

Question 2.

2 di. $z_1 = 3 + 7j$, $z_2 = 2 - 10j$, $z_3 = -4 - j$

i. $z_1 + z_3 = (3 + 7j) + (-4 - j)$
 $= -1 + 6j$

ii. $z_1 - z_3 = (3 + 7j) - (-4 - j)$
 $= 7 + 8j$

ii. $\frac{z_1}{z_2} = \frac{3 + 7j}{2 - 10j}$

$$= \frac{3 + 7j}{2 - 10j} \times \frac{2 + 10j}{2 + 10j}$$

$$= \frac{(3 + 7j)(2 + 10j)}{(2 - 10j)(2 + 10j)}$$

$$= \frac{6 + 44j + 70j^2}{4 - 100j^2}$$

$$= \frac{6 + 44j + 70(-1)}{4 - 100(-1)}$$

$$= \frac{-64 + 44j}{104}$$

$$= -\frac{64}{104} + \frac{44j}{104} = -\frac{8}{13} + \frac{11j}{26}$$

$$\text{iv. } z_1 - \bar{z}_2 = (3+7j) - (2+10j) \\ = 1-3j$$

$$\text{v. } (z_3)^3 - \bar{z}_1 = (-4-j)^3 - (3-7j) \\ = (-4-j)(-4-j)(-4-j) - (3-7j) \\ = (16+8j+j^2)(-4-j) - (3-7j) \\ = (16+8j+(-1))(-4-j) - (3-7j) \\ = (15+8j)(-4-j) - (3-7j) \\ = (-60-15j-32j-8j^2) - (3-7j) \\ = (-60-47j-8(-1)) - (3-7j) \\ = -52-47j-3+7j \\ = -55-40j$$

$$\text{b. } 8x^2 = -3x - 18$$

$$0 = 8x^2 + 3x + 18$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(3) \pm \sqrt{3^2 - 4(8)(18)}}{2(8)}$$

$$\therefore = \frac{-3 \pm \sqrt{9 - 576}}{16}$$

$$= \frac{-3 \pm \sqrt{-567}}{16}$$

$$= \frac{-3 \pm \sqrt{-1} \sqrt{567}}{16}$$

$$= \frac{-3 \pm 23.81j}{16}$$

$$= \frac{-3 + 23.81j}{16}$$

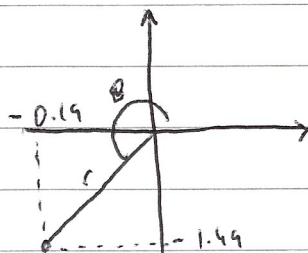
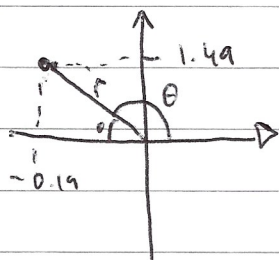
$$= \frac{-3 - 23.81j}{16}$$

$$= \frac{-3}{16} + \frac{23.81}{16}j$$

$$= \frac{-3}{16} - \frac{23.81}{16}j$$

$$= -0.19 + 1.49j$$

$$= -0.19 - 1.49j$$



$$r = \sqrt{0.19^2 + 1.49^2} = 1.50$$

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$$\theta = 180 - \tan^{-1}\left(\frac{1.49}{0.19}\right) = 97.27$$

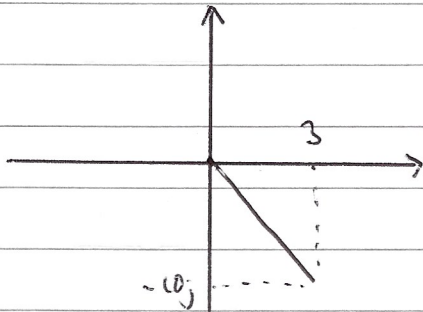
$$\theta = 180 + \tan^{-1}\left(\frac{1.49}{0.19}\right) = 262.73$$

Ans

$$= 1.50 (\cos(262.73) + j \sin(262.73))$$

$$\therefore 1.50 (\cos(97.27) + j \sin(97.27))$$

c. $(3 - 10j)^6$



$$r = \sqrt{3^2 + (-10)^2} = 10.44$$

$$\theta = 360 - \tan^{-1}\left(\frac{10}{3}\right) = 286.70$$

$$r^n (\cos n\theta + j \sin n\theta)$$

$$= (10.44)^6 (\cos(6 \times 286.70) + j \sin(6 \times 286.70))$$

$$= 1294800.40 (\cos(1720.2) + j \sin(1720.2))$$

$$= 229289.48 - 1274337.36j$$

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d. $j^5 \times j^3$

$$= j^8 = j^4 \times j^4 = 1 \times 1 = 1$$

$$= 1$$

e. $z_1 = 9 + 4j$, $z_2 = -7 - 5j$, $z_3 = 2 - 5j$

$$z_1 z_2 \overline{z_3}$$

$$= (9 + 4j)(-7 - 5j)(2 + 5j)$$

$$\therefore = (9-4j)(-7-5j)(2-5j)$$

$$= (-63 - 17j + 20j^2)(2-5j)$$

$$= (-63 - 17j + 20(-1))(2-5j)$$

$$= (-83 - 17j)(2-5j)$$

$$= -166 + 415j ~~42-20j~~ - 34j + 85j^2$$

$$= -166 + 381j + 85(-1)$$

$$= -251 + 381j$$

$j \cdot \bar{X}$ denotes a conjugate of X

Question 3.

ai

$$\int (x^5 - 2x)(5x^3 + 6x) dx$$

$$= \int 5x^8 + 6x^6 - 10x^4 - 12x^2 dx$$

$$= \frac{5x^9}{9} + \frac{6x^7}{7} - \frac{10x^5}{5} - \frac{12x^3}{3} + c$$

$$= \frac{5x^9}{9} + \frac{6x^7}{7} - 2x^5 - 4x^3 + c$$

ii

$$\int (e^{5x} + 1)(e^{3x} + 2) dx$$

$$= \int e^{8x} + 2e^{5x} + e^{3x} + 2 dx$$

$$= \frac{1}{8}e^{8x} + \frac{2}{5}e^{5x} + \frac{1}{3}e^{3x} + 2x + c$$

$$\text{iii} \int_0^2 (3x^3 + 2x)^3 + 4x + 3 \, dx$$

$$\int_0^2 (3x^3 + 2x)(3x^3 + 2x)(3x^3 + 2x) + 4x + 3 \, dx$$

$$\int_0^2 (9x^6 + 6 \cdot 12x^4 + 4x^2)(3x^3 + 2x) + 4x + 3 \, dx$$

$$\int_0^2 (27x^9 + 36x^7 + 12x^5 + 18x^7 + 24x^5 + 8x^3) + 4x + 3 \, dx$$

$$\int_0^2 27x^9 + 54x^7 + 36x^5 + 8x^3 + 4x + 3 \, dx$$

$$= \left[\frac{27x^{10}}{10} + \frac{54x^8}{8} + \frac{36x^6}{6} + \frac{8x^4}{4} + 4x^2 + 3x \right]_0^2$$

$$= \left[\frac{27}{10}x^{10} + \frac{27}{4}x^8 + 6x^6 + 2x^4 + 4x^2 + 3x \right]_0^2$$

$$= \left[\frac{27}{10}(2)^{10} + \frac{27}{4}(2)^8 + 6(2)^6 + 2(2)^4 + 4(2)^2 + 3(2) \right] - \left[\frac{27}{10}(0)^{10} + \frac{27}{4}(0)^8 + 6(0)^6 + 2(0)^4 + 4(0)^2 + 3(0) \right]$$

$$= [2764.8 + 1728 + 384 + 32 + 16 + 6] - [0 + 0 + 0 + 0 + 0 + 0]$$

$$= 4930.8 - 0$$

$$= 4930.8$$

$$\text{iv. } \int_2^4 (2x+3)(x+5) \, dx$$

$$\int_2^4 2x^2 + 13x + 15$$

$$= \left[\frac{2x^3}{3} + \frac{13x^2}{2} + 15x \right]_2^4$$

$$= \left[\frac{2(4)^3}{3} + \frac{13(4)^2}{2} + 15(4) \right] - \left[\frac{2(2)^3}{3} + \frac{13(2)^2}{2} + 15(2) \right]$$

$$= [42.66 + 104 + 60] - [5.33 + 26 + 30]$$

$$= 206.66 - 61.33$$

$$= 145.33$$

b i. $\int 4x \cos 3x \, dx$

$$U = 4x$$

$$UV - \int v \frac{du}{dx}$$

$$V = \frac{1}{3} \sin 3x$$

$$du = 4$$

$$dv = \cos 3x$$

$$= \frac{4}{3} x \sin 3x - \int \left(\frac{1}{3} \sin 3x \right) (4)$$

$$= \frac{4}{3} x \sin 3x - \frac{4}{3} \int \sin 3x$$

$$= \frac{4}{3} x \sin 3x - \frac{4}{3} \left(-\frac{1}{3} \cos 3x \right) + C$$

$$= \frac{4}{3} x \sin 3x + \frac{4}{9} \cos 3x + C$$

ii $\int 3x e^{2x}$

$$U = 3x$$

$$UV - \int v \frac{du}{dx}$$

$$V = \frac{1}{2} e^{2x}$$

$$du = 3$$

$$dv = e^{2x}$$

$$= \frac{3x}{2} e^{2x} - \int \left(\frac{1}{2} e^{2x} \right) (3)$$

$$= \frac{3}{2} x e^{2x} - \frac{3}{2} \int e^{2x}$$

$$= \frac{3}{2} x e^{2x} - \frac{3}{2} \left(\frac{1}{2} e^{2x} \right)$$

$$= \frac{3}{2} x e^{2x} - \frac{3}{4} e^{2x} + C$$

$$c) \int \frac{21x^2 + 10x - 3}{7x^3 + 5x^2 - 3x - 1} dx$$

$$= \ln(7x^3 + 5x^2 - 3x - 1) + C$$

$$ii) \int \frac{8}{3x+10} dx$$

$$= \frac{8}{3} \ln(3x+10) + C$$

$$iii) \int \frac{1}{x^2+9} dx$$

$$= \cancel{\frac{1}{3}} \frac{1}{3} \tan^{-1}\left(\frac{x}{3}\right) + C$$



$$d. \int_{\frac{\pi}{5}}^{\frac{\pi}{2}} \frac{7}{9} \sin x \cos x dx, \text{ let } u = \sin x$$

$$\therefore du = \cos x$$

$$\therefore \frac{dx}{\cos x} = \frac{du}{\cos x}$$

$$\therefore \int_{\frac{\pi}{5}}^{\frac{\pi}{2}} \frac{7}{9} u \cos x \frac{du}{\cos x}$$

$$\int_{\frac{\pi}{5}}^{\frac{\pi}{2}} \frac{7}{9} u \, du$$

$$= \frac{7}{9} \left[\frac{u^2}{2} \right]_{\frac{\pi}{5}}^{\frac{\pi}{2}}$$

$$= \left[\frac{7 (\sin x)^2}{18} \right]_{\frac{\pi}{5}}^{\frac{\pi}{2}}$$

$$= \left[\frac{7 \sin^2\left(\frac{\pi}{2}\right)}{18} \right] - \left[\frac{7 \sin^2\left(\frac{\pi}{5}\right)}{18} \right]$$

$$= \frac{7}{18} (0.38 - 0.13)$$

$$= 0.2858 \approx 0.26$$

Q. $\int \frac{3x}{\sqrt{2x-1}} \, dx$, let $u^2 = 2x-1$

$$\therefore u = \sqrt{2x-1} = (2x-1)^{\frac{1}{2}}$$

$$du = \frac{1}{2}(2x-1)^{-\frac{1}{2}} =$$

$$dx = \frac{du}{\frac{1}{2}(2x-1)^{-\frac{1}{2}}} = \frac{2(2x-1)^{\frac{1}{2}}}{du}$$

$$\int \frac{3x}{u} \cdot \frac{du}{\frac{1}{2}(2x-1)^{\frac{1}{2}}}$$

$$\therefore \frac{3x}{u} \cdot \frac{2(2x-1)^{\frac{1}{2}}}{du}$$