma^* = \sigma / \sigma_{HEL}$, where σ is the actual von Mises equivalent stress and σ_{HEL} is the equivalent stress at the Hugoniot elastic limit (HEL), at which a one-dimensional (uniaxial strain) shock wave exceeds the elastic limit of the material. The model assumes that the normalized intact and fractured stresses can be expressed as functions of the pressure and strain rate as:

$$\sigma_i^* = A(P^* + T^*)^N (1 + C \ln \dot{\epsilon}^*) \leq \sigma_i^{\max}$$

$$\sigma_f^* = B(P^*)^M (1 + C \ln \dot{\epsilon}^*) \leq \sigma_f^{\max}$$

The material parameters are A , B , C , M , N , and the optional limits for the strengths $\sigma_i^{\max}$ and $\sigma_f^{\max}$. The normalized pressure is defined as $P^* = P / P_{HEL}$, where P is the actual pressure and P_{HEL} is the pressure at the HEL. Similarly, the normalized maximum tensile hydrostatic pressure $T^* = T / P_{HEL}$, where T is the maximum tensile pressure that the material can withstand.

Rate effects are included using the Johnson-Cook strain-rate dependence criterion. The dimensionless strain rate is given as $\dot{\epsilon}^* = \dot{\epsilon}^{pl} / \dot{\epsilon}_0$, where $\dot{\epsilon}^{pl}$ is the equivalent plastic strain rate and $\dot{\epsilon}_0$ is the reference (or threshold) strain rate.

Plastic flow is assumed to be isochoric (volume preserving) using a Mises flow surface. The evolution of the plastic strain tensor is given as $\dot{\epsilon}^{pl} = \dot{\epsilon}^{pl} \mathbf{n}$ where $\mathbf{n}$ is the normal to the Mises flow surface.

1.2. Damage

The model uses a damage accumulation criterion that is similar to that in the Johnson-Cook fracture model. The damage initiation parameter, ω , accumulates with plastic strain according to

$$\omega = \sum \frac{\Delta \bar{\epsilon}^{pl}}{\bar{\epsilon}_f^{pl}(P)}$$

Here $\Delta \bar{\epsilon}^{pl}$ is the increment in equivalent plastic strain and $\bar{\epsilon}_f^{pl}(P)$ is the equivalent plastic strain to fracture under constant pressure, defined as

$$\bar{\epsilon}_f^{pl} = D_1 (P^* + T^*)^{D_2}; \quad \bar{\epsilon}_{f,min}^{pl} \leq \bar{\epsilon}_f^{pl} \leq \bar{\epsilon}_{f,max}^{pl}$$

where D_1 and D_2 are material constants. The optional parameters $\bar{\epsilon}_{f,min}^{pl}$ and $\bar{\epsilon}_{f,max}^{pl}$ are provided for additional flexibility to limit the minimum and maximum values of the fracture strain.

The JH-2 model assumes that the damage variable increases progressively with plastic deformation by setting $D = \omega$. In an earlier version of the model (known as the JH-1 model), the material was assumed to fail instantaneously by setting $D = 1$ when $\omega = 1$. The different damage models are summarized in the table below.

Damage Evolution	
JH-1	JH-2
$D = 0$ if $\omega < 1$	$D = \omega$
$D = 1$ if $\omega = 1$	

Both options are supported in the Abaqus implementation. The flag `1Damage` is used to specify the damage option. Set `1Damage=0` (or leave blank) to specify the JH-2 progressive damage model, and `1Damage=1` to specify the JH-1 (instantaneous) damage model.

1.3. Pressure

The pressure-density relationship is assumed to be given by an equation of state of the form

$$P = \begin{cases} K_1 \mu + K_2 \mu^2 + K_3 \mu^3 & \text{if } \mu \geq 0 \text{ (compression)} \\ K_1 \mu & \text{if } \mu < 0 \text{ (expansion)} \end{cases}$$

with $\mu = \frac{\rho}{\rho_0} - 1$

where K_1 , K_2 and K_3 are constants (K_1 is the initial bulk modulus), ρ is the current density and ρ_0 is the reference density.

The model includes the effects of dilation (or bulking) that occur when brittle materials fail by including an additional pressure increment, ΔP , such that

$$P = K_1\mu + K_2\mu^2 + K_3\mu^3 + \Delta P$$

The pressure increment is determined from energy considerations. The decrease in strength when the material undergoes damage (as it goes from an intact state to a failed state) produces a decrease in the deviatoric elastic energy, ΔU . This loss of elastic energy is converted into potential hydrostatic energy by incrementally increasing ΔP according to (refer to the references for more detail)

$$\Delta P_{t+\Delta t} = -K_1\mu_{t+\Delta t} + \sqrt{(K_1\mu_{t+\Delta t} + \Delta P_t)^2 + 2\beta K_1\Delta U}$$

where β is the fraction of the elastic energy loss converted to potential hydrostatic energy ($0 \leq \beta \leq 1$).

An important experimental data point commonly used for determining the intact strength is the *HEL*. The *HEL* is the net compressive stress (containing both pressure and deviator stress components) at which a one-dimensional (uniaxial strain) shock wave exceeds the elastic limit of the material. Thus:

$$HEL = P_{HEL} + \frac{2}{3}\sigma_{HEL}$$

This expression can be expanded using the above pressure-density relation and the elastic relations for uniaxial strain conditions to obtain

$$HEL = K_1\mu_{HEL} + K_2\mu_{HEL}^2 + K_3\mu_{HEL}^3 + \frac{4}{3}G \frac{\mu_{HEL}}{1 + \mu_{HEL}}$$

where μ_{HEL} is the volumetric strain at the HEL. This expression can be used to compute μ_{HEL} from *HEL* and then substitute back in the pressure-density relation to obtain P_{HEL} . This calculation is performed automatically by the VUMAT if the user specifies only *HEL* and leaves P_{HEL} unspecified.

2. User interface

To activate the JH-2 ceramic model the material name must start with ABQ_JH2; e.g., ABQ_JH2_1. A synopsis of the interface is shown below. The number of solution-dependent variables (under *DEPVAR) is 8, and the DELETE parameter is equal to 8. Refer to Table 1 for a detailed description of each material constant specified in the keyword interface.

```
*MATERIAL, NAME=ABQ_JH2
*DENSITY
   $\rho$ 
*USER MATERIAL, CONSTANTS=32
** Line 1:
   $\rho_0, G, A, N, B, M, C, \dot{\epsilon}_0$ 
** Line 2:
   $T, \sigma_i^{\max}, \sigma_f^{\max}, HEL, P_{HEL}, \beta$ 
** Line 3:
   $D_1, D_2, \bar{\epsilon}_{f,\max}^{pl}, \bar{\epsilon}_{f,\min}^{pl}, FS, lDamage$ 
** Line 4:
   $K_1, K_2, K_3$ 
*DEPVAR, DELETE=8
8
1, PEEQ      , "Equivalent plastic strain"
2, PEEQ_RATE, "Equivalent plastic strain rate"
3, DUCTCRT   , "Ductile damage initiation criterion"
4, DAMAGE    , "Damage variable"
5, DELTAP    , "Pressure increment due to bulking"
6, YIELD     , "Yield strength"
7, EVOL      , "Volumetric Strain, Mu"
8, MPSTATUS  , "Material point status"
```

Table 1: Constants for the JH-2 material model
LINE: 1

Pos. (Prop.)	Symbol	Description
1 (1)	ρ_0	Reference density of the material
2 (2)	G	Shear modulus
3 (3)	A	Intact normalized strength parameter
4 (4)	N	Intact strength parameter (pressure exponent)
5 (5)	B	Fractured normalized strength parameter
6 (6)	M	Fractured strength parameter (pressure exponent)
7 (7)	C	Strength parameter for strain-rate dependence
8 (8)	$\dot{\epsilon}_0$	Reference strain rate

LINE: 2

Pos. (Prop.)	Symbol	Description
1 (9)	T	Maximum tensile hydrostatic stress
2 (10)	$\sigma_i^{\max}$	Maximum normalized intact strength
3 (11)	$\sigma_f^{\max}$	Maximum normalized fractured strength
4 (12)	HEL	Net compressive stress at Hugoniot elastic limit (HEL)
5 (13)	p_{HEL}	Pressure component at the HEL. If not specified, it will be derived from HEL
6 (14)	β	Fraction of elastic energy loss converted to hydrostatic energy
7-8 (15-16)		<i>Not used</i>

LINE: 3

Pos. (Prop.)	Symbol	Description
1 (17)	D_1	Parameter for plastic strain to fracture
2 (18)	D_2	Parameter for plastic strain to fracture (exponent)
3 (19)	$\bar{\epsilon}_{f,\max}^{pl}$	Maximum failure strain
4 (20)	$\bar{\epsilon}_{f,\min}^{pl}$	Minimum failure strain
5 (21)	FS	Failure criteria, FS FS < 0 $\Rightarrow$ fail if $(P^* + T^*) < 0$ (tensile failure) FS = 0 $\Rightarrow$ no failure (default) FS > 0 $\Rightarrow$ fail if PEEQ > FS
6 (22)	lDamage	Damage flag: lDamage = 0 $\Rightarrow$ progressive damage (JH-2) lDamage = 1 $\Rightarrow$ instantaneous damage (JH-1)
7-8 (23-24)		<i>Not used</i>

Table 1: (Continued)
LINE: 4

Pos. (Prop.)	Symbol	Description
1 (25)	K_1	Bulk modulus
2 (26)	K_2	Second pressure coefficient
3 (27)	K_3	Third pressure coefficient
4-8 (28-32)		<i>Not used</i>

3. Output

In addition to the standard (material-independent) output variables in Abaqus/Explicit for stress-displacement elements (such as stress, S, strain, LE, element STATUS, etc.) the following output variables have a special meaning for the JH-2 ceramic model:

Output Variable	Symbol	Description
SDV1	$\bar{\epsilon}^{pl}$	Equivalent plastic strain PEEQ
SDV2	$\dot{\bar{\epsilon}}^{pl}$	Equivalent plastic strain rate
SDV3	ω	Damage initiation criterion
SDV4	D	Damage variable
SDV5	ΔP	Pressure increment due to bulking
SDV6	σ	Yield strength
SDV7	μ	Volumetric strain, $\mu = \rho / \rho_0 - 1$
SDV8	MpStatus	Material point status: 1 if active, 0 if failed.

4. References

- [1] G. R. Johnson and T. J. Holmquist, in Proceedings of Explomet Conference, San Diego, August 1990.
- [2] G. R. Johnson and T. J. Holmquist, in *High Pressure Science and Technology—1993*, edited by S. C. Schmidt, J. W. Schaner, G. A. Samara, and M. Ross (AIP, New York, 1994)
- [3] T. J. Holmquist, D. W. Templeton, and K. D. Bishnoi, Constitutive modeling of aluminum nitride for large strain, *International Journal of Impact Engineering*, **25** (2001) 211-231