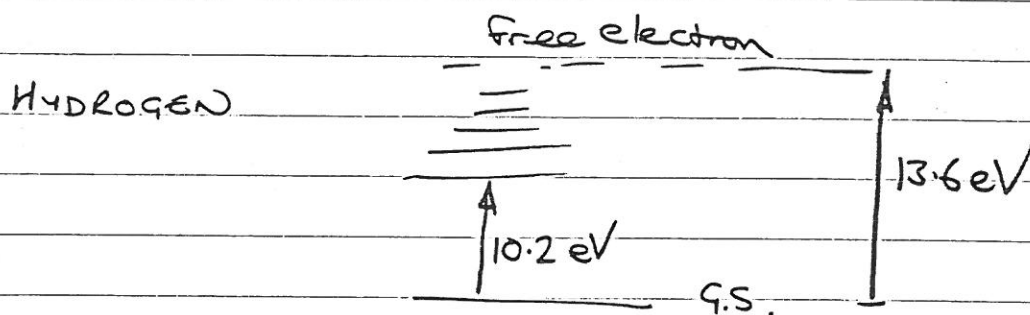


LASERS

Radiation predominantly produced by decelerating electrons.

At optical frequencies for trans. of electrons from one state to another.



He 24 eV

IONISATION ENERGY.

When bang an electron up through the states will get a whole range of emissions as it works its way back down the states.

In a semiconductor electron states are assoc. with whole matrices not single atoms.

$$h\nu = E_2 - E_1 \quad \dots \dots \dots \textcircled{1}$$

~~equiv~~ ~~inside atom~~ ~~oscillating~~. Treat atom as a black box containing osc. ~~or~~ or a collection of oscillators.

How long is an emitted photon?

~~$E_2 - E_1$~~ = ~~energy~~ of exc. st

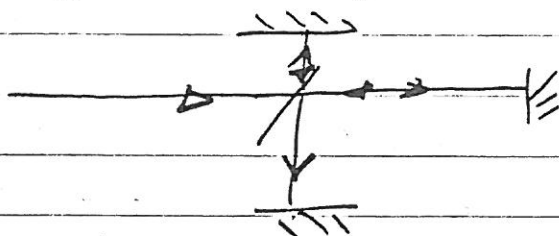
If ~~I~~ know lifetime can calculate it

$$\text{lifetime} \approx 10^{-8} \text{ s}$$

$$\therefore \text{length} \approx 3 \text{ m}$$

How could we measure lengths?

look for interference.



when ~~the~~ path difference between two beams greater than photon length interference will stop.

Due to truncation of photon by F.A. there will be a frequency spread.

$$\Delta t \Delta \nu = 1$$

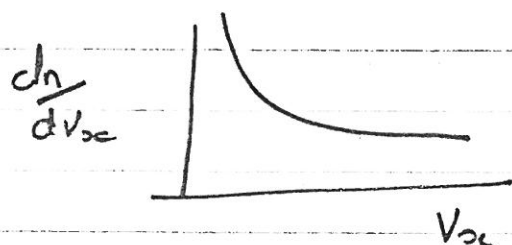
$$\Delta \nu \approx 100 \text{ MHz}.$$

Also DOPPLER As when ~~the~~ radiation emitted freq. depends on the motion of the atom when it emits the atom

$$\begin{array}{l} \text{If } v = 0 \quad \nu_0 = \text{rad freq.} \\ \text{IF } \nu - \nu_0 = \frac{v}{c} \cdot \nu_0 \quad \dots \quad (2) \end{array}$$

Maxwell Boltzmann

$$dn(\nu_{sc}) = \left(\frac{n}{\sqrt{\pi}} \right) \exp - \frac{m \nu_{sc}^2}{2kT} d\nu_{sc}$$

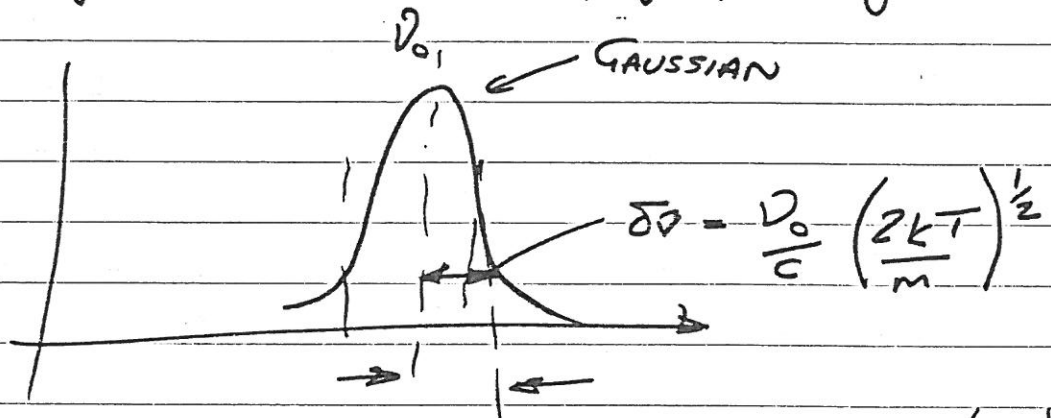


can change into a frequency dist.

using ②

$$dn(\nu) = \frac{n}{\sqrt{\pi}} \exp. -\frac{mc^2}{2kT} \left(\frac{\nu - \nu_0}{\nu_0} \right)^2 \cdot \frac{c}{\nu} d\nu$$

(no emitted in an interval i.e. strength of signal at selected frequency)



$$\Delta\nu = \frac{2\nu_0}{c} \left(\frac{2kT}{m} \right)^{1/2}$$

if take gas at $300\text{ K} = T$

$$\nu_0 = 5 \times 10^{14}$$

$$m = m_{\text{Ne}} = 4 \times 10^{-26} \text{ kg}$$

$$\Delta\nu = 3 \times 10^9 \text{ Hz}$$

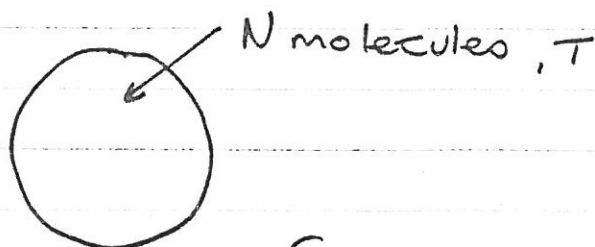
Freq. width observed likely to be doppler width not lifetime width (this is for a gas only)

Eq. DENSITY

$$\rho(\nu) = \frac{8\pi h \nu^3}{c^3} \left(\exp \frac{h\nu}{kT} - 1 \right) d\nu$$

Planck first calc.

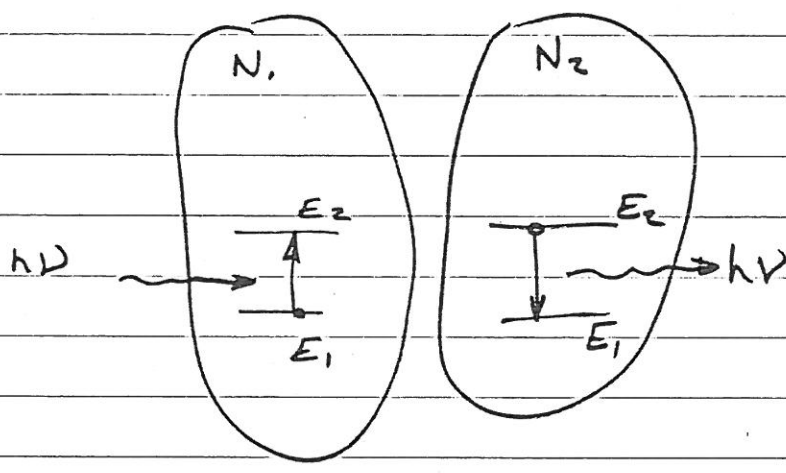
Use this result to understand atom, radiation interaction: —



$$E_z = N_z = N_0 \exp \left(-\frac{E_z - E_0}{kT} \right)$$

etc.

just conc. on two energy states E_2, E_1



only two interactive proc. between E_1 and E_2 and $h\nu$.

No of atoms in each state vnt. constant with time (i.e. steady state)

$A \rightleftharpoons B \rightleftharpoons C$ overall + detailed balance, Detailed balance is a req. of thermodynamic equil.

If derive equations for rates of emission / absorption could derive Plancks black body radiation law.

Rate of spontaneous emission = $N_2 A_{21}$

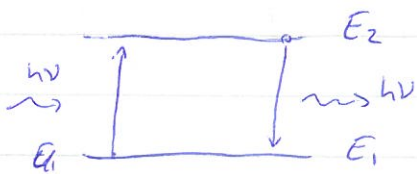
how often a charact. of atom.

Rate of absorption = $N_1 \rho(\nu) B_{12}$

(A) $\therefore N_2 A_{21} = N_1 \rho(\nu) B_{12}$, $\rho(\nu) = \frac{A_{21}}{B_{12}} \frac{N_2}{N_1}$

$= \frac{A_{21}}{B_{12}} \frac{1}{N_1/N_2}$

No of photons at freq prop to energy density



$$N B_{12} \rho(\nu) = N_2 A_{21}$$

Compare with Planck's black body radiation law

$$\rho(\nu) = \frac{8\pi h \nu^3}{c^3} \frac{1}{\left(\exp \frac{h\nu}{kT} - 1\right)} = \left(\right) \frac{1}{\left(\frac{N_1}{N_2} - 1\right)}$$

$$= \left(\right) \frac{N_2}{(N_1 - N_2)}$$

$$N_1 \rho(\nu) - N_2 \rho(\nu) = \left(\right) N_2$$

↑ absorption term ↑ emission term

Comparing,

Missing is an emission term.

- $N_2 \rho(\nu)$

The emission is governed by the photon density $\rho(\nu)$, i.e. a photon knocks the electron from higher to a lower state. This is stimulated emission.

If photon has energy $= (E_2 - E_1)$, it will drive the higher state electron to emit its energy. This is a "resonant" effect - by assuming that the photon is of the resonant frequency of the electron in the state, the emission occurs ~~in sympathy with~~ due to the driving photons. (STIMULATED EMISSION).

There must be a well defined phase relation between incoming and emitted photon: they must be in phase; and, ^{as such} cannot be distinguished, therefore get apparently a photon of $h\nu$ going in, but one of $2h\nu$ coming out.

This may repeat, to get $3h\nu$, $4h\nu$, $5h\nu$ etc. given favourable collisions.

actually call into as $N_1 A_{21} d\nu$ (compensates for $d\nu$ left out in Planck equation)

$$B_{12} N_1 \rho(\nu) = N_2 A_{21} + N_2 B_{21} \rho(\nu)$$

$$\text{then } \rho(\nu) = \frac{N_2 A_{21}}{(B_{12} N_1 + B_{21} N_2)} = \frac{A_{21}}{B_{12} \left(\frac{N_1}{N_2} - 1 \right)}$$

hence agreement with Planck provided $B_{12} = B_{21}$
and $\frac{A_{21}}{B_{12}} = \frac{8\pi h \nu^3}{c^3}$

$$\frac{\text{Rate of spontaneous emission}}{\text{Rate of stimulated emission}} = \frac{N_2 A_{21}}{N_2 B_{21} \rho(\nu)} = \left(\exp\left(\frac{h\nu}{kT}\right) - 1 \right)$$

look at frequency dependence

microwaves $\nu = 10^{10} \text{ Hz}$ ratio = 2×10^{-3}

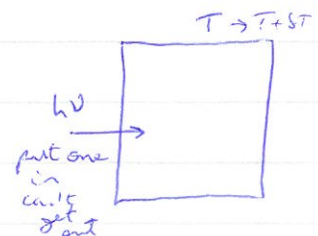
MASER is an amplifier

optical $\nu = 5 \times 10^{14} \text{ Hz}$ ratio = e^{67}

- LASER is an oscillator

$B_{12} = B_{21}$ has important implications

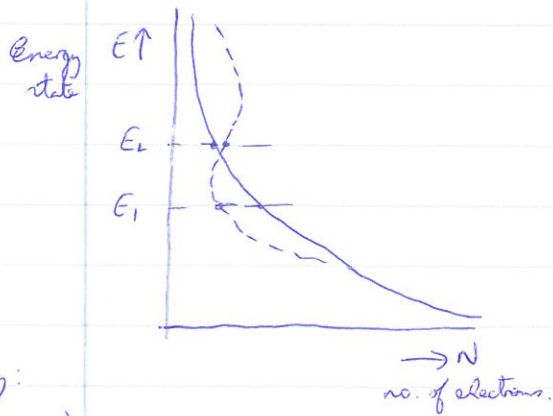
Absorption probability $\propto N_1 B_{12}$
Emission " $\propto N_2 B_{21}$
 $B_{12} = B_{21}$ $N_1 > N_2$ is inequality



To get net growth as in an amplifier, need $N_2 > N_1$
and for amplifier

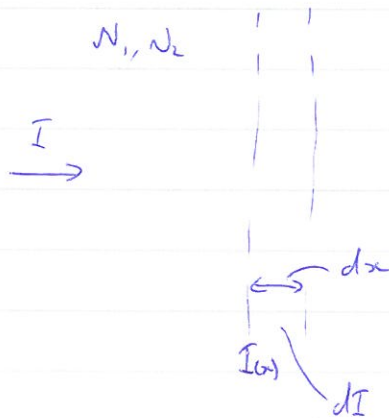
for gain in equilibrium, $\therefore$ you need a $-ve T$ i.e. a negative temperature ($N_2 = N_1 \exp \frac{-h\nu}{kT}$)
Can't be done.

∴ To get gain, what is required is population inversion.
Want a higher no. in E_2 than E_1 .



(reaching:
Try Diagram)

○ (repeated) for gain, need $N_2 > N_1$,
gain at ν , $h\nu = E_2 - E_1$



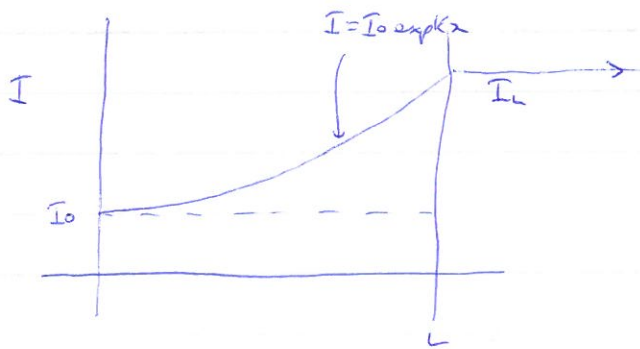
$$dI = -kI dx$$

$$I = I_0 e^{-kx}$$

In case of gain, k -ve

$$I = I_0 \exp kx$$

$\uparrow$
 $F \propto (N_2 - N_1)$



ii



R = reflectivity

for an oscillator, need loop gain = +1

$$\therefore R_1 R_2 e^{2kL_1} \geq 1$$

$$R_1 R_2 = e^{-2\delta}$$

$$\text{so } k \geq \frac{\delta}{L_1}$$

He-Ne $\rightarrow$ red

$$R_1 \approx R_2 \approx 98\%$$

$$L_{\text{min}} \approx 20 \text{ cm.} \quad (\text{typically slightly longer})$$

$$\Rightarrow \delta = 0.02$$

$$\therefore \text{need } k \approx 10^{-3} / \text{cm}$$

but also need in phase, so need ~~stake~~ ^{mirrors} $\sim$ whole
no. of $\frac{1}{2}$ wavelengths apart.

CO_2 typically has $k \approx 50\%$

He-Ne example
shows basically how low
gain can be.

for in phase,

$$\frac{n\lambda_n}{2} = L_2$$

(L_2 = separation of mirrors)
(λ_n = wavelength of n^{th} mode)

$$\lambda = \frac{2L}{n}$$

$$\frac{c}{\nu_n} = \frac{2L}{n}$$

$$\nu_n = \frac{nc}{2L}$$

(frequency of mode)

$\therefore$ what is frequency difference between successive modes

$$\Delta\nu = \nu_{n+1} - \nu_n = \left(\frac{c}{2L} \right)$$

$$\approx 3 \times 10^8 \text{ Hz (for 50 cm)}$$

This frequency is significantly smaller than Doppler-shift frequency therefore there will be several modes available. (typically get three).

in Ne-He



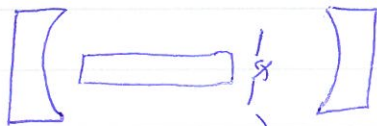
Spectrum analyzer



as sweep voltage, sweep resonant frequency of cavity

If can lock the phase of the various modes relative to each other, can get a well defined pulse train. Called mode-locking. Not done with He-Ne or on three mode, but with say dye-lasers, can get pulse lengths of ~~10~~ 1 ps (10^{-12} s) because because many more modes are present. Get ~~also~~ total energy in pulse, so more modes, shorter pulse, $\therefore$ higher energy density in pulse.

Mode-locking produces pulses. Not all lasers produce pulses. Need a non-linearity in the cavity to produce phase relation.



open & close on ~~an~~ round trip time. Not a real switch, a conceptual switch

if 0.5m length, transit time = $\frac{1}{3 \times 10^8}$ sec = 3 ns

Standard device is an acousto-optic device.

Generally of scientific interest, as can get very high short term powers.

Aside { Is some question as to whether it is better to have a continuous wave or pulse train for communication use.

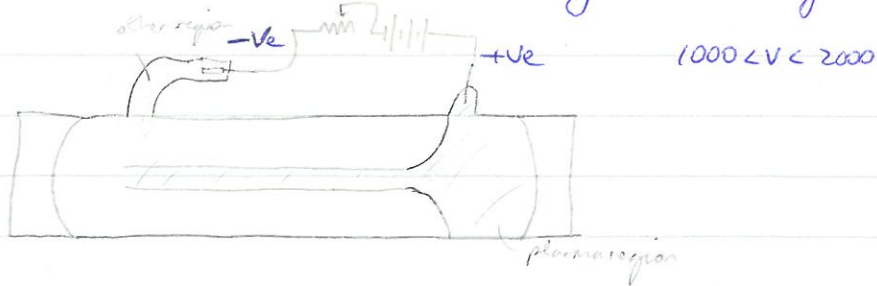
$$+I \rightarrow K \rightarrow N_2 - N_1 \downarrow$$

The no. N_2 will decrease due to the increase in photons $\therefore$ self-limiting gain.

Types of laser

Gas
 Liquid-dye
 Solid
 (Semiconductor)

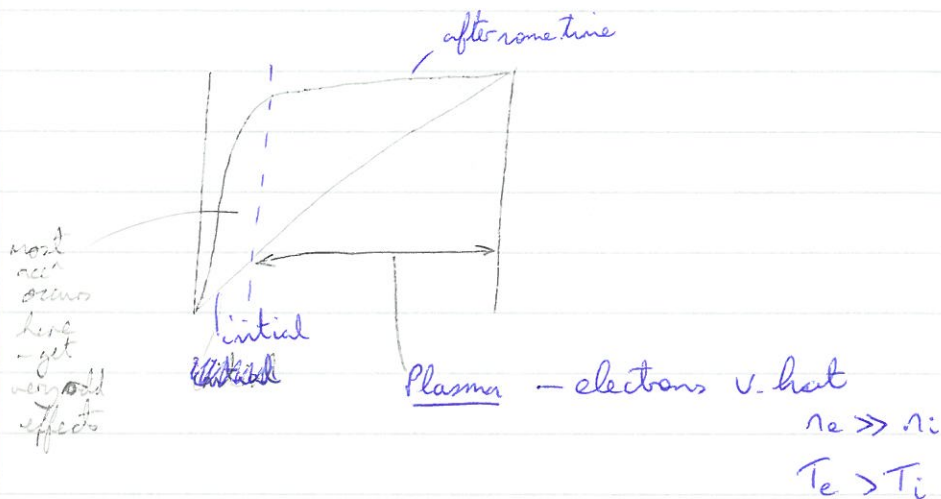
gas - most lasers. Based on gas discharge, Glow discharges



get multiplication of electrons between anode & cathode (due to collision ionisation). mass of ion several thousand times > mass of electron.

∴ electrons don't lose much energy in collision with ions.

Ions take a relatively long time to reach cathode,



Reference: LASER ELECTRONICS

J.T. Verdeyen: Prentice Hall

+ Q switching ↔ Made book p 222-230, Chapter 10 laser excitation; scan chapter 11 (gas discharge), p 352 onwards (quantum view of interaction of atom & field)

Also: Introduction to Opt. elec. Yariv, chap. 7

Also: Lengyel & Siegman

Current flowing through tube is basically controlled by load resistance

$$I \propto n_e$$

∴ have some control over electron density in +ve column

electron temperature

$$T_e \rightarrow E \text{ V/m}$$

can't really effect the field by just changing the applied voltage

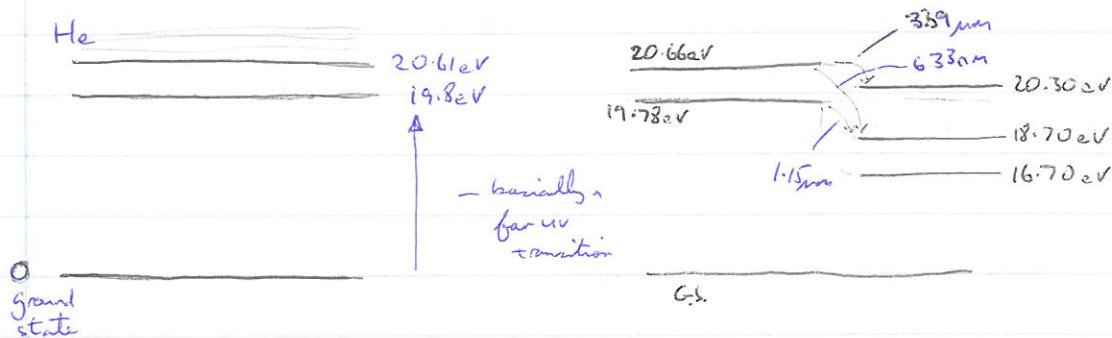
Microscopically

T_e controlled by need for steady state in plasma. To increase T_e , need to increase the rate of electron loss. This gives rise to a need for a higher ionisation rate, hence higher T_e .

Main mode of loss is due to wall recombination, To make wall recombination easier, two methods

- i) Reduce wall recombination (i.e. radius of tube)
- ii) Increase gas pressure so diffusion coefficient goes up, so more wall collisions

He-Ne laser (red)



(for reference kT at room temperature $= 0.03 eV$) Both helium levels are meta-stable: once you get an electron there, it tends to stay there.

Hence if He collides with Ne, will tend to excite Ne to 20.66 eV or 19.78 eV, as well as normal excitation mode. Three possible lasing transitions.

$$\begin{array}{llll} 20.66 \text{ eV} & \text{to} & 20.30 \text{ eV} & = 3.9 \mu\text{m} \\ 19.78 & \text{to} & 18.7 & = 1.15 \mu\text{m} \\ 20.66 & \text{to} & 18.7 & = \text{Red light } 633 \text{ nm} \end{array} \left. \vphantom{\begin{array}{l} 20.66 \text{ eV} \\ 19.78 \\ 20.66 \end{array}} \right\} \text{I.R.}$$

Most electrons go from 18.7 to 16.7, and then to the wall.

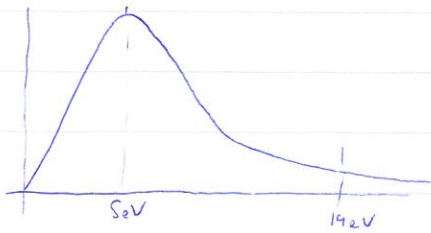
Pressure $\approx 1 \text{ mm Hg}$

tube dia. 1-3 mm

Gas ratio He:Ne = 8:1

Need T_e of approx 5eV

($\approx 50,000$ K) gas temp ≈ 300 K



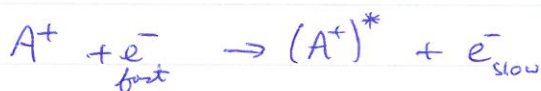
Bad efficiency $\approx 0.01\%$
Energy lost in resistor and cathode fall. Then to maintain energy distribution, only a small quantity have ^{the} 19eV needed for interaction

Output = 5mW, wall dissipation $\approx$ several watts.

Argon Ion laser

Population inversion exists between two states of the Argon ion, not atom. Excitation by.

(practically any gas will lose)



} $\therefore$ need high electron density
+ high field
high $h\nu \rightarrow I \uparrow$

Electron will need to be very fast to excite ion. (ion holds rest of electrons more tightly)
 $\therefore$ high T_e & hence high $E \uparrow$

This means there will be tremendous dissipation in the column
Needs cooling



Tend to use an indirectly heated cathode (thermionic), large, 10's of ~~amps~~ amps.

Dominant lines	488nm	- blue	} 10's of Watts
	514nm	- green	
	350nm	- UV	

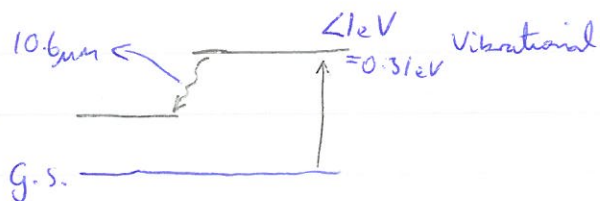
high cost : 10W cost £16,000

Green/blue give higher photon energy (3eV ish)

low efficiency
c/w

CO₂ laser

Since CO₂ is a molecule, can get vibrational excitations at lower levels than excited electronic state



Emission IR (10.6 μm). Must keep gas cool to avoid ramping level. Due to low energy gap, many more electrons can make the jump $\therefore$ much higher efficiency

efficiency 20% .

May give several kW power output. Used in welding, cutting.

Water cooling used to keep temperature low

also get

T.E.A. CO₂

Transverse excited atmospheric pressure $\rightarrow$ gives higher power for smaller & shorter tube.

Don't need gas discharge to excite. Can be done otherwise, say by using large gas collision or causing engines. May get over 100 kW output.

Considered
for weapons
application

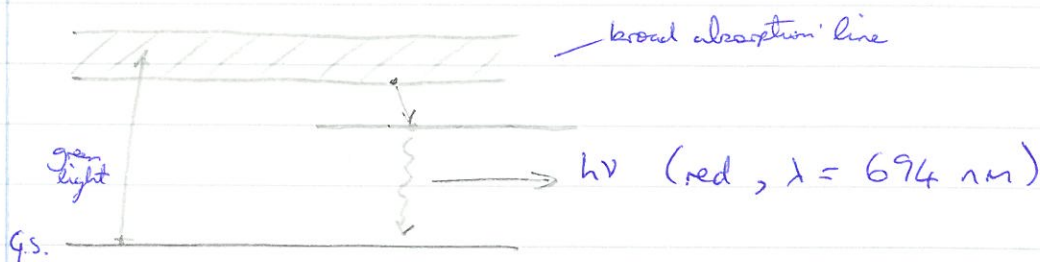
Solid-State Lasers

- ① Ruby
- ② Neodymium - YAG Yttrium Aluminium Garnet (Glass)

Contain relatively small quantities of impurity ions. Impurity ions excited + form population inversion.

- ① Ruby - contains a small amount of chromium in Al_2O_3

Look at energy levels of Cr^{3+} ion



If illuminate with an intense green light, can get more than half the electrons out of the ground state.



Illuminate with a gas discharge flash tube. An enclosing vessel elliptical, all light emitted ^{by flash} goes through the opposite focus. Generally pulsed (discharge tube driven by capacitor - need recharging time). A bit of superseded by YAG.



Called a 3-state laser
(because energy levels involved)

② look at Nd^{3+}



Doubling = harmonic-generating + therefore frequency doubling etc. Harmonic may be up to 50% efficient.

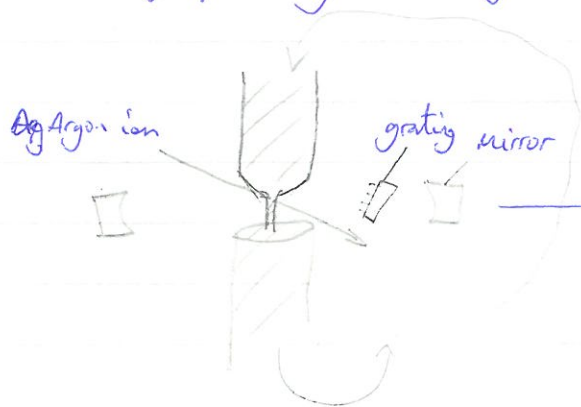
Considered as a control source for thermonuclear fusion. Can get a lot of energy from a neodymium laser. Pulsed. May be ns duration. Can get a moderate laser producing μJ . Large scale may get many joules.

Excitation of the same sort as ruby laser.

Liquid Lasers

Various complicated organic compounds in solution (dyes) can lase. Can get population inversion across very wide bands.

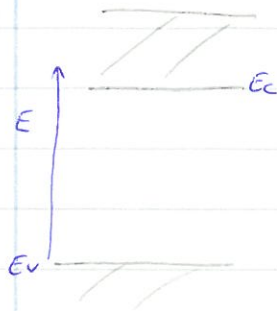
Rhodamine 6G can lase from 500-650 nm : a very wide range of lasing wavelength.



Generally driven by argon ion laser

$\rightarrow$ may get sub-picosecond pulses because of wide bandwidth.

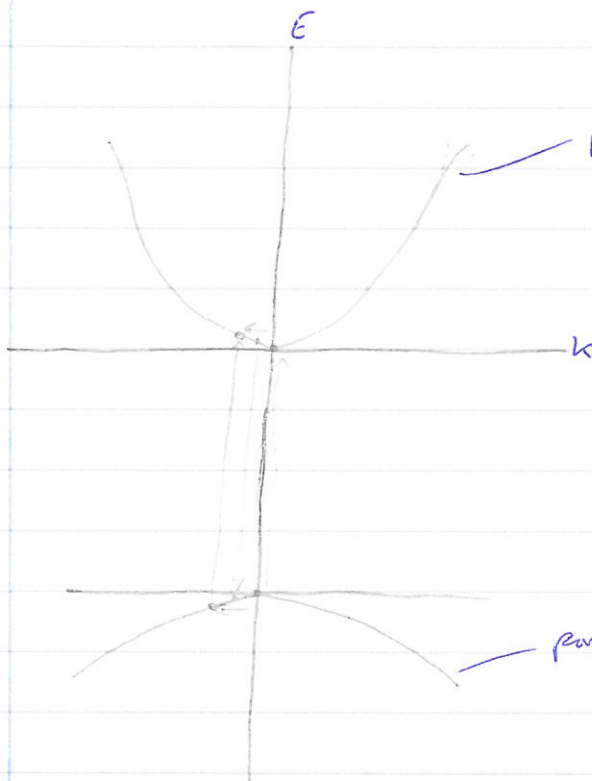
Semiconductor Laser



$$p = \frac{h}{\lambda} = \frac{h}{2\pi} \cdot \frac{2\pi}{\lambda} = \left(\frac{h}{2\pi} \right) k$$

$$E = \frac{p^2}{2m} = \left(\frac{\hbar^2}{2m^*} \right) k$$

Ev vs k



Direct Band gap

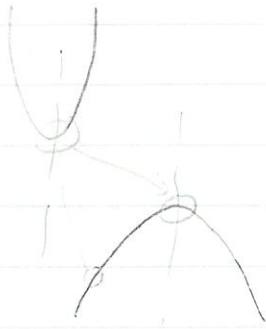
because min E_c is directly above max E_v .

if electron falls from E_c to E_v , ~~very~~ emits a photon = $h\nu$.

k value of electron in valence band = k value of electron in conduction band (vice-versa)

ie changes in energy, but no change in momentum. This is because photon carries very little momentum ($= \frac{h\nu}{c}$), $\therefore$ little k change.

Most are not direct band gap (eg. silicon). This is why there are no silicon or germanium LED, or laser. Recombination occurs on impurities or traps.



Semiconductor lasers thus are direct band gap. Gallium Arsenide (GaAs) is the prime example of semiconductor with direct band gap. Typically radiatively emit $0.8\mu\text{m}$. P-N junctions emit when forward biased.

E-k diagram



GaAs - Direct band gap. $\lambda \approx 0.8\mu\text{m}$

$\hbar\omega$

If look at electron-hole pair operate as a simple harmonic oscillator. Then incoming photon can cause greater oscillation and hence stimulated emission. ie if incoming $h\nu$ matches gap, will cause emission.



Population inversion? Must have sufficient electrons in the conduction band. As increase electron density in conduction band, get greater prob probability of s.e. Defined in terms of Fermi levels. Degenerate level = Fermi level in conduction band (or valence band).



s.e.
= stimulated
emission

For population inversion needs to be doubly degenerate. So cannot achieve in equilibrium.

So need heavily doped ~~semiconductor~~ semiconductor, to get single degeneracy, then heavy forward bias to get sufficient of the other type of carrier* in the region to get second degeneracy.

*non-equilibrium carrier

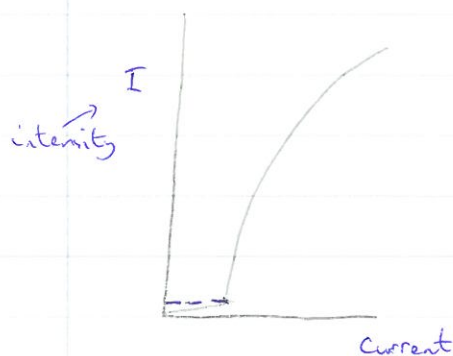
Equilibrium

forward bias



See texts. (Do need strong forward bias).

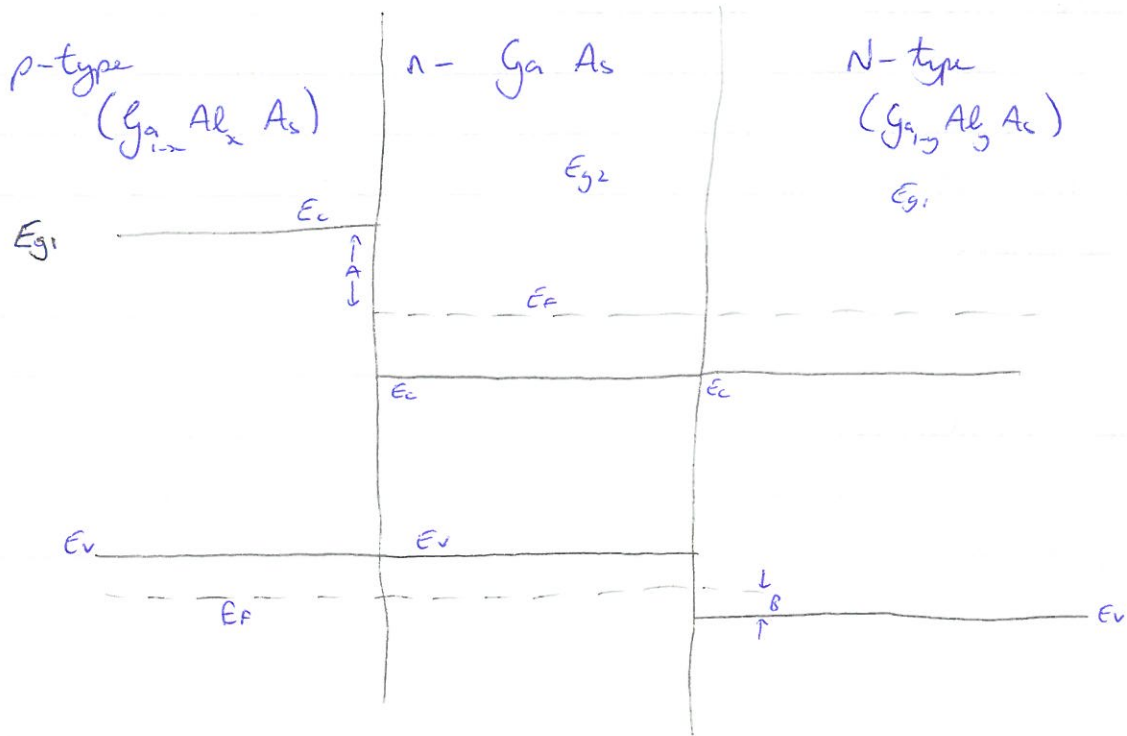
For low forward bias, get an LED. For higher bias, get a lasing effect.



Because of current used in this mode, need to cool the diode in (say) liquid nitrogen. Want therefore to reduce the current needed to cause lasing. The old mode is inefficient.

Can reduce current needed by using double-heterostructure.

Double heterostructure

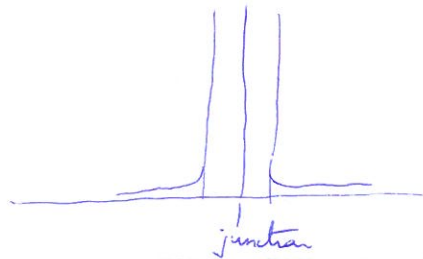


A stops electrons getting into E_c
 B stops holes getting into E_v

So, instead of standard,

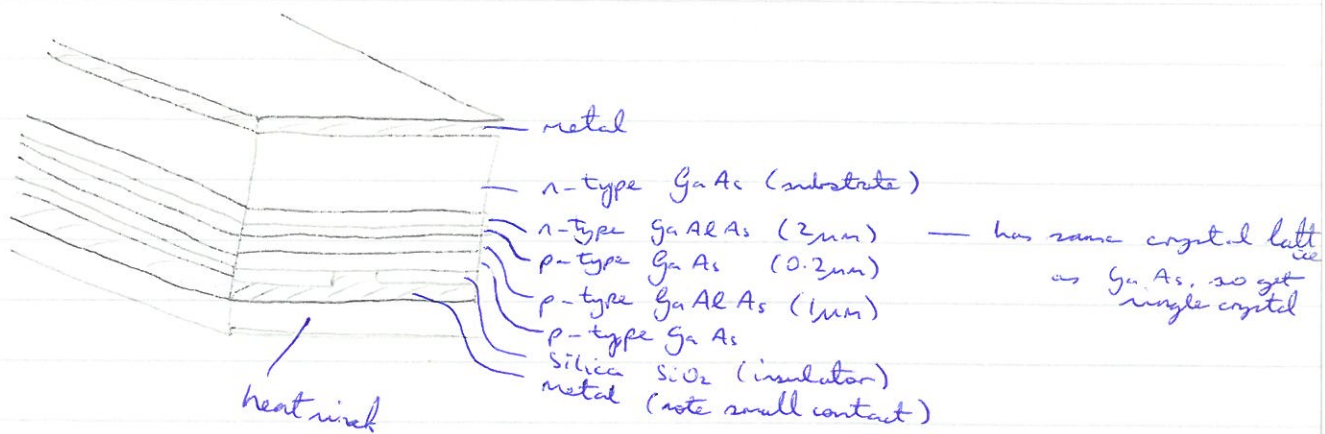


get



ie trap much more carriers in region of junction,
 and so get many more carriers in GaAs by trapping, and hence
 need much less current for lasing.

Also GaAs has a higher refractive index than GaAlAs, so
 tends to keep radiation in by total internal reflection.



Because of narrow current flow, gap can restrict current to a narrow path through the diode.

Note:

- ① Radiation out of very small spot $\therefore$ get wide spreading beam at large angle
- ② Energy levels not so closely defined as He-Ne $\therefore$ not so narrow band of emission : $\frac{1}{2}$ nm spread of λ .
- ③ Light output of GaAs laser directly proportional to current : easy therefore to modulate very quickly (up to 5 GHz). Can't modulate HeNe above kHz.

But: 0.8 μm not really good for communication. Really want 1.3 μm or 1.5 μm .

To achieve this, modern work now uses quaternary compound of Ga, Al, As, P (GaAlAsP). Tailoring is possible to achieve the desired λ (due to E).

Typical power output (c/w) = 85 mW.

Optical Communication

$$\text{Frequency} \approx 5 \times 10^{14} \text{ Hz}$$

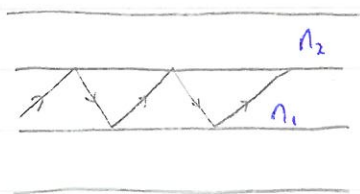
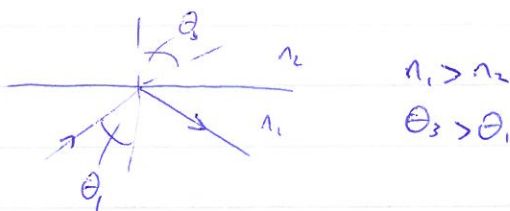
$$\text{one telephone band} = 3 \times 10^3 \text{ Hz}$$

Seems to imply you could get 10" telephone calls down one channel: utter ~~pile~~ bollocks - it ain't that simple.

First tried free-space communication. Many problems. The signal is weather dependent, firstly just rain and fog may interfere, and then differences in air pressure cause changes in refractive index, and hence signal path. So free space is not feasible for civilian communication.

Then tried just waveguides - many problems of loss (conductivity of waveguide not good enough at optical frequency). Then tried using a waveguide with lenses to focus ~~beam~~ beam away from the walls: but highly fragile as any movement of the guide will ruin the direction.

1965 idea came to try as dielectric waveguide, which relies on total internal reflection. Provide incident angle is greater than the critical angle, get all power reflected



— glass
— glass

Using glass in glass in intimate contact, the core is protected from contact with an outside "clasp" with which losses could occur.

First proposed by Kao. at STC.

Two obvious problems:

① Loss

Best loss available in glass in 1965 was 600 dB/km. They calculated that 20 dB/km was needed. ~~Reasonably~~ Reasonably quickly 20 dB/km was obtained. Now best loss $\approx \frac{1}{2}$ dB/km

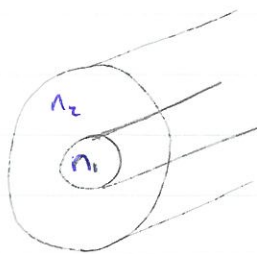
② Dispersion (Way in which pulses broaden)

i.e. $\text{rect} \rightarrow \text{tri}$

Governs data rate. Original guides had data rates ≈ 10 Mbit/sec over a ~~km~~ km. Most of fibre now is ground can handle 140 Mbit/sec over 10 km. Currently 256 Mbit/sec over more than 100 km is technically possible.

Good co-ax can do 140 Mbit/sec over a maximum of 2 km between repeaters.

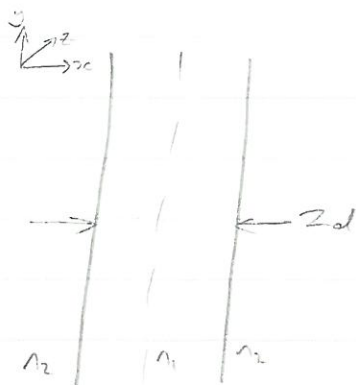
Scheme suggested by Kao was



n_1 high index, n_2 low index glass.

Circular waveguides use cylindrical polar coordinates, and give Bessel function solutions.

So use a slab waveguide for ~~analysis~~ analysis



see sheet

z-direction

Wave of form $\exp j(\omega t - \beta z)$

$\beta = \text{propagation const}$

$$\frac{\partial}{\partial y} = 0 \quad \text{and} \quad \frac{\partial}{\partial z} = -j\beta$$

$$\begin{cases} v_{ph} = \omega/\beta \\ v_g = \frac{d\omega}{d\beta} \end{cases}$$

Now, from Maxwell eqns,

$$\text{Curl } \underline{H} = \frac{\partial \underline{D}}{\partial t} = \epsilon_0 \mu^2 \frac{\partial \underline{E}}{\partial t} \quad (*)$$

$$\text{curl } \underline{E} = -\mu_0 \frac{\partial \underline{H}}{\partial t}$$

from (*)

$$\text{curl } \underline{H} = i \left(\frac{\partial H_x}{\partial y} - \frac{\partial H_y}{\partial z} \right) + j \left(\frac{\partial H_z}{\partial z} - \frac{\partial H_z}{\partial x} \right) + k \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right)$$

$$= i j \beta H_y + j \left(-j \beta H_x - \frac{\partial H_z}{\partial x} \right) + k \frac{\partial H_y}{\partial x}$$

$$= \epsilon_0 \mu^2 j \omega \underline{E}$$

$$= \epsilon_0 \mu^2 j \omega \left[i E_x + j E_y + k E_z \right]$$

so

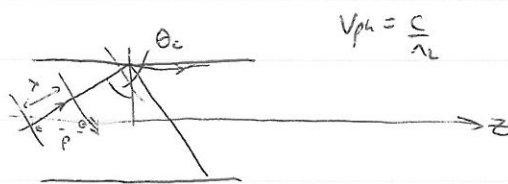
$$\epsilon_0 \mu^2 j \omega E_x = j \beta H_y$$

$$\epsilon_0 \mu^2 j \omega E_y = -j \beta H_x - \frac{\partial H_z}{\partial x}$$

$$\epsilon_0 \mu^2 j \omega E_z = \frac{\partial H_y}{\partial x}$$

and can get 3 more equations from other curl equations.
(see sheet). Rather than 6 eqns in 6 unknowns, actually see that
you get 2 sets of 3 eqns in 3 unknowns i.e. two independent sets.

Physical argument for top of p. 3



$$v_{ph} = \frac{c}{n_2}$$

$$\beta = n_2 k_0$$

$$\beta = n_1 k_0$$

$$p \sin \theta = \lambda$$

$$\beta = \frac{2\pi}{p} = \frac{2\pi}{\lambda} \sin \theta$$

$$\frac{\omega}{k_0} = c \quad \text{in free space}$$

in a material

$$\frac{\omega}{k} = \frac{c}{n}$$

$$\text{from } \frac{\omega}{k_0} = c$$

$$\frac{\omega}{c} = k_0 = \frac{k}{n}$$

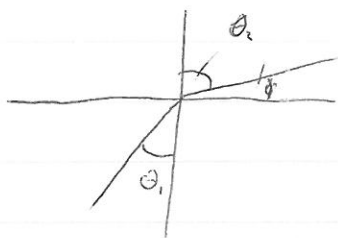
$$\frac{2\pi}{\lambda} = n \frac{2\pi}{\lambda_0}$$

$$\lambda = \frac{\lambda_0}{n}$$

$$\text{see top sheet 3 : } \frac{2\pi}{\lambda} = \beta = \frac{2\pi}{\lambda} \sin \theta$$

$$\beta = \frac{2\pi}{\lambda_0} n_1 \sin \theta$$

$$\beta_{\max} = \frac{2\pi}{\lambda_0} n_1 = k_0 n_1$$



for critical θ_c (i.e. θ_c)
 $\sin \theta_c = \left(\frac{n_2}{n_1} \right)$

Condition $\beta < n_1 k_0$ corresponds to a radiative mode, i.e. not guided.

$\beta > n_1 k_0$ gives a discontinuity at the origin and does not solve ~~div $\underline{B}$~~ ~~div $\underline{H}$~~ is ~~not~~

$$\text{div } \underline{H} = 0$$

$$\text{div } \underline{E} = 0$$

in Maxwell's equations