

12:00pm, 6th January

ACTHAWOOD, 1902731

Question One

$$A = \begin{pmatrix} 8 & -1 & 7 \\ -5 & 2 & 5 \\ 3 & 0 & 2 \end{pmatrix}$$

$$B = \begin{pmatrix} -2 & 8 & 0.5 \\ 2 & 5 & 2 \\ 0 & -9 & 1 \end{pmatrix}$$

$$C = \begin{pmatrix} 2 & 5 \\ -6 & 0 \\ 1 & 12 \end{pmatrix}$$

$$D = \begin{pmatrix} 8 \\ 0.25 \end{pmatrix}$$

$$i) AB = \begin{pmatrix} -16 & -2 & 0 & 64 & -5 & \cancel{63} & 4 & -2 & 7 \\ 10 & 4 & 0 & -40 & 10 & -45 & -2.5 & 4 & 5 \\ -6 & 0 & 0 & 24 & 0 & -18 & 1.5 & 0 & 2 \end{pmatrix}$$

$$AB = \begin{pmatrix} -18 & -4 & 9 \\ 14 & -75 & 6.5 \\ -6 & 6 & 3.5 \end{pmatrix}$$

$$ii) BA = \begin{pmatrix} -2 & 8 & 0.5 \\ 2 & 5 & 2 \\ 0 & -9 & 1 \end{pmatrix} \begin{pmatrix} 8 & -1 & 7 \\ -5 & 2 & 5 \\ 3 & 0 & 2 \end{pmatrix}$$

$$BA = \begin{pmatrix} -16 & -40 & 1.5 & 2 & 16 & 0 & -14 & 40 & 1 \\ 16 & -25 & 6 & -2 & 10 & 0 & 14 & 25 & 4 \\ 0 & 45 & 3 & 0 & -18 & 0 & 0 & -45 & 2 \end{pmatrix}$$

$$BA = \begin{pmatrix} -54.5 & 18 & 27 \\ -3 & 8 & 43 \\ 48 & -18 & -43 \end{pmatrix}$$

12:00, 6th January
AC11A1000, 1902731

$$\text{iii) } A = \begin{pmatrix} 8 & -1 & 7 \\ -5 & 2 & 5 \\ 3 & 0 & 2 \end{pmatrix} \quad C = \begin{pmatrix} 2 & 5 \\ -6 & 0 \\ 1 & 12 \end{pmatrix}$$

$R \times C$ $R \times C$
 3×3 3×2

$$\begin{array}{ll} AC = 16 + 6 + 7 & 40 + 0 + 84 \\ -10 + -12 + 5 & -25 + 0 + 60 \\ 6 + 0 + 2 & 15 + 0 + 24 \end{array}$$

$$AC = \begin{pmatrix} 29 & 124 \\ -17 & 35 \\ 8 & 39 \end{pmatrix}$$

$$\text{iv) } CA = \begin{matrix} R \times C \\ 3 \times 2 \end{matrix} \quad \begin{matrix} R \times C \\ 3 \times 3 \end{matrix}$$

Not possible to calculate.

$$\text{v) } A = \begin{pmatrix} 8 & -1 & 7 \\ -5 & 2 & 5 \\ 3 & 0 & 2 \end{pmatrix} \quad \begin{array}{l} |A| = 8(4 - 0) + 1(-10 - 15) + 7(0 - 6) \\ |A| = 32 - 25 - 42 \\ |A| = \underline{35} \end{array}$$

$$\text{vi) } B = \begin{pmatrix} -2 & 8 & 0.5 \\ 2 & 5 & 2 \\ 0 & -9 & 1 \end{pmatrix} \quad \begin{array}{l} |B| = -2(5 + 18) - 8(2 - 0) + 0.5(-18 - 0) \\ |B| = -20 - 16 - 9 \\ |B| = \underline{-45} \end{array}$$

12:00, 6th January
ACHA4000, 1202231

$$\text{Vii) } A+B = \begin{pmatrix} 8 & -1 & 1 \\ -5 & 2 & 5 \\ 3 & 0 & 2 \end{pmatrix} + \begin{pmatrix} -2 & 8 & 0.5 \\ 2 & 5 & 2 \\ 0 & -9 & 1 \end{pmatrix}$$

$$A+B = \begin{pmatrix} 6 & 7 & 1.5 \\ -3 & 7 & 7 \\ 3 & -9 & 3 \end{pmatrix}$$

$$\text{Viii) } AB = \begin{matrix} R \times C \\ 3 \times 3 \end{matrix} \quad \begin{matrix} R \times C \\ 3 \times 3 \end{matrix} + D = \begin{matrix} R \times C \\ 2 \times 2 \end{matrix}$$

Not possible to calculate.

$$\text{b. } \begin{pmatrix} 15x^2 + 4x - 17 & 1 & 11 \\ 2 & 0 & -2 \\ 2 & -2 & x^4 \end{pmatrix} = \begin{pmatrix} y & 1 & 11 \\ 2 & 5 & -2 \\ 2 & -2 & y^2 \end{pmatrix}$$

$$0 = 15x^2 + 4x - 17$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(15)(-17)}}{2(15)}$$

$$x = \frac{-4 \pm \sqrt{16 + 1020}}{30}$$

$$x = \frac{-4 + 32.18}{30}$$

$$x = \frac{-4 - 32.18}{30}$$

$$x = 0.94$$

$$x = -1.206$$

$$c) w = \begin{pmatrix} 2 & 4x & -5 \\ 4 & x & 0 \\ 1 & (x+1) & 3 \end{pmatrix}$$

$$2(3x) - 4x(12) - 5(4x+4) - x = 2$$

$$6x - 48x - 20x - 20 + 5x = 2$$

$$-57x - 20 = 2$$

$$-57x = 2 + 20$$

$$x = \frac{22}{-57} = -0.39$$

$$d) \begin{pmatrix} 2x + 3y + 2z = 14 \\ x + 2y + 5z = 4 \\ 3x + y - 2z = 19 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 3 & 2 \\ 1 & 2 & 5 \\ 3 & 1 & -2 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 14 \\ 4 \\ 19 \end{pmatrix}$$

$$|A| = 2(-4 - 5) - 3(-2 - 15) + 2(1 - 6)$$

$$|A| = -18 + 57 - 10$$

$$|A| = \underline{\underline{29}}$$

$$A_1 = \begin{pmatrix} 14 & 3 & 2 \\ 4 & 2 & 5 \\ 19 & 1 & -2 \end{pmatrix} \quad |A_1| = 14(-4 - 5) - 3(-8 - 95) + 2(4 - 38)$$
$$|A_1| = -126 + 309 - 68$$
$$|A_1| = \underline{\underline{115}}$$

12:00, 6th January
ACT/A 4000, 1902731

Page 5 of 9

$$A_2 = \begin{pmatrix} 2 & 14 & 2 \\ 1 & 4 & 5 \\ 3 & 19 & -2 \end{pmatrix} \begin{array}{l} |A_2| = 2(-8-95) - 14(-2-15) + 2(19-12) \\ |A_2| = -206 + 238 + 14 \\ |A_2| = 46 \end{array}$$

$$A_3 = \begin{pmatrix} 2 & 3 & 14 \\ 1 & 2 & 4 \\ 3 & 1 & 19 \end{pmatrix} \begin{array}{l} |A_3| = 2(38-4) - 3(19-12) + 14(1-6) \\ |A_3| = 68 - 21 - 20 \\ |A_3| = -23 \end{array}$$

$$x = \frac{|A_1|}{|A|}$$

$$y = \frac{|A_2|}{|A|}$$

$$z = \frac{|A_3|}{|A|}$$

$$x = \frac{115}{29}$$

$$y = \frac{46}{29}$$

$$z = \frac{-23}{29}$$

$$x = 3.97$$

$$y = 1.58$$

$$z = -0.79$$

Question Two

$$\text{i) } z_1 + z_3 = (3+7j) + (-4-j) \\ = \underline{-1+6j}$$

$$\text{ii) } z_1 - z_3 = (3+7j) - (-4-j) \\ = \underline{7+8j}$$

$$\text{iii) } \frac{z_1}{z_2} = \frac{3+7j}{2-10j} \times \frac{2+10j}{2+10j} \\ = \frac{(3+7j)(2+10j)}{(2-10j)(2+10j)} = \frac{6+30j+14j+70j^2}{4+20j-20j-100j^2} \\ = \frac{-64+44j}{104} = \frac{64}{104} + \frac{44j}{104} \\ = \underline{0.62+0.42j}$$

$$\text{iv) } z_1 - \bar{z}_2 = (3+7j) - (2+10j) \\ = \underline{1-3j}$$

$$\text{v) } (z_3)^3 - \bar{z}_1 = (-4-j)(-4-j)(-4-j) - (3-7j) \\ = (16+4j+4j+j^2)(-4-j) - (3-7j) \\ = (15+8j)(-4-j) - (3-7j) \\ = (-60-15j-32j-8j^2) - (3-7j) \\ = (-52-47j) - (3-7j) \\ = \underline{-55-40j}$$

$$b) 8x^2 = -3x - 18$$

$$0 = 8x^2 + 3x + 18$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(8)(18)}}{2(8)}$$

$$x = \frac{-3 \pm \sqrt{9 - 576}}{16}$$

$$x = \frac{-3 \pm \sqrt{-1} \times \sqrt{567}}{16}$$

$$x = \frac{-3 \pm j \times 23.81}{16}$$

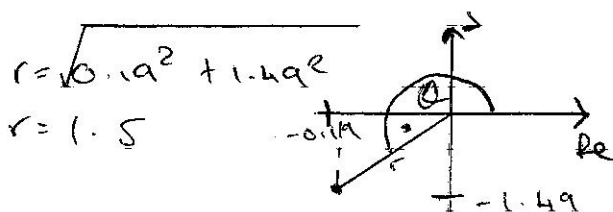
$$x = \frac{-3 - 23.81j}{16}$$

$$x = \frac{-3 + 23.81j}{16}$$

$$x = \frac{-3}{16} - \frac{23.81j}{16}$$

$$x = \frac{-3}{16} + \frac{23.81j}{16}$$

$$x = -0.19 - 1.49j$$



$$\tan \theta = \frac{+1.49}{0.19}$$

$$\theta = \tan^{-1} \left(\frac{1.49}{0.19} \right)$$

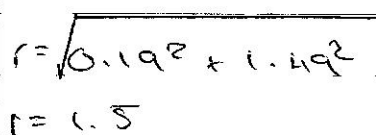
$$\theta = 82.73^\circ$$

$$\theta = 180 + 82.73 \quad \theta = 262.73$$

$$1.5 (\cos 262.73 + j \sin 262.73)$$

$$1.5 (-0.13 - 0.99j)$$

$$x = -0.19 + 1.49j$$



$$\tan \theta = \frac{1.49}{0.19}$$

$$\theta = \tan^{-1} \left(\frac{1.49}{0.19} \right)$$

$$\theta = 82.73^\circ$$

$$\theta = 360 - 82.73$$

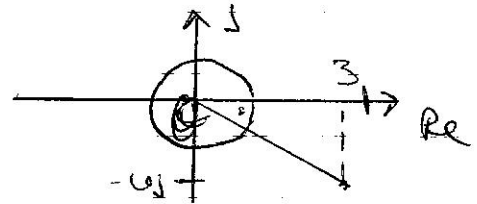
$$\theta = 277.27^\circ$$

$$1.5 (\cos 277.27 + j \sin 277.27)$$

$$1.5 (-0.13 + 0.99j)$$

~~1.5 cos~~

c) i) $(3 - j\omega)^6$



$$r = \sqrt{3^2 + \omega^2}$$

$$r = 10.44$$

$$\tan \theta = \frac{\omega}{3}$$

$$\theta = \tan^{-1} \left(\frac{\omega}{3} \right)$$

$$\theta = 73.3^\circ$$

$$\theta = 360^\circ - 73.3^\circ$$

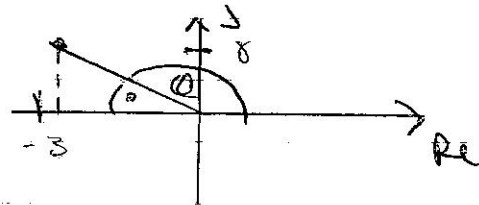
$$\theta = 286.69^\circ$$

$$(10.44)^6 (\cos 6 \times 286.69 + j \sin 6 \times 286.69)$$

$$1294800.89 (0.17 - j 0.98)$$

$$= 220116.15 - j 1243008.85$$

ii) $(-3 + j8)^5$



$$r = \sqrt{3^2 + 8^2}$$

$$r = 8.54$$

$$\tan \theta = \frac{8}{3}$$

$$\theta = \tan^{-1} \left(\frac{8}{3} \right)$$

$$\theta = 180^\circ - 69.44^\circ$$

$$\theta = 110.56^\circ$$

$$\theta = 69.44^\circ$$

$$(8.54)^5 (\cos 5 \times 110.56 + j \sin 5 \times 110.56)$$

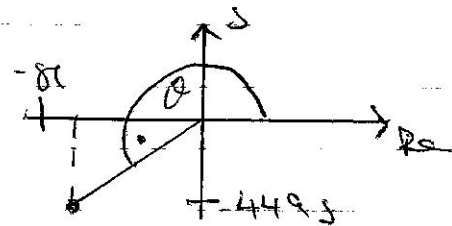
$$4245424.42 (-0.98 - j 0.22)$$

$$-44515.93 - j 9293.37$$

$$d | \sqrt{1} \times -\sqrt{-1}$$

$$\begin{aligned} e) \quad \bar{z}_1 \cdot \bar{z}_2 \cdot \bar{z}_3 &= (9-4j)(-7-5j)(2+5j) \\ &= (-63-45j+28j+20j^2)(2+5j) \\ &= (-83-17j)(2+5j) \\ &= -166-415j-34j-85j^2 \\ &= -81-449j \end{aligned}$$

$$\begin{aligned} r &= \sqrt{81^2 + 449^2} \\ r &= 456.25 \end{aligned}$$



$$\tan \theta = \frac{449}{81}$$

$$\begin{aligned} \theta &= \tan^{-1} \left(\frac{449}{81} \right) = 79.77^\circ \\ \theta &= 180^\circ + 79.77^\circ \\ \theta &= 259.77^\circ \end{aligned}$$

$$\begin{aligned} &456.25 (\cos 259.77^\circ + j \sin 259.77^\circ) \\ &456.25 (-0.18 - 0.98j) \end{aligned}$$

f)