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Y Drindod Dewi Sant
University of Wales
Trinity Saint David

Yr Athrofa
Centre for Education
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Partneriaeth Dysgu Proffesiynol **Yr Athrofa**
Yr Athrofa Professional Learning Partnership

Trawsnewid Addysg Gyda'n Gilydd Transforming Teaching Together



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ECAD 6015

Arwain y Dysgu (gyda SAC): Beth, sut a pham ydw i'n addysgu?

Leading the Learning (with QTS): what, how and why am I teaching?

**Sesiwn 1 – Egwyddorion addysgu
Mathemateg a Rhifedd effeithiol**

**Session 1 – The Principles of effective
Mathematics and Numeracy teaching**

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Amcanion

- I adnabod egwyddorion addysgu mathemateg a rhifedd effeithiol
- Trafod yn feirniadol beth sy'n gwneud gwers mathemateg a rhifedd da
- I ail-grynhoi'r 5 hyfedredd

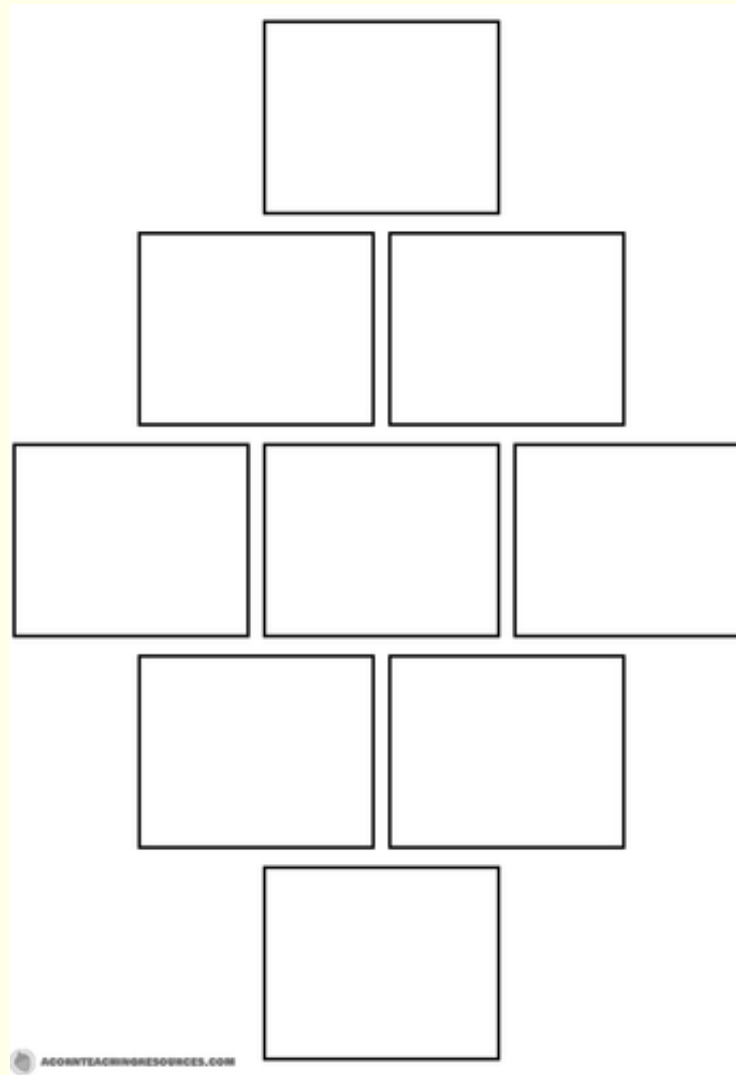
Objectives

- To identify the principles of effective mathematics and numeracy teaching
- To critically discuss what makes a good maths and numeracy lesson
- To re-cap the 5 proficiencies





- Prioritise qualities of a successful young learner
- Prioritise aspects for your subject area



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200
1822 - 2022

*Trawsnewid Addysg, Trawsnewid Bywydau
Transforming Education, Transforming Lives*

Nodi egwyddorion addysgu mathemateg a rhifedd effeithiol

Identify the principles of teaching effective mathematics and numeracy

Mae 5 egwyddor allweddol o addysgu mathemateg a rhifedd effeithiol (Learning with Mr.Lee)

1. Deall bod Mathemateg yn Hierarchaidd iawn
2. Mae angen Dealltwriaeth Gysyniadol a Gweithdrefnol arnom
3. Defnyddiwch gynrychioliadau Concrit, Darluniadol a Haniaethol (CPA) yn seiliedig ar y anghenion y dysgwr
4. I feddwl fel mathemategydd, mae angen i ni ddysgu mathemateg iddynt
5. Anelwch at ddysgu cynhyrchol, yn hytrach na brwydr gynhyrchol

There are 5 key principles of teaching effective mathematics and numeracy (Learning with Mr.Lee)

1. Understand that Mathematics is Highly Hierarchical
2. We need both Conceptual and Procedural Understanding
3. Use Concrete, Pictorial and Abstract (CPA) representations based on the needs of the learner
4. To think like a mathematician, we need to teach them maths
5. Aim for productive learning, rather than productive struggle

1. Deall bod Mathemateg yn Hierarchaidd lawn

Yn yr ystafell ddosbarth cynradd, mae cysyniadau mathemategol sylfaenol yn chwarae rhan hanfodol wrth lunio'r daith fathemategol i'n dysgwyr ifanc. O gyfrif sylfaenol a chydabod rhifau i weithrediadau hanfodol, mae'r cysyniadau hyn yn gwasanaethu fel blociau adeiladu ar gyfer sgiliau mathemategol mwy datblygedig (Price, Mazzocco & Ansari, 2013).

Yn union fel y mae ffoneg yn gosod y sylfaen ar gyfer darllen a sillafu, mae meistrolï'r sgiliau mathemateg sylfaenol hyn yn darparu sylfaen gadarn ac yn gwella galluedd datrys problemau myfyrwyr.



Tasg - Sut fydddech chi'n sicrhau cynnydd yn y modd hwn?

Meddyliwch am enghreifftiau o sut y bydddech chi'n defnyddio'r blociau adeiladu hyn ar gyfer sgiliau mathemategol mwy datblygedig.

Task - How would you ensure progression in this way?

Think of examples of how you would use these building blocks for more advanced mathematical skills.

1. Understand that Mathematics is Highly Hierarchical

In the primary classroom, fundamental mathematical concepts play a crucial role in shaping the mathematical journey for our young learners. From basic counting and number recognition to essential operations, these concepts serve as the building blocks for more advanced mathematical skills (Price, Mazzocco & Ansari, 2013).

Just as phonics lays the groundwork for reading and spelling, mastering these fundamental maths skills provides a solid foundation and enhances students' problem-solving abilities.



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2. Mae angen Dealltwriaeth Gysyniadol a Gweithdrefnol arnom

Mae dealltwriaeth gynhwysfawr o fathemateg yn gofyn am ddealltwriaeth gysyniadol a rhuglder gweithdrefnol. Mae dealltwriaeth gysyniadol yn cynnwys deall yr egwyddorion sylfaenol a'r cysylltiadau rhesymegol mewn mathemateg, tra bod dealltwriaeth weithdrefnol yn canolbwyntio ar weithredu gweithdrefnau mathemategol penodol yn gywir.

Er bod dealltwriaeth gysyniadol a gweithdrefnol yn hanfodol, gall ceisio canolbwyntio arnynt ar yr un pryd fod yn llethol i fyfyrwyr. Gall arwain at ddryswch a rhwystro profiad dysgu cyffredinol myfyrwyr. Fodd bynnag, nid yw'n ymwneud â'r naill neu'r llall neu un cyn y llall, ond yn hytrach maent yn ddwyochrog a gellir eu cydblethu (Rittle-Johnson, et al, 2015).

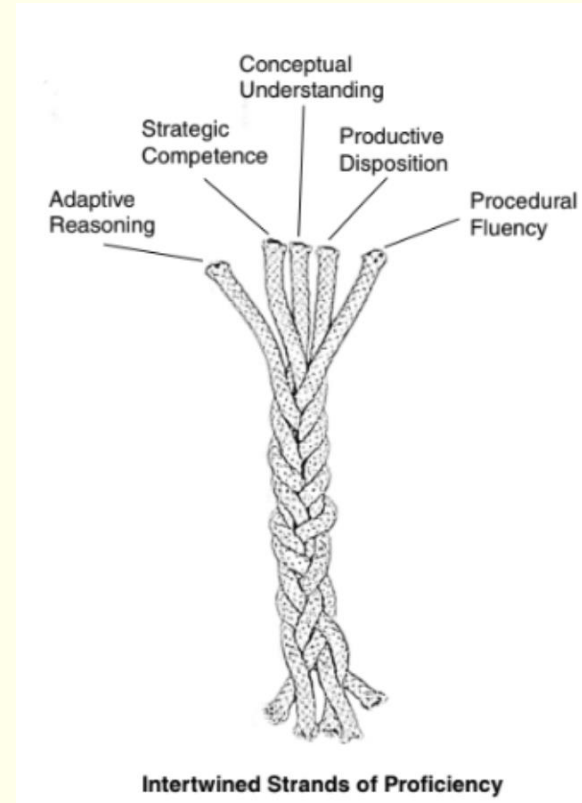
2. We need both Conceptual and Procedural Understanding

A comprehensive understanding of mathematics requires both conceptual understanding and procedural fluency. Conceptual understanding involves grasping the underlying principles and logical connections in math, while procedural understanding focuses on executing specific mathematical procedures accurately.

While both conceptual and procedural understanding are vital, attempting to focus on them simultaneously can be overwhelming for students. It can lead to confusion and hinder students' overall learning experience. However, it is not about one or the other or one before the other, but rather they are bidirectional and can be intertwined (Rittle-Johnson, et al, 2015).

Mae'r graffeg hwn o Kilpatrick et al, (2001) yn dangos sut mae'r pum llinyn (rhesymu rhesymegol, cymhwysedd strategol, dealltwriaeth gysyniadol, cyfathrebu á symbolau a rhuglder gweithdrefnol) wedi'u plethu ac yn rhyng-ddibynnol.

Yn debyg i 'Scarborough's Reading Rope', bydd yr hyn rydych chi'n canolbwyntio arno yn dibynnu ar gam taith fathemategol y myfyriwr e.e. yn y blynyddoedd cynnar, mae angen treulio mwy o amser ar ddatblygu eu synnwyr rhif (deall rhifau) ac yna ffeithiau mathemateg sylfaenol, cyn newid i gysyniadau mwy cymhleth fel ffracsiynau ac Algebra.



This graphic from Kilpatrick et al, (2001) shows how the five strands (**adaptive reasoning, strategic competence, conceptual understanding, productive disposition and procedural fluency**) are interwoven and interdependent.

Similar to Scarborough's Reading Rope, what you focus on **will be dependent on the stage of the student's mathematical journey** e.g. in the early years, more time needs to be spent on developing their number sense (understanding numbers) and then basic maths facts, before a shift is made onto more complex concepts like fractions and Algebra.

3. Defnyddio cynrychioliadau Concrit, Darluniadol a Haniaethol (CPA) yn seiliedig ar anghenion y dysgwr

Concrit: yn cynnwys gwrthrychau corfforol neu manipulatives y gall myfyrwyr eu cyffwrdd a rhyngweithio â nhw, gan eu helpu i ddatblygu sylfaen gadarn o ddealltwriaeth.

Darluniadol / Cynrychioliadol : fel diagramau a siartiau, yn darlunio syniadau mathemategol yn weledol ac yn gwasanaethu fel pont rhwng y concrit a'r haniaethol. Maent yn helpu myfyrwyr i ddelweddu perthnasoedd a phatrymau, gan feithrin dealltwriaeth gysyniadol ddyfnach.

Haniaethol: yn cynnwys symbolau a nodiant mathemategol, yn ffyrdd cryno ac effeithlon o gynrychioli syniadau mathemateg. Maent yn hanfodol ar gyfer datblygu rhuglder mewn iaith fathemategol, cyfrifiadau, a datrys problemau. Fodd bynnag, mae'n bwysig cydnabod y gall dibynnu ar gynrychioliadau haniaethol yn unig fod yn heriol i rai myfyrwyr sydd angen mwy o amlygiad i gynrychioliadau concrit neu ddarluniadol cyn y gallant ei ddelweddu eu hunain.

3. Use Concrete, Pictorial and Abstract (CPA) representations based on the needs of the learner

Concrete: involves physical objects or manipulatives that students can touch and interact with, helping them develop a solid foundation of understanding. These manipulatives make math concepts more tangible and accessible.

Pictorial/Representational: such as diagrams and charts, visually depict mathematical ideas and serve as a bridge between the concrete and abstract. They help students visualise relationships and patterns, fostering a deeper conceptual understanding.

Abstract: includes symbols and mathematical notation, are concise and efficient ways to represent math ideas. They are essential for developing fluency in mathematical language, calculations, and problem-solving. However, it's important to recognise that relying solely on abstract representations may be challenging for some students who need more exposure to concrete or pictorial representations before they can visualise it themselves.



4. I feddwl fel mathemategydd, mae angen i ni ddysgu mathemateg iddynt

Efallai y bydd mathemategwyr yn mwynhau sefyllfaoedd heriol. Gallant fod yn chwilfrydig a gallant fonitro eu meddwl eu hunain. Fodd bynnag, nid yw dechreuwr yn ychydig o arbenigwr (Chi et al, 1981; Willingham, 2021). Er y gall deall sut mae mathemategydd yn meddwl ddarparu mewnwelediadau i'w strategaethau a'u dulliau datrys problemau, **nid yw'n gwarantu y bydd rhywun yn meddwl yn well neu'n dod yn well mathemategydd.**

Mae dod yn fathemategydd medrus yn gofyn am ymarfer, dealltwriaeth ddofn o gysyniadau mathemategol, a'r gallu i **gymhwyso meddwl rhesymegol a dadansoddol.**

4. To think like a mathematician, we need to teach them maths

Mathematicians might relish in challenging situations. They can be curious and can monitor their own thinking. However, a new learner is not a little expert (Chi et al, 1981; Willingham, 2021). While understanding how a mathematician thinks can provide insights into their problem-solving strategies and approaches, **it does not guarantee that one will automatically think better or become a better mathematician.**

Becoming a proficient mathematician requires practice, a deep understanding of mathematical concepts, and the ability to **apply logical and analytical thinking.**



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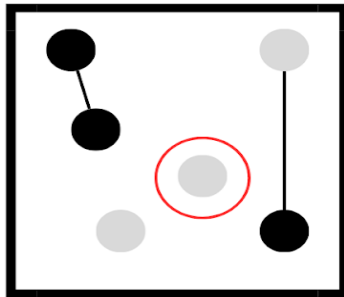
Trafodwch – sut fydddech chi'n dehongli'r diagram/graffig hwn?

- Beth allai'r dotiau du ei gynrychioli?
- Beth allai'r dotiau llwyd ei gynrychioli?
- Beth allai'r cylch coch ei gynrychioli?
- Beth allai'r llinellau du ei gynrychioli?

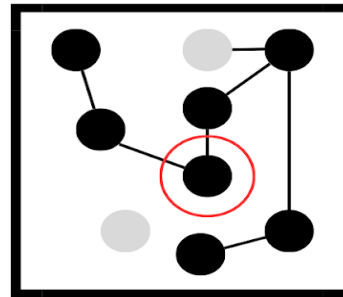
Discuss – how would you interpret this diagram/graphic?

- What could the black dots represent?
- What could the grey dots represent?
- What could the red circle represent?
- What could the black lines represent?

How learning happens



Novice



Expert

learnwithlee.net

Learn with Mr.Lee



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Pam bod gosod sylfaeni yn mathemateg mor bwysig?

Why are the foundations in mathematics so important?



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5. Aim for productive learning, rather than productive struggle

Anxiety can affect anyone and be about anything. Maths anxiety isn't so much a whole separate thing, but rather just something people can get anxious about. Often, it can be connected to being in uncomfortable situations, such as not being able to complete a problem. Researchers have found that **having to understand mathematical concepts intuitively and the difficulty in conjuring up the spatial imagery is actually what causes the feelings of anxiety** (Krasa et al, 2022)

This is where “productive struggle” can actually lead to destructive struggle. For new learners, using the problem-first approach also **has ramifications on their self-efficacy, motivation and potential to pick up misconceptions**. Another negative of relying on students to discover things for themselves is **that they can actually “learn” how to do things the wrong way and embed these misconceptions into their long-term memory**. This then becomes another barrier for the teacher to overcome (Roy et al, 2019).

Motivation and connection aren't dependent on the task, it's dependent on the students' ability to make connections to their schema, feelings of previous success and an understanding of the task.

5. Anelwch at ddysgu cynhyrchiol, yn hytrach na brwydr gynhyrchiol

Gall gorbryder effeithio ar unrhyw un a bod am unrhyw beth. Nid yw pryder mathemateg yn beth hollol ar wahân, ond yn hytrach dim ond rhywbeth y gall pobl fod yn bryderus amdano. Yn aml, gall fod yn gysylltiedig â bod mewn sefyllfaoedd anghyfforddus, fel peidio â gallu cwblhau problem. Mae ymchwilyr wedi darganfod **mai gorfod deall cysyniadau mathemategol yn reddfod a'r anhawster i greu'r delweddau gofodol yw'r hyn sy'n achosi'r teimladau o bryder mewn gwirionedd** (Krasa et al, 2022)

Dyma lle gall "brwydr gynhyrchiol" arwain at frwydr ddinistriol.

I ddysgwyr newydd, mae defnyddio'r dull problem-first hefyd **yn cael effaith ar eu hunan-effeithiolrwydd, eu cymhelliant a'u potensial i godi camsyniadau**. Peth negyddol arall o ddibynnu ar fyfyrwyr i ddarganfod pethau dros eu hunain **yw eu bod yn gallu "dysgu" sut i wneud pethau yn y ffordd anghywir ac ymgorffori'r camsyniadau hyn yn eu cof hirdymor**. Mae hyn wedyn yn dod yn rhwystr arall i'r athro ei oresgyn (Roy et al, 2019).

Nid yw cymhelliant a chysylltiad yn dibynnu ar y dasg, mae'n dibynnu ar allu'r myfyrwyr i wneud cysylltiadau â'u sgema, teimladau o lwyddiant blaenorol a dealltwriaeth o'r dasg.



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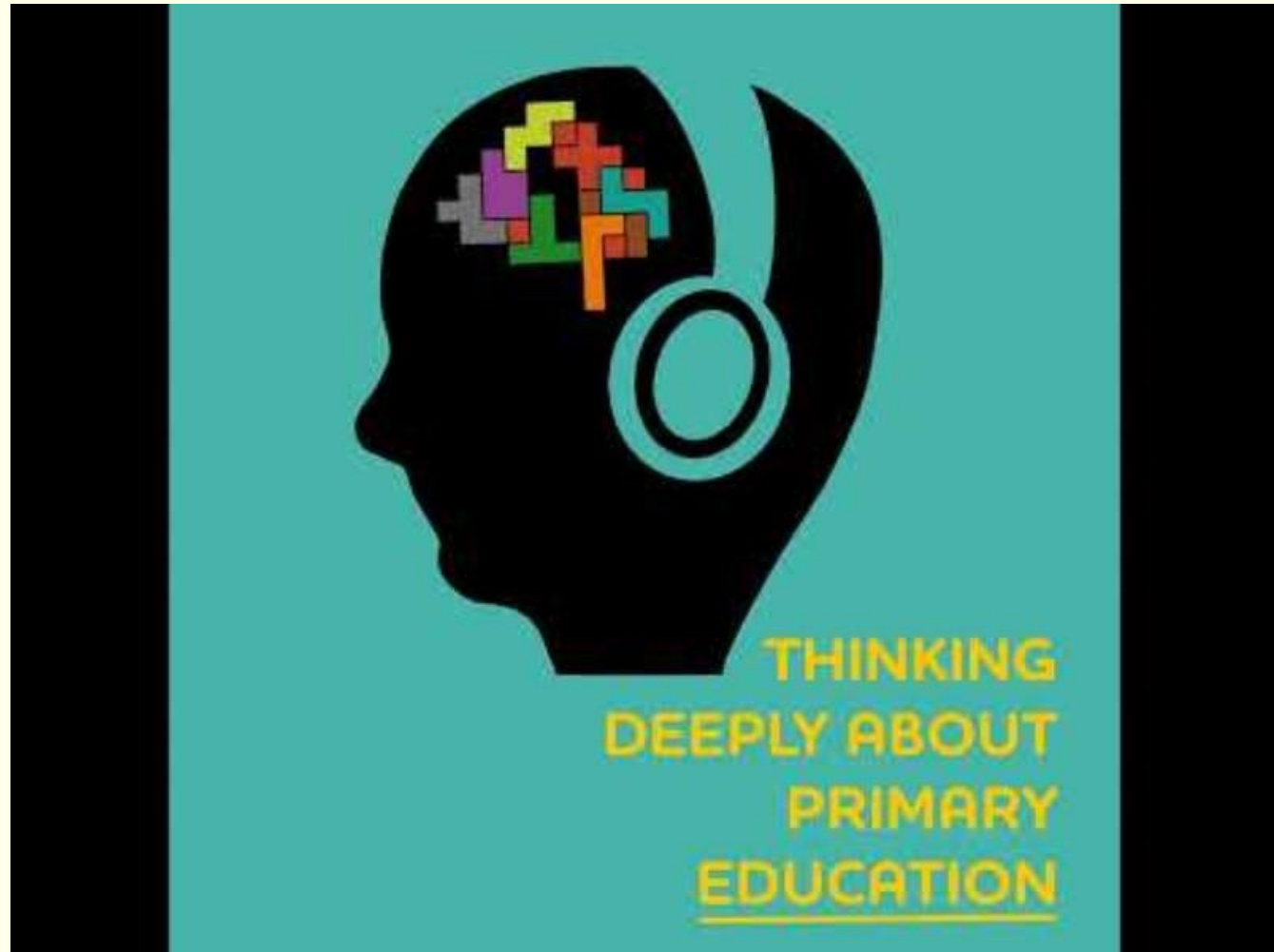
Trafodaeth feirniadol – Beth sy'n gwneud gwers Mathemateg/Rhifedd da?

- Beth yw gwers 'dda'?
- Dysgu v Perfformiad
- Beth yw'r cyffredin rhagdybiaethau?
- Mewnbwn a chwestiynu athrawon

Critical discussion – What makes a good Mathematics/Numeracy lesson?

- What is a 'good' lesson?
- Learning v Performance
- What are the common assumptions?
- Teacher input and questioning

[What are the four most prominent features of an effective mathematics lesson? - YouTube](#)



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Cwestiwn: - Pa sgiliau sydd eu hangen er mwyn ateb y broblem eiriol hon?

Mae llyfrgell ysgol yn trefnu llyfrau mathemateg newydd.

Mae **3 silff**, ac mae pob silff yn dal **24 llyfr**. Yna, mae'r llyfrgellydd yn rhoi **18 llyfr** i ysgol arall.

Cwestiwn:

Faint o lyfrau mathemateg sydd ar ôl yn y llyfrgell?

Question - What skills are needed in order to answer this word problem?

A school library is organizing new mathematics books.

There are **3 shelves**, and each shelf holds **24 books**.

Later, the librarian donates **18 books** to another school.

Question:

How many mathematics books are left in the library?

What skills does a learner require in order to complete a word problem?

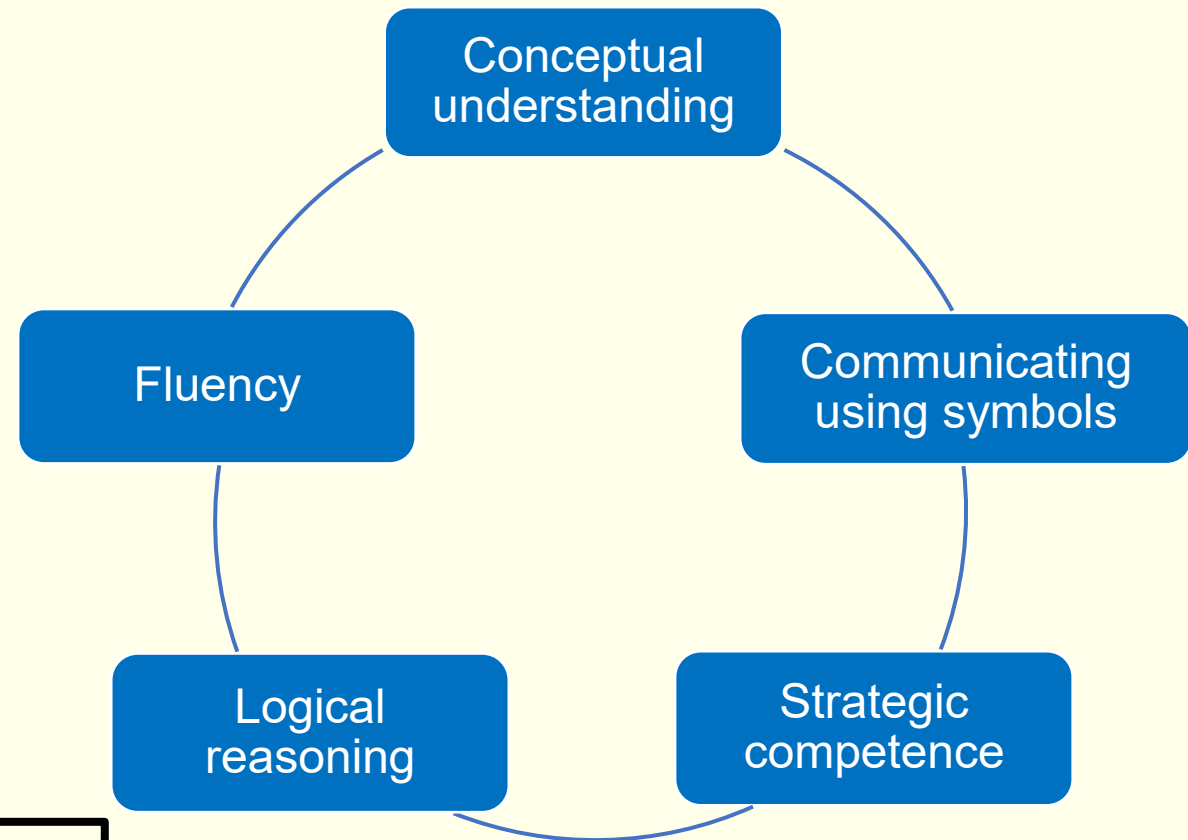
Reasoning

Reading Comprehension

Mathematical vocabulary

Problem-Solving Strategies

Fluency in Mathematical Operations



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Reading Comprehension

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Reasoning

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Question:

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Fluency in Mathematical Operations

Problem-Solving Strategies



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Dealltwriaeth o ddarllen

Geirfa mathemategol

Mae llyfrgell ysgol yn trefnu llyfrau mathemateg newydd.

Mae **silff**, ac mae **pob silff** yn dal **24 llyfr**.

Yna, mae'r llyfrgellydd yn **rhoi 18 llyfr** i ysgol arall.

Rhesymu

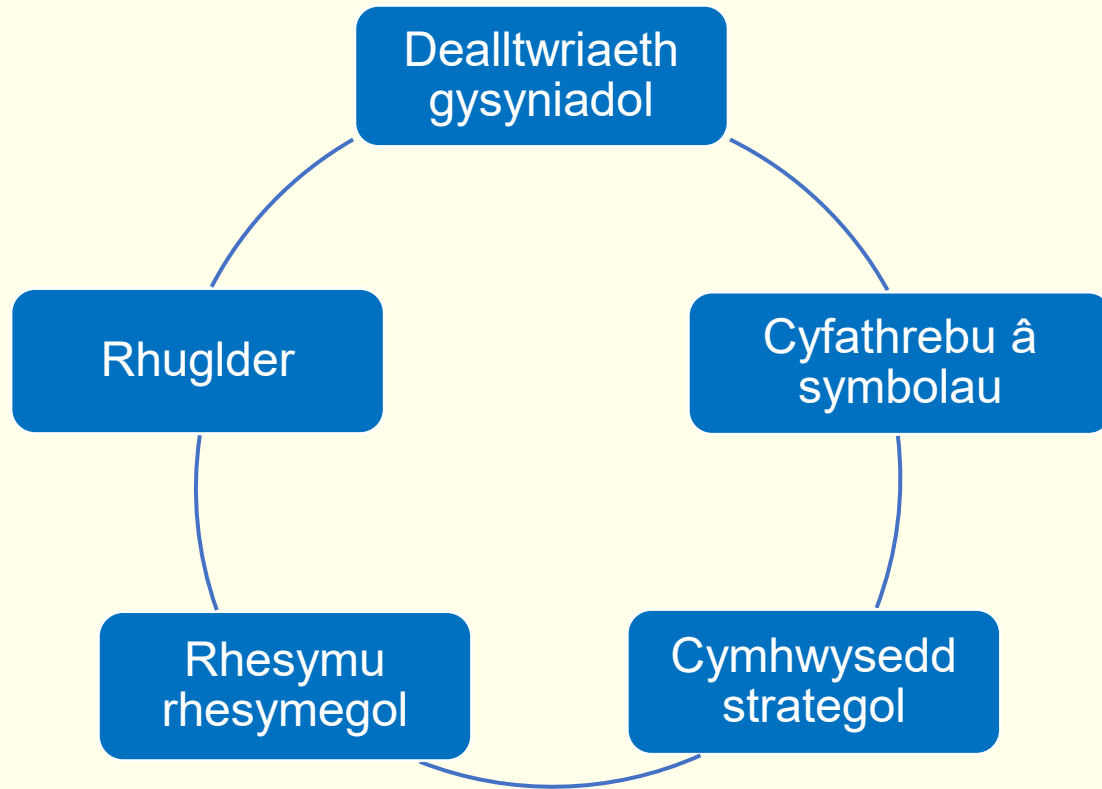
Rhuglder mewn Gweithrediadau Mathemategol

Cwestiwn:

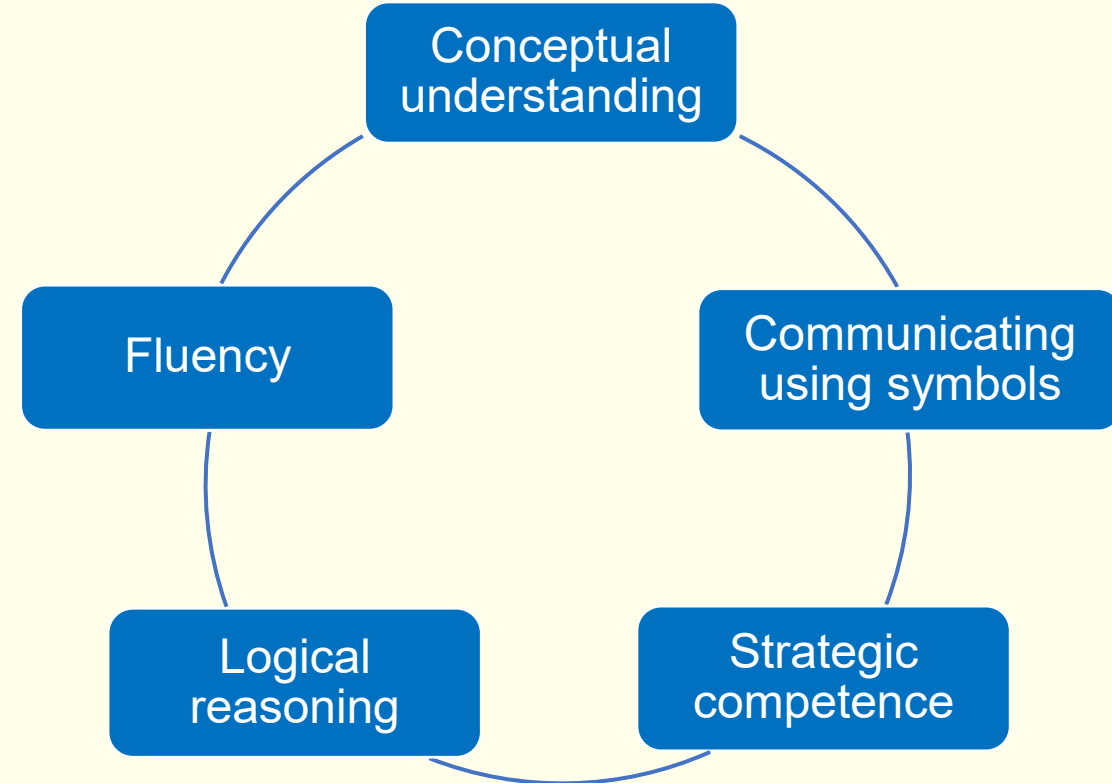
Faint o lyfrau mathemateg sydd **ar ôl** yn y llyfrgell?

Strategaethau Datrys Problemau

Hyfedreddau mathemategol



Mathematical proficiencies



Mathematical concepts and ideas should be built on, deepened and connected as learners experience increasingly complex mathematical ideas. Learners demonstrate conceptual understanding through being able to explain and express concepts, find examples (or *non-examples*) and by being able to represent a concept in different ways, flowing between different representations including verbal, *concrete*, visual, digital and *abstract*.

As learners experience increasingly complex concepts, they should also develop an understanding of the relationships between and within these concepts. They should apply logical reasoning about these relationships and be able to justify and prove them. Justifications and *proof* should become increasingly abstract, moving from verbal explanations, visual or concrete representations to abstract representations involving symbols and conventions.

Can you match the proficiency with its definition?

Learners should understand that the symbols they are using are abstract representations and should develop greater flexibility with the application and manipulation of an increasing range of symbols, understanding the conventions of the symbols they are using.

Learners should become increasingly independent in recognising and applying the underlying mathematical structures and ideas within a problem, in order to develop strategies to be able to solve them.

As learners experience, understand and effectively apply increasingly complex concepts and relationships, fluency in remembering facts, relationships and techniques should grow, meaning that facts, relationships and techniques learned previously should become firmly established, memorable and usable.

Conceptual understanding

Mathematical concepts and ideas should be built on, deepened and connected as learners experience increasingly complex mathematical ideas. Learners demonstrate conceptual understanding through being able to explain and express concepts, find examples (or *non-examples*) and by being able to represent a concept in different ways, flowing between different representations including verbal, *concrete*, visual, digital and *abstract*.

Communication using symbols

Learners should understand that the symbols they are using are abstract representations and should develop greater flexibility with the application and manipulation of an increasing range of symbols, understanding the conventions of the symbols they are using.

Strategic competence

Learners should become increasingly independent in recognising and applying the underlying mathematical structures and ideas within a problem, in order to develop strategies to be able to solve them.

Logical reasoning

As learners experience increasingly complex concepts, they should also develop an understanding of the relationships between and within these concepts. They should apply logical reasoning about these relationships and be able to justify and prove them. Justifications and *proof* should become increasingly abstract, moving from verbal explanations, visual or concrete representations to abstract representations involving symbols and conventions.

Fluency

As learners experience, understand and effectively apply increasingly complex concepts and relationships, fluency in remembering facts, relationships and techniques should grow, meaning that facts, relationships and techniques learned previously should become firmly established, memorable and usable.

Dylid myfyrio ar syniadau a chysyniadau mathemategol, ynghyd â'u datblygu a'u cysylltu â'i gilydd, wrth i ddysgwyr gael profiad o syniadau mathemategol cynyddol gymhleth. Mae dysgwyr yn dangos dealltwriaeth gysyniadol drwy egluro a mynegi cysyniadau, dod o hyd i enghreifftiau (neu anenghreifftiau) a thrwy gynrychioli cysyniad mewn gwahanol ffyrdd, gan gynnwys cynrychioliadau llafar, diriaethol, gweledol, digidol a haniaethol.

Wrth i ddysgwyr brofi cysyniadau cynyddol gymhleth, dylent hefyd feithrin dealltwriaeth o'r berthynas rhwng y cysyniadau hyn ac o fewn y cysyniadau hyn. Dylent gymhwyso dulliau rhesymu rhesymegol wrth ymdrin â'r perthnasau hyn a gallu eu cyfiawnhau a'u profi. Dylai'r cyfiawnhad a'r dystiolaeth ddod yn gynyddol haniaethol, gan symud o esboniadau llafar, gweledol neu ddiriaethol i gynrychioliadau haniaethol sy'n cynnwys symbolau a chonfensiynau.

A allwch chi gydweddu pob hyfedredd â'i ddiffiniad?

Dylai dysgwyr ddeall bod y symbolau y maent yn eu defnyddio yn gynrychioliadau haniaethol a dylent ddatblygu mwy o hyblygrwydd wrth iddynt gymhwyso a defnyddio amrywiaeth gynyddol o symbolau, gan ddeall confensiynau'r symbolau y maent yn eu defnyddio.

Wrth i ddysgwyr brofi, deall a chymhwyso cysyniadau a pherthnasau sy'n gynyddol fwy cymhleth, dylai eu rhuglder wrth gofio ffeithiau, perthnasau a thechnegau wella. O ganlyniad, dylai ffeithiau, perthnasau a thechnegau a ddysgwyd ynghynt fod wedi'u hymsefydlu'n gadarn, yn gofiadwy ac yn ddefnyddiadwy.

Dylai dysgwyr ddod yn gynyddol annibynnol wrth adnabod a chymhwyso'r strwythurau a'r syniadau mathemategol sydd wrth wraidd problem, er mwyn gallu ei datrys.

Dealltwriaeth gysyniadol

Dylid myfyrio ar syniadau a chysyniadau mathemategol, ynghyd â'u datblygu a'u cysylltu â'i gilydd, wrth i ddysgwyr gael profiad o syniadau mathemategol cynyddol gymhleth. Mae dysgwyr yn dangos dealltwriaeth gysyniadol drwy egluro a mynegi cysyniadau, dod o hyd i enghreifftiau (neu anenghreifftiau) a thrwy gynrychioli cysyniad mewn gwahanol ffyrdd, gan gynnwys cynrychioliadau llafar, diriaethol, gweledol, digidol a haniaethol.

Cyfathrebu â symbolau

Dylai dysgwyr ddeall bod y symbolau y maent yn eu defnyddio yn gynrychioliadau haniaethol a dylent ddatblygu mwy o hyblygrwydd wrth iddynt gymhwyso a defnyddio amrywiaeth gynyddol o symbolau, gan ddeall confensiynau'r symbolau y maent yn eu defnyddio.

Cymhwysedd strategol

Dylai dysgwyr ddod yn gynyddol annibynnol wrth nodi a chymhwyso'r strwythurau a'r syniadau mathemategol sydd wrth wraidd problem, er mwyn gallu ei datrys.

Rhesymu rhesymegol

Wrth i ddysgwyr brofi cysyniadau cynyddol gymhleth, dylent hefyd feithrin dealltwriaeth o'r berthynas rhwng y cysyniadau hyn ac o fewn y cysyniadau hyn. Dylent gymhwyso dulliau rhesymu rhesymegol wrth ymdrin â'r perthnasau hyn a gallu eu cyfiawnhau a'u profi. Dylai'r cyfiawnhad a'r dystiolaeth ddod yn gynyddol haniaethol, gan symud o esboniadau llafar, gweledol neu ddiriaethol i gynrychioliadau haniaethol sy'n cynnwys symbolau a chonfensiynau.

Rhuglder

Wrth i ddysgwyr brofi, deall a chymhwyso cysyniadau a pherthnasau sy'n gynyddol fwy cymhleth, dylai eu rhuglder wrth gofio ffeithiau, perthnasau a thechnegau wella. O ganlyniad, dylai ffeithiau, perthnasau a thechnegau a ddysgwyd ynghynt fod wedi'u hymsefydlu'n gadarn, yn gofiadwy ac yn ddefnyddiadwy.