



Examination Answer Books

12 Pages

Instructions to Candidates

1. Please give the information requested in the boxes below.
2. Enter your name in the space provided to the right, fold over and seal down (using the label provided) to hide your name.
3. The answers must be legibly written. The Examiners can either ignore illegible script or have it typed and the expense of typing charged to the candidate.
4. Write your answers in ink.
5. Begin each answer on a fresh page. Do all rough work in the answer book and cross it through.
6. No part of this book is to be torn out; it must be handed in complete.
7. If more books than one are used, fasten them together using the tags available from the invigilator(s).
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1	Student Number	2114210
2	Title of Examination Paper	Advanced Digital Signal processing System
3	Module Code	ACDA6018
4	Date and Time of Examination	12:00 08/01/2025
5	Examination Venue	IQ 310
6	Seat Number	063
7	Number of Answer Books handed in	

List the numbers of the questions you have attempted	Office Use
Q1	22
Q2	19
Q3	16
Q4	25
Q1	22
Q2	19
Q3	16
Q4	25
Total	82/100

1.) a)
$$f(t) = \begin{cases} 1 & -2 \leq t \leq -1 \\ 2 & -1 \leq t \leq 1 \\ 1 & 1 \leq t \leq 2 \end{cases}$$

$$\begin{aligned}
 F(j\omega) &= \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt \\
 &= \int_{-2}^{-1} e^{-j\omega t} dt + 2 \int_{-1}^1 e^{-j\omega t} dt + \int_1^2 e^{-j\omega t} dt \\
 &= \frac{1}{-j\omega} \left[\left[e^{-j\omega t} \right]_{-2}^{-1} + 2 \left[e^{-j\omega t} \right]_{-1}^1 + \left[e^{-j\omega t} \right]_1^2 \right] \\
 &= \frac{1}{-j\omega} \left[e^{j\omega} - e^{j2\omega} + 2e^{-j\omega} - 2e^{j\omega} + e^{-j2\omega} - e^{-j\omega} \right] \\
 &= \frac{1}{-j\omega} \left[-e^{j\omega} - e^{j2\omega} + e^{-j\omega} + e^{-j2\omega} \right] \\
 &= \frac{1}{j\omega} \left[e^{j2\omega} - e^{j\omega} + e^{-j\omega} - e^{-j2\omega} \right] \\
 &= \frac{2}{\omega} \left[\frac{e^{j2\omega} - e^{-j2\omega}}{2j} + \frac{e^{j\omega} - e^{-j\omega}}{2j} \right] \\
 &= \frac{2}{\omega} [\sin(2\omega) + \sin(\omega)] \\
 &= 2 [\sin(2\omega) + \sin(\omega)] \\
 &= \frac{2\sin(2\omega)}{\omega} + \frac{2\sin(\omega)}{\omega} \\
 &= \frac{4\sin(2\omega)}{2\omega} + \frac{2\sin(\omega)}{\omega} \\
 &= 4 \operatorname{sinc}(2\omega) + 2 \operatorname{sinc}(\omega) \\
 &= \text{option C}
 \end{aligned}$$

2.
②

1.
①

3.
③

1.
①

3.
③

3.

2.
②

1.) b.) $F(j\omega) = \frac{1}{\omega^2 + 1} e^{-\frac{2\omega^2}{\omega^2 + 1}}$

$f(t-t_0) \Rightarrow F(j\omega) e^{-j\omega t_0}$

$f(t-2) \Rightarrow F(j\omega) e^{-j2\omega} = \frac{1}{\omega^2 + 1} e^{-\frac{2\omega^2}{\omega^2 + 1}} e^{-j2\omega}$ (1)

$f(t) e^{j\omega_0 t} \Rightarrow F(j\omega - j\omega_0)$

$f(t) e^{jt} \Rightarrow F(j\omega - j)$ (2)

~~$f(t-2) e^{jt} = \frac{1}{(\omega+1)^2 + 1} e^{-\frac{2(\omega+1)^2}{(\omega+1)^2 + 1}} e^{-j2\omega}$~~

~~$= \frac{1}{\omega^2 + 2\omega + 2} e^{-2}$~~

Using (1) and (2):

$f(t-2) e^{jt} \Rightarrow \frac{1}{(\omega-1)^2 + 1} e^{-\frac{2(\omega-1)^2}{(\omega-1)^2 + 1}} e^{-j2(\omega-1)}$

= option B

$Q_1 = 2+1+3+1+3+5+5+2 = 22$

22/25

2) a)



$$x(n) = \begin{cases} \frac{1}{5} & -1 \leq n \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

$$N=3$$

$$x(n) = \left\{ \frac{1}{5}, \frac{1}{5}, \frac{1}{5} \right\} \quad \text{for } n=-1, 0, 1$$

$$X(k) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} \quad \text{at } \omega = \frac{2\pi k}{N} \quad k=0, 1, 2$$

$$X(k) = \sum_{n=-\infty}^{\infty} x(n) e^{-j \frac{2\pi k n}{3}}$$

$$= \frac{1}{5} e^{-j(0)} + \frac{1}{5} e^{-j \frac{2\pi k}{3}} + \frac{1}{5} e^{-j \frac{2\pi k}{3}}$$

$$X(k) = \frac{1}{5} e^{+j \frac{2\pi}{3} k} + \frac{1}{5} e^{(0)} + \frac{1}{5} e^{-j \frac{2\pi}{3} k}$$

$$= \frac{1}{5} \left[1 + e^{j \frac{2\pi}{3} k} + e^{-j \frac{2\pi}{3} k} \right]$$

$$X(k) = \frac{1}{5} \left[1 + 2 \cos\left(\frac{2\pi k}{3}\right) \right]$$

$$X(0) ?$$

$$X(1) ?$$

$$X(k) = \{ \} ? \quad X(2) ?$$

1 -

(1)

✱

(4)

1 -

(5)

b.) i.) The problem with ideal FIR filters is that they are non-causal and infinite in length.

☆

The length can be made finite by truncating, and that does not effect the amplitude of the signal, but it does produce oscillations at the pass band.

These ~~sensors~~ filters could be used in communications or RADAR

ii.) $x(n) = \{12.5, 8, 8, 12.5\}$ $N=4$

$$w(n) = 0.54 - 0.46 \cos\left(\frac{2\pi n}{N-1}\right)$$

$$w(0) = 0.54 - 0.46 \cos\left(\frac{2\pi(0)}{3}\right) = 0.08$$

$$w(1) = 0.54 - 0.46 \cos\left(\frac{2\pi(1)}{3}\right) = 0.77$$

$$w(2) = 0.54 - 0.46 \cos\left(\frac{2\pi(2)}{3}\right) = 0.77$$

$$w(3) = 0.54 - 0.46 \cos\left(\frac{2\pi(3)}{3}\right) = 0.08$$

$$w(n) = \{0.08, 0.77, 0.77, 0.08\}$$

$$y(n) = x(n) * w(n)$$

$$= \{12.5 \times 0.08, 8 \times 0.77, 8 \times 0.77, 12.5 \times 0.08\}$$

$$\underline{y(n) = \{1, 6.16, 6.16, 1\}}$$

$$Q_2 = 1 + 5 + 2 + 1 + 4 + 6 = 19$$

3.)
a)S(t):

$A = 3$

$f = 1200$

$\varphi = \frac{\pi}{6}$

LO:

$A_b = 2$

$f = 1150$

i.)

$$S(t) = A \cos(2\pi f_m t + \varphi)$$

$$= 3 \cos(2\pi(1200)t + \frac{\pi}{6})$$

$$\underline{S(t) = 3 \cos(2400\pi t + \frac{\pi}{6})}$$

LO:

I_n-phase $2 \cos(2\pi(1150)t)$

Q-phase $-2 \sin(2\pi(1150)t)$

$$I(t) = S(t) \cdot LO_{I\text{-phase}}(t)$$

$$= 3 \cos(2400\pi t + \frac{\pi}{6}) \cdot 2 \cos(2300\pi t)$$

$$= 6 \times \frac{1}{2} [\cos(2400\pi t + \frac{\pi}{6} + 2300\pi t) + \cos(2400\pi t + \frac{\pi}{6} - 2300\pi t)]$$

$$\underline{I(t) = 3 [\cos(4700\pi t + \frac{\pi}{6}) + \cos(100\pi t + \frac{\pi}{6})]}$$

$$Q(t) = S(t) \cdot LO_{Q\text{-phase}}(t)$$

$$= 3 \cos(2400\pi t + \frac{\pi}{6}) \cdot -2 \sin(2300\pi t)$$

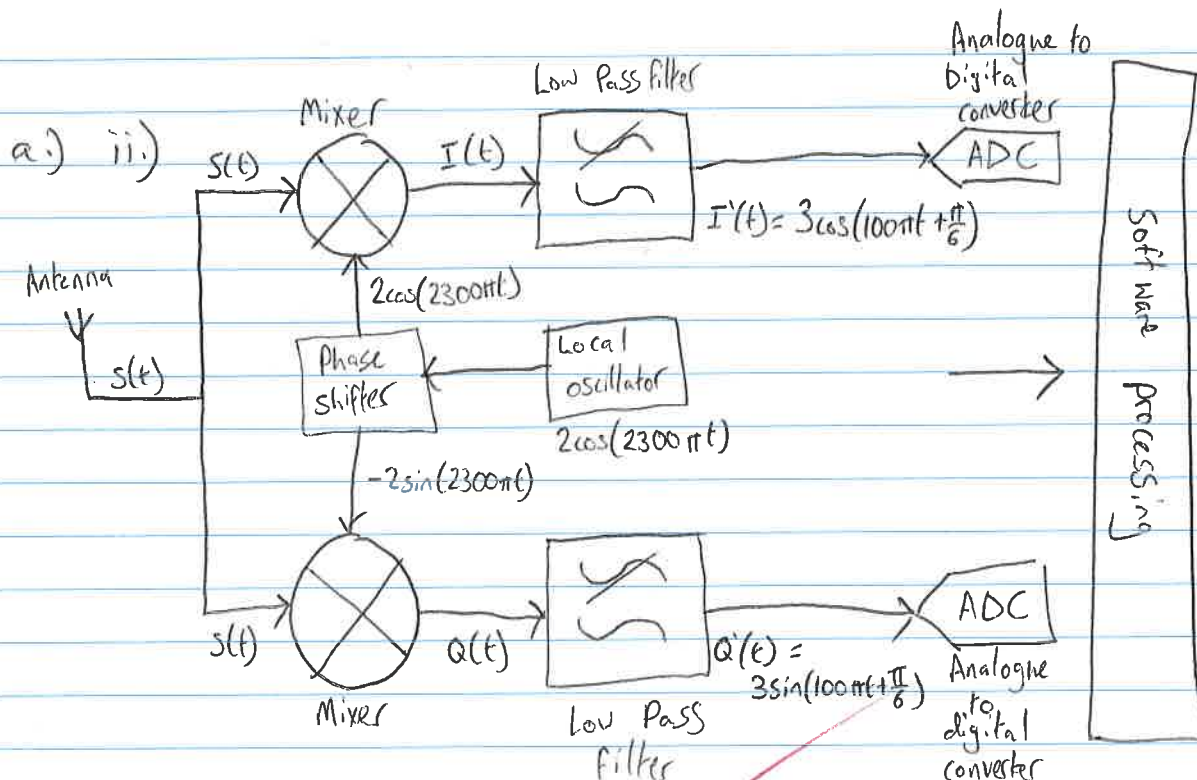
$$= -6 \times \frac{1}{2} [\sin(2400\pi t + \frac{\pi}{6} + 2300\pi t) - \sin(2400\pi t + \frac{\pi}{6} - 2300\pi t)]$$

$$\underline{Q(t) = -3 [\sin(4700\pi t + \frac{\pi}{6}) - \sin(100\pi t + \frac{\pi}{6})]}$$

$$Z(t) = I(t) + jQ(t)$$

$$\underline{Z(t) = 3 (\cos(4700\pi t + \frac{\pi}{6}) + \cos(100\pi t + \frac{\pi}{6})) - 3j (\sin(4700\pi t + \frac{\pi}{6}) - \sin(100\pi t + \frac{\pi}{6}))}$$

3.)



For values of $I(t)$, $Q(t)$, and $s(t)$, please refer to answer for Q 3, A, i

iii.) The signal $s(t)$ is first captured on the antenna, and is multiplied by the carrier wave and -90° phase shift of the carrier wave ($2\cos(2300\pi t)$ and $-2\sin(2300\pi t)$ respectively). This produces the In-phase and Quadrature-phase waves ($I(t)$ and $Q(t)$).

Both of these are passed through a low-pass filter, removing the high frequency components, leaving only the baseband signals ($I'(t)$ and $Q'(t)$).

These are converted to digital form by Analogue to Digital Converters so that they can both be processed digitally.

3.) b.) i.)

$$I_{\text{interference high}} = 1230 + 1150 \text{ Hz} = 2380 \text{ Hz}$$

$$I_{\text{interference low}} = 1230 - 1150 \text{ Hz} = 80 \text{ Hz}$$

The lower frequency will be retained in the baseband after the low pass filter because its frequency is below the pass band.

ii.) The output frequency with interference will be lower than the output frequency without interference.

iii.) The output ^{signal} filter must be filtered with an effective band-pass filter with ~~N=4~~ a high Q-factor ($N \geq 4$).

$$Q_3 = 3 + 1 + 2 + 4 + 2 + 4 = 16$$

1

1

1

1

(4)

$$\frac{16}{25}$$

4

4.) a) $y(n) = \{1, 2, 3, 4, 4, 3, 2, 1\}$

i) $y_{even}(n) = \{1, 3, 4, 2\}$

$y_{odd}(n) = \{2, 4, 3, 1\}$

$y_{new}(n) = \{1, 4, 3, 2, 2, 3, 4, 1\}$

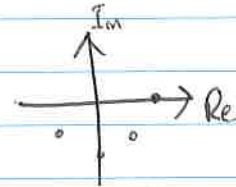
ii) $W_8^0 = e^{-j0} = 1$

$W_8^1 = e^{-j\frac{\pi}{4}} = 0.707 - j0.707$

$W_8^2 = e^{-j\frac{\pi}{2}} = -j$

$W_8^3 = e^{-j\frac{3\pi}{4}} = -0.707 - j0.707$

$W_N^k = e^{-j\frac{2\pi}{N}k} \quad W_8^k = e^{-j\frac{\pi}{4}k}$



b) Please see answer sheet for butterfly diagram

$Y(0) = 10 + 10 = 20$

$Y(1) = -3 - j + (-1 - 3j)(0.707 - j0.707)$
 $= -3 - j - 0.707 + j0.707 - j2.121 - 2.121$
 $= -5.828 - 2.414j$

$Y(2) = 0 + 0 = 0$

$Y(3) = -3 + j + (-1 + 3j)(-0.707 - j0.707)$
 $= -3 + j + 0.707 + j0.707 - 2.121j + 2.121$
 $= -0.172 - j0.414$

$Y(4) = 10 - 10 = 0$

$Y(5) = -3 - j - (-1 - 3j)(0.707 - j0.707)$
 $= -3 - j - (-0.707 + j0.707 - j2.121 - 2.121)$
 $= -3 - j + 0.707 - j0.707 + j2.121 + 2.121$
 $= -0.172 + j0.414$

$Y(6) = 0 + 0 = 0$

$Y(7) = -3 + j + (-1 + 3j)(0.707 + j0.707)$
 $= -3 + j - 0.707 - j0.707 + j2.121 - 2.121$
 $= -5.828 + j2.414$

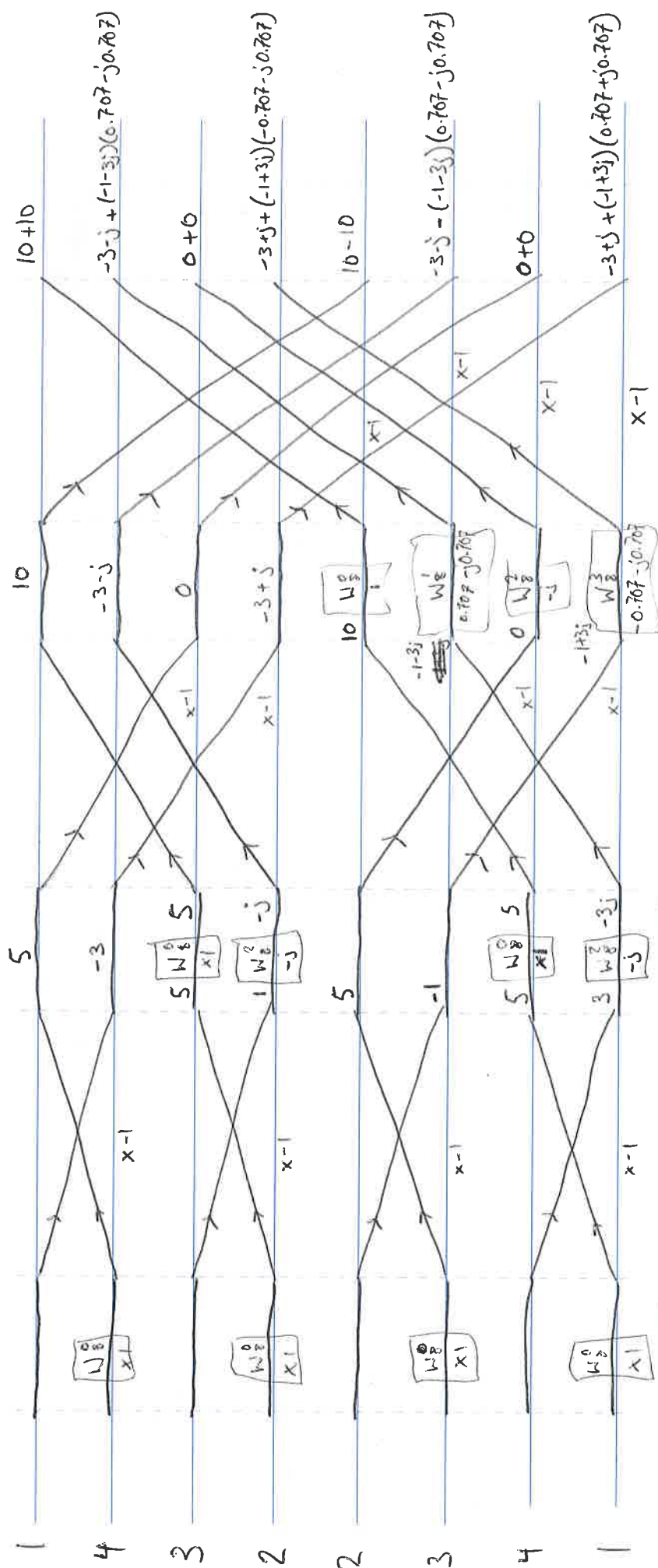
$Y(k) = \{20, -5.828 - j2.414, 0, -0.172 - j0.414, 0, -0.172 + j0.414, 0, -5.828 + j2.414\}$

25
25

16

$25 = 2^2 + 2 + 16 + 5 = 25$

②



④

