

$\int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$

Instructions to Candidates

1. Please give the information requested in the boxes below.
2. Enter your name in the space provided to the right, fold over and seal down (using the label provided) to hide your name.
3. The answers must be legibly written. The Examiners can either ignore illegible script or have it typed and the expense of typing charged to the candidate.
4. Write your answers in ink.
5. Begin each answer on a fresh page. Do all rough work in the answer book and cross it through.
6. No part of this book is to be torn out; it must be handed in complete.
7. If more books than one are used, fasten them together using the tags available from the invigilator(s).
8. **Every book must be handed in. In no circumstances may answer books, used or unused, be taken out of the examination venue.**

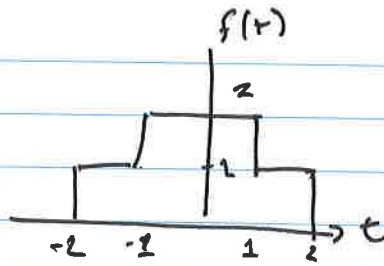
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6	Seat Number	060
7	Number of Answer Books handed in	

List the numbers of the questions you have attempted	Office Use
Q 1	23
Q 2	17
Q 3	19
Q 4	25
29	23
22	17
23	19
24	25
Total	84/100

$\frac{84}{100}$

(Gweler yr ochr arall am y fersiwn Cymraeg)

1) a)



$$f(t) = \begin{cases} 2, & \text{for } -1 < t < 1 \\ 1, & \text{for } -2 < t < -1 \\ 1, & \text{for } 1 < t < 2 \end{cases}$$

$$F(j\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

$$F(j\omega) = \int_{-2}^{-1} 1 e^{-j\omega t} dt + \int_{-1}^1 2 e^{-j\omega t} dt + \int_1^2 1 e^{-j\omega t} dt$$

$$F(j\omega) = \left[\frac{e^{-j\omega t}}{-j\omega} \right]_{-2}^{-1} + \left[\frac{2e^{-j\omega t}}{-j\omega} \right]_{-1}^1 + \left[\frac{e^{-j\omega t}}{-j\omega} \right]_1^2$$

$$F(j\omega) = \frac{1}{-j\omega} \left[\left[\frac{e^{-j\omega t}}{-j\omega} \right]_{-2}^{-1} + \left[\frac{2e^{-j\omega t}}{-j\omega} \right]_{-1}^1 + \left[\frac{e^{-j\omega t}}{-j\omega} \right]_1^2 \right]$$

$$F(j\omega) = \frac{1}{-j\omega} \left[\left[2e^{-j\omega(1)} - 2e^{-j\omega(-1)} \right] + \left[e^{-j\omega(1)} - e^{-j\omega(-2)} \right] \right]$$

$$+ \left[e^{-j\omega(2)} - e^{-j\omega(1)} \right]$$

$$F(j\omega) = \frac{1}{-j\omega} \left[\left[2e^{-j\omega} - 2e^{j\omega} \right] + \left[e^{j\omega} - e^{-2j\omega} \right] + \left[e^{-2j\omega} - e^{-j\omega} \right] \right]$$

$$F(j\omega) = \frac{1}{-j\omega} \left[2(e^{-j\omega} - e^{j\omega}) + (e^{j\omega} - e^{-2j\omega}) + (e^{-2j\omega} - e^{-j\omega}) \right]$$

$$F(j\omega) = \frac{1}{-j\omega} \left[\frac{4}{2}(e^{-j\omega} - e^{j\omega}) + \frac{2}{2}(e^{j\omega} - e^{-2j\omega}) + \frac{2}{2}(e^{-2j\omega} - e^{-j\omega}) \right]$$

$$F(j\omega) = \frac{1}{j\omega} \left[4 - \frac{4}{2} (e^{-j\omega} - e^{j\omega}) - \frac{2}{2} (e^{j\omega} - e^{-j\omega}) - \frac{2}{2} (e^{-2j\omega} - e^{2j\omega}) \right]$$

$$F(j\omega) = \frac{1}{j\omega} \left[\frac{4}{2} (e^{j\omega} - e^{-j\omega}) - \frac{2}{2} (e^{j\omega} - e^{-j\omega}) + \frac{2}{2} (e^{2j\omega} - e^{-2j\omega}) \right]$$

$$\sin(\omega) = \frac{e^{j\omega} - e^{-j\omega}}{2j}$$

$$F(j\omega) = \frac{1}{\omega} \left[4 \left(\frac{e^{j\omega} - e^{-j\omega}}{2j} \right) - 2 \left(\frac{e^{j\omega} - e^{-j\omega}}{2j} \right) + 2 \left(\frac{e^{2j\omega} - e^{-2j\omega}}{2j} \right) \right]$$

$$F(j\omega) = \frac{1}{\omega} \left[4\sin(\omega) - 2\sin(\omega) + 2\sin(2\omega) \right]$$

$$\text{Sinc}(\omega) = \frac{\sin(\omega)}{\omega}$$

$$F(j\omega) = \frac{2\sin(\omega)}{\omega} + \frac{2\sin(2\omega)}{\omega} \quad \leftarrow \times \frac{2}{2} \quad (1)$$

$$F(j\omega) = \frac{2\sin(\omega)}{\omega} + \frac{4\sin(2\omega)}{2\omega}$$

$$F(j\omega) = 2\text{sinc}(\omega) + 4\text{sinc}(2\omega)$$

C

$$1) b) \quad F(j\omega) = \frac{1}{\omega^2 + 1} e^{-\frac{2\omega^2}{\omega^2 + 1}} \quad f(t-2)e^{jt} \quad f(t-2)e^{jt}$$

$$t_0 = 2$$

$$f(t-t_0) \xleftrightarrow{FT} F(j\omega) e^{-j\omega t_0}$$

$$\frac{f(t-t_0)}{F(j\omega)} = \frac{1}{\omega^2 + 1} e^{-\frac{2\omega^2}{\omega^2 + 1}} \cdot e^{-j2\omega}$$

$$f(t) e^{j\omega_0 t} \leftrightarrow F(j\omega - j\omega_0) \quad \omega_0 = 1$$

$$\frac{F(j\omega - j\omega_0)}{F(j\omega)} = \frac{1}{(\omega-1)^2 + 1} e^{-\frac{2(\omega-1)^2}{(\omega-1)^2 + 1}} \cdot e^{-j2(\omega-1)}$$

B

$$f(t-2)e^{jt} \xleftrightarrow{FT} = \frac{1}{(\omega-1)^2 + 1} e^{-\frac{2(\omega-1)^2}{(\omega-1)^2 + 1}} \cdot e^{-j2(\omega-1)}$$

23
25

$$P_1 = 1 + 2 + 3 + 1 + 3 + 3 + 2 + 1 + 2 + 3 + 2 = 23$$

2)a) DFT

$$x(n) = \begin{cases} \frac{1}{5}, & \text{for } -1 \leq n \leq 1 \\ 0, & \text{otherwise} \end{cases} \quad \checkmark \quad \underline{N=3}$$

$$k = 0, 1, 2, \quad \underline{\underline{N=3}}$$

$$F(\omega) = \sum_{n=0}^{N-1} x(n) e^{-j\omega n}$$

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\omega n}$$

$$X(k) = \frac{1}{5} e^{-j\omega(-1)} + \frac{1}{5} e^{-j\omega(0)} + \frac{1}{5} e^{-j\omega(1)}$$

$$X(k) = \frac{1}{5} e^{j\omega} + \frac{1}{5} + \frac{1}{5} e^{-j\omega}$$

$$X(k) = \frac{1}{5} (1 + (e^{j\omega} + e^{-j\omega})) \quad 2\cos(\omega) = e^{j\omega} + e^{-j\omega}$$

$$X(k) = \frac{1}{5} (1 + 2\cos(\omega)) \quad \omega = \frac{2\pi k}{N}$$

$$X(k) = \frac{1}{5} \left(1 + 2\cos\left(\frac{2\pi \times 0}{3}\right) \right) + \frac{1}{5} \left(1 + 2\cos\left(\frac{2\pi \times 1}{3}\right) \right) + \frac{1}{5} \left(1 + 2\cos\left(\frac{2\pi \times 2}{3}\right) \right)$$

$$X(k) = \frac{1}{5} (1 + 2) + \frac{1}{5} \left(1 + 2\left(-\frac{1}{2}\right) \right) + \frac{1}{5} \left(1 + 2\left(-\frac{1}{2}\right) \right)$$

$$\frac{1}{5} (3) + \frac{1}{5} (0) + \frac{1}{5} (0)$$

$$\underline{\underline{X(k) = \frac{3}{5}}}$$

5)i)
$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n} d\omega$$

- Impulse Response
- Casual or non-casual
- No of taps
- Utilised in SDK Digital Filtering
- Selectivity / drop-off

b)ii)
$$x(n) = [12.5, 8, 8, 12.5] \quad n = 0, 1, 2, 3$$

Hamming Window

$$N=4$$

$$w(n) = 0.54 - 0.46 \cos\left(\frac{2\pi n}{N-1}\right)$$

$$w(0) = 0.54 - 0.46 \cos\left(\frac{2\pi(0)}{3}\right)$$

$$0.54 - 0.46(1) = 0.08$$

$$w(1) = 0.54 - 0.46 \cos\left(\frac{2\pi \times 1}{3}\right)$$

$$0.54 - 0.46 \cos\left(\frac{2\pi}{3}\right) = 0.77$$

$$w(2) = 0.54 - 0.46 \cos\left(\frac{2\pi \times 2}{3}\right) = 0.77$$

$$w(3) = 0.54 - 0.46 \cos\left(\frac{2\pi \times 3}{3}\right) = 0.08$$

$$y(n) = w(n) \times x(n)$$

$$y(0) = 0.08 \times 12.5 = 1$$

$$y(1) = 0.77 \times 8 = 6.16$$

$$y(2) = 0.77 \times 8 = 6.16$$

$$y(3) = 0.08 \times 12.5 = 1$$

$$y(n) = [1, 6.16, 6.16, 1]$$

$$P_2 = 1 + 4 + 1 + 1 + 4 + 1 + 4 + 1 = 19$$

$$3) a) i) A \cos(2\pi f t + \phi)$$

$$s(t) = 3 \cos(2\pi(1200)t + \frac{\pi}{6})$$

$$L_{O_{in}} = 2 \cos(2\pi(1150)t)$$

$$L_{O_q} = -2 \sin(2\pi(1150)t)$$

$$\cos A \cos B = \frac{1}{2} [\cos(A+B) + \cos(A-B)]$$

$$\sin A \cos B = \frac{1}{2} [\sin(A+B) - \sin(A-B)]$$

$$I(t) = \frac{6 \times 2}{2} \left[\cos(2\pi(2350)t + \frac{\pi}{6}) + \cos(2\pi(50)t + \frac{\pi}{6}) \right]$$

$$Q(t) = \frac{6 \times 2}{2} \left[\sin(2\pi(2350)t + \frac{\pi}{6}) - \sin(2\pi(50)t + \frac{\pi}{6}) \right]$$

LPF

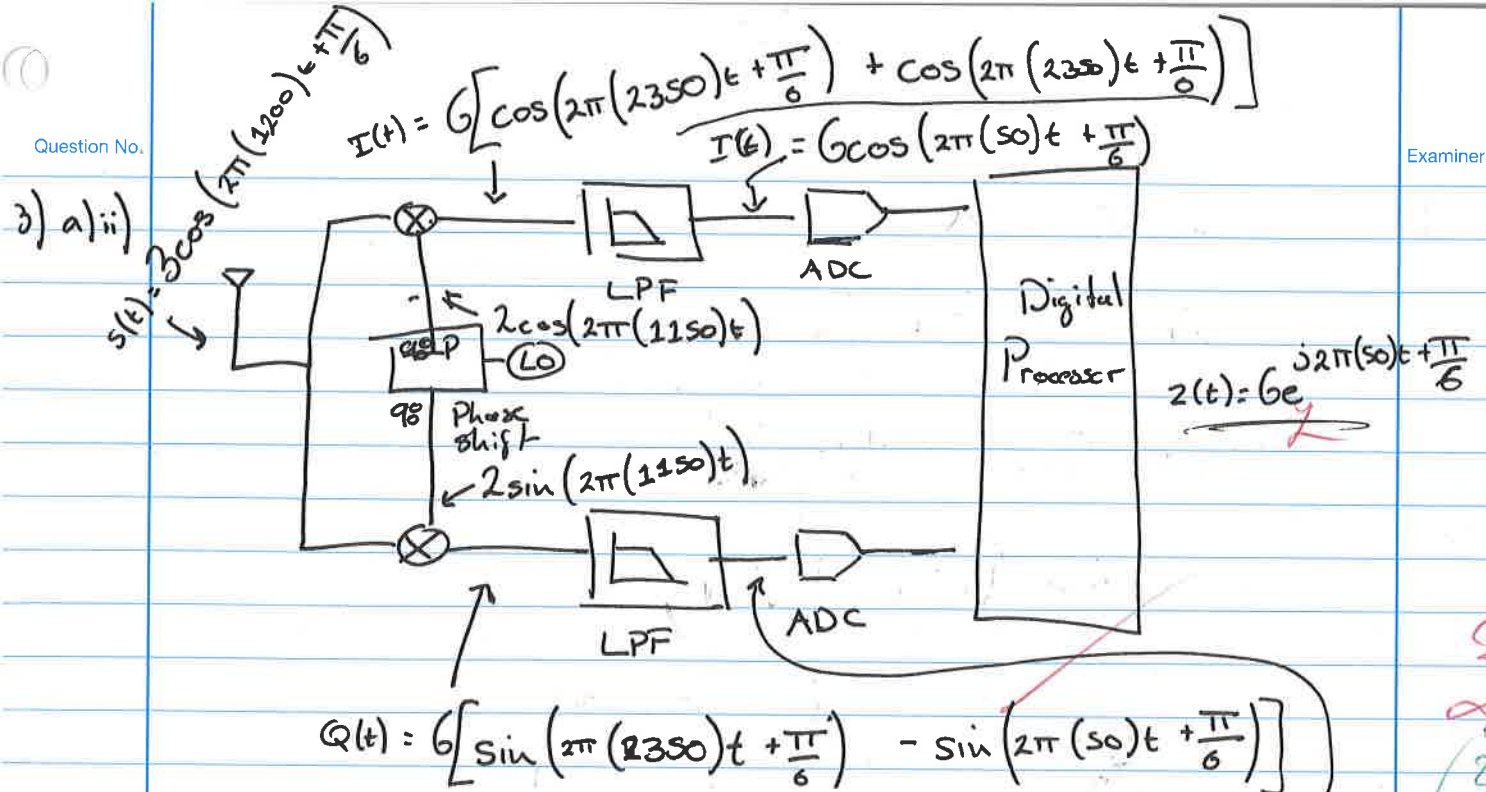
$$I'(t) = 6 \left[\cos(2\pi(50)t + \frac{\pi}{6}) \right]$$

$$Q'(t) = 6 \left[\sin(2\pi(50)t + \frac{\pi}{6}) \right]$$

$$z(t) = 6 \cos(2\pi(50)t + \frac{\pi}{6}) + 6j \sin(2\pi(50)t + \frac{\pi}{6})$$

$$z(t) = \cancel{6e^{j2\pi(50)t + \frac{\pi}{6}}} \cancel{6e^{j2\pi(50)t + \frac{\pi}{6}}}$$

$$z(t) = 6e^{j2\pi(50)t + \frac{\pi}{6}}$$



3) a) iii) When $S(t)$ mixes with $I(t)$ & $Q(t)$ it creates two replicas of the signal at sum and difference frequencies (2350Hz & 50Hz). The LPF rejects the high frequency leaving only 50Hz.
 3) a) iii) Interference 1230Hz Low freq 50Hz variant

3) b) i)

$$A \cos(2\pi f t + \alpha) = \cos(2\pi(1230)t)$$

RF I

$$\cos A \cos B$$

$$R_{FI} S(t) = 6 \left[\cos(2\pi(2430)t + \frac{\pi}{6}) + \cos(2\pi(30)t + \frac{\pi}{6}) \right]$$

passed through LPF

$$L_{FI} = 2 \left[\cos(2\pi(2380)t) + \cos(2\pi(80)t) \right]$$

3) b) ii) LPF Rejects 2380Hz & depending on the cut-off frequency of the LPF would either allow or reject 80Hz. the message signal is at 50Hz. Ideally Reject 80Hz

(2)

3)b)i) and so a good ~~the~~ LPF design would be close to 50Hz. ~~IF~~ the LPF does not reject the 80Hz it will ~~be~~ sum with message creating interference & degrade ~~the~~ message signal/quality. If it does then the interference would be rejected keeping the clean message signal.

2

1

3)b)iii) ~~2)b)i)~~ → Have sharp LPF drop-off from 50Hz → 60Hz to attenuate and reject the interference at 80Hz from degrading the message signal.

→ Forward error correction if signal is digital to help recover from any loss due to interference

→ Higher Gain Antenna - pointed towards transmitter → Better received signal

→ LNA Before Mixer to increase signal strength

→ Band-pass filter after LPF for greater selectivity

→ ~~Adapt~~ more Robust modulation schemes such as FHSS.

1

19/25

$$Q_3 = 3 \times 2^2 + 1 + 3 + 2 + 1 + 1 + 2 + 1 + 1 = 19$$

4

4)a)i) DIT FFT $y(n) = [1, 2, 3, 4, 4, 3, 2, 1]$

even: $[x(0), x(2), x(4), x(6)]$

odd: $[x(1), x(3), x(5), x(7)]$

even = $[1, 3, 4, 2]$

odd = $[2, 4, 3, 1]$

4)a)ii)

Twiddle factor = $W_N^k = e^{-j \frac{2\pi k}{N}}$

$N = 8$

$k = 0, 1, 2, 3$

$W_8^0 = e^{-j \frac{2\pi \times 0}{8}} = 1$

$W_8^1 = e^{-j \frac{2\pi}{8}} = 0.707 - 0.707j$

$W_8^2 = e^{-j \frac{4\pi}{8}} = -j$

$W_8^3 = e^{-j \frac{6\pi}{8}} = -0.707 - 0.707j$

4)b) $Y(0) = 20$

$Y(1) = -5.828 - 2.414j$

$Y(2) = 0$

$Y(3) = -0.172 - 0.414j$

$Y(4) = 0$

$Y(5) = -0.172 + 0.414j$

$Y(6) = 0$

$Y(7) = -5.828 + 2.414j$

$Y(k) = [20, -5.828 - 2.414j, 0, -0.172 - 0.414j, 0, -0.172 + 0.414j, 0, -5.828 + 2.414j]$

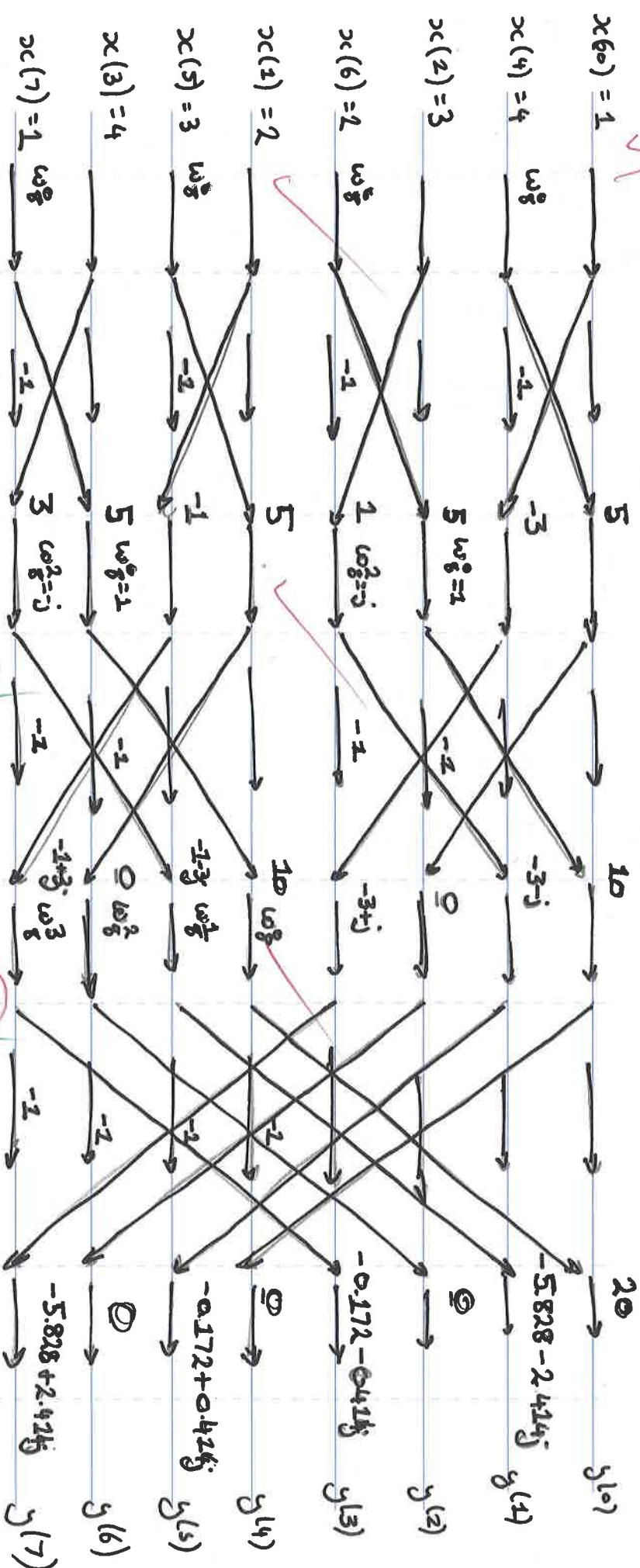
$Q_4 = 2 + 2 + 4 + 5 = 25$

4) 5) DIT FFT

Student ID = 2114762



Power = 0



$$W_5^0 = 1$$

$$W_5^1 = 0.707 - 0.707j$$

$$W_5^2 = -j$$

$$W_5^3 = -0.707 - 0.707j$$