



Examination Answer Books

12 Pages

Instructions to Candidates

1. Please give the information requested in the box.
2. Enter your name in the space provided to the right, for the invigilator to see, and seal down (using the label provided) to hide your name.
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1	Student Number	2111738
2	Title of Examination Paper	Advanced Digital Signal processing
3	Module Code	ACDA6018
4	Date and Time of Examination	8 th Jan 2025 12:00
5	Examination Venue	1Q 310
6	Seat Number	064
7	Number of Answer Books handed in	1

List the numbers of the questions you have attempted	Office Use
Q1	22
Q2	19
Q3	14
Q4	17
Q1	22
Q2	19
Q3	17
Q4	14
Total	72/100

72/100

PL 1
11/01/25

1 a $F(j\omega) = 2\text{Sinc}(\omega) + 4\text{Sinc}(2\omega)$

$$f(t) = \begin{cases} 1, & -2 \leq t \leq -1 \\ 2, & -1 \leq t \leq 1 \\ 1, & 1 \leq t \leq 2 \end{cases}$$

$$F(j\omega) = \int_{-2}^{-1} e^{-j\omega t} dt + \int_{-1}^1 2e^{-j\omega t} dt + \int_1^2 e^{-j\omega t} dt$$

$$= \left[\frac{e^{-j\omega t}}{-j\omega} \right]_{-2}^{-1} + \left[\frac{2e^{-j\omega t}}{-j\omega} \right]_{-1}^1 + \left[\frac{e^{-j\omega t}}{-j\omega} \right]_1^2$$

$$= \frac{e^{-j\omega(-2)} - e^{-j\omega(-1)}}{-j\omega} + \frac{2e^{-j\omega(1)} - 2e^{-j\omega(-1)}}{-j\omega} + \frac{e^{-j\omega(2)} - e^{-j\omega(1)}}{-j\omega}$$

$$= \frac{1}{-j\omega} [e^{j2\omega} - e^{j\omega} + 2e^{-j\omega} - 2e^{j\omega} + e^{-j2\omega} - e^{-j\omega}]$$

$$= \frac{1}{-j\omega} [e^{j2\omega} + e^{-j2\omega} - e^{j\omega} - e^{-j\omega} + 2e^{-j\omega} - 2e^{j\omega}]$$

$$= \frac{1}{-j\omega} \cdot \frac{2}{2} [e^{j2\omega} + e^{-j2\omega} - e^{j\omega} - e^{-j\omega} - 2e^{j\omega} + 2e^{-j\omega}]$$

$$= \frac{1}{-j\omega} \left[\frac{2(e^{j2\omega} + e^{-j2\omega})}{2} \right]$$

~~At this point~~

$$= \frac{1}{\omega} \left[\frac{2(e^{j2\omega} - e^{-j2\omega})}{2j} + \frac{2(e^{j\omega} - e^{-j\omega})}{2j} + \frac{2 \cdot 2(e^{-j\omega} - e^{j\omega})}{2j} \right]$$

$$= \frac{1}{\omega} (2\text{Sinc}(2\omega) + 2\text{Sinc}(\omega) + 4\text{Sinc}(\omega))$$

$$= \frac{1}{\omega} (2\text{Sinc}(2\omega) + 2\text{Sinc}(\omega))$$

$$= \frac{1}{\omega} (2\text{Sinc}(2\omega) + 2\text{Sinc}(\omega))$$

$$= \frac{1}{\omega} \left(\frac{2}{2} \cdot 2\text{Sinc}(2\omega) + 2\text{Sinc}(\omega) \right)$$

$$= \frac{4\text{Sinc}(2\omega)}{2\omega} + \frac{2\text{Sinc}(\omega)}{\omega} = 4\text{Sinc}(2\omega) + 2\text{Sinc}(\omega)$$

$$\text{Sinc } x = \frac{\sin x}{x}$$

$$16 \quad \bar{F}(j\omega) = \frac{1}{\omega^2 + 1} e^{\frac{-2\omega^2}{(\omega^2 + 1)}}$$

$$f(t-2)e^{j\omega t}$$

$$f(t-t_0) \xleftrightarrow{FT} F(j\omega) e^{-j\omega t_0}$$

$$t_0 \rightarrow 2$$

$$\therefore F(j\omega) e^{-j\omega(2)}$$

$$\Rightarrow F(j\omega) = \frac{1}{\omega^2 + 1} e^{\frac{-2\omega^2}{(\omega^2 + 1)}} e^{-j2\omega}$$

$$f(t)e^{j\omega_0 t} \xleftrightarrow{FT} F(j\omega - j\omega_0)$$

$$\omega_0 \rightarrow 1$$

$$\therefore F(j\omega - 1)$$

$$\Rightarrow \frac{1}{(\omega-1)^2 + 1} e^{\frac{-2(\omega-1)^2}{(\omega-1)^2 + 1}}$$

$$f(t-2)e^{j\omega t} \xleftrightarrow{FT} \frac{1}{(\omega-1)^2 + 1} e^{\frac{-2(\omega-1)^2}{(\omega-1)^2 + 1}} e^{-j2(\omega-1)}$$

$$Q_1 = 2 + 3 + 1 + 6 + 2 + 3 + 3 + 2 = 22$$

2 a $X(k) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$ at $\omega = \frac{2\pi k}{N}$, $k = 0, 1, \dots, N-1$

$$x_n = \frac{1}{5}, \frac{1}{5}, \frac{1}{5}$$

$$\begin{aligned} X(k) &= \frac{1}{5} e^{-j\omega(-1)} + \frac{1}{5} e^{-j\omega(0)} + \frac{1}{5} e^{-j\omega(1)} \\ &= \frac{1}{5} (e^{j\omega} + 1 + e^{-j\omega}) \\ &= \frac{1}{5} (1 + 2\cos(\omega)) \\ &= \frac{1}{5} + \frac{2}{5} \cos(\omega) \end{aligned}$$

$$\omega = \frac{2\pi k}{N} \quad \text{where } N=3$$

$$\frac{1}{5} + \frac{2}{5} \cos\left(\frac{2\pi k}{3}\right)$$

$$X(0) = \frac{3}{5}$$

$$X(1) = 0$$

$$X(2) = 0$$

2b i: Ideal FIR filters are difficult and expensive to create due to the computing power required. They are typically utilised in image processing and high quality signal transmissions.

2b ii $x(n) = 12.5, 8, 8, 12.5$

$$W(n) = 0.54 - 0.46 \cos\left(\frac{2\pi n}{N-1}\right)$$

$$W(0) = 0.54 - 0.46 \cos\left(\frac{2\pi \cdot 0}{3}\right) = 0.08$$

$$W(1) = 0.77$$

$$W(2) = 0.77$$

$$W(3) = 0.08$$

2b ii $x_n \cdot w_n = y_n$

$x(0) \cdot w(0) = y(0)$ ✓

$12.5 \times 0.08 = 1 = y(0)$ ✓

$8 \times 0.77 = 6.16 = y(1)$ ✓

$8 \times 0.77 = 6.16 = y(2)$ ✓

$12.5 \times 0.08 = 1 = y(3)$ ✓

$y(n) = \{1, 6.16, 6.16, 1\}$ ✓

$Q_2 = 1+4+3+5+6 = 19$

Q4 ai Even = $\{1, 3, 4, 2\}$ ✓

Odd = $\{2, 4, 3, 1\}$ ✓

ii $w_N^k = e^{-j\frac{2\pi}{N}k}$

$w_8^0 = e^{-j\frac{2\pi}{8}0} = 1$

$w_8^1 = e^{-j\frac{2\pi}{8}1} = e^{-j\frac{\pi}{4}} = 0.707 - j0.707$

$w_8^2 = e^{-j\frac{2\pi}{8}2} = e^{-j\frac{\pi}{2}} = -j$ ✓

$w_8^3 = e^{-j\frac{2\pi}{8}3} = e^{-j\frac{3\pi}{4}} = -0.707 - j0.707$

Q4b butterfly diagram

$x(k) = \{20, -1-2j, 0, -7+2j, 0, -5, 0, 1\}$

$Q_4 = 1+2+11 = 14$

$\frac{14}{25}$

Q3a

$$S(t) = A \cos(f_m t + \phi)$$

$$L_{o_{in}} \cos(f_{lo} t)$$

$$L_{o_{quad}} - \sin(f_{lo} t)$$

$$S(t) = 3 \cos(1200 + \frac{1}{6}\pi)$$

$$L_o = 2 \cos(1150)$$

$$= -2 \sin(1150)$$

$$i \quad I(t) = 3 \cos(1200 + \frac{1}{6}\pi) \cdot 2 \cos(1150)$$

$$\cos A \cos B = \frac{1}{2} (\cos(A+B) + \cos(A-B))$$

$$I(t) = \frac{1}{2} \times 6 (\cos(1200 + \frac{1}{6}\pi + 1150) + \cos(1200 + \frac{1}{6}\pi - 1150))$$

$$= 3 (\cos(50 + \frac{1}{6}\pi) + \cos(2350 + \frac{1}{6}\pi))$$

$$Q(t) = 3 \cos(1200 + \frac{1}{6}\pi) \cdot (-2 \sin(1150))$$

$$\cos A \sin B = \frac{1}{2} (\sin(A+B) - \sin(A-B))$$

$$Q(t) = \frac{1}{2} \times -6 (\sin(1200 + \frac{1}{6}\pi + 1150) - \sin(1200 + \frac{1}{6}\pi - 1150))$$

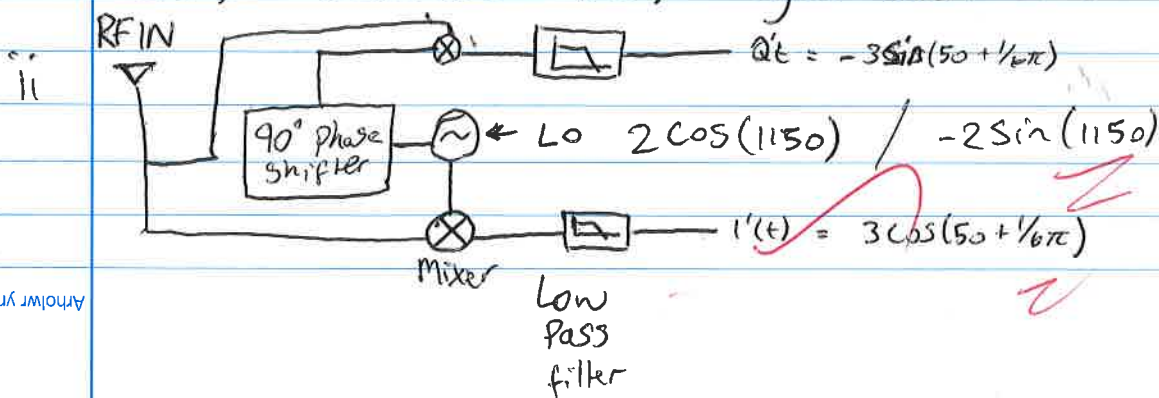
$$= -3 (\sin(2350 + \frac{1}{6}\pi) - \sin(50 + \frac{1}{6}\pi))$$

$$Z(t) = I'(t) + j Q'(t)$$

$$I'(t) = 3 \cos(50 + \frac{1}{6}\pi)$$

$$Q'(t) = -3 \cos(50 + \frac{1}{6}\pi)$$

$$Z(t) = 3 \cos(50 + \frac{1}{6}\pi) - 3j \cos(50 + \frac{1}{6}\pi)$$



Q3a:iii The RF Signal is received and filtered by an RF Image rejection filter, after being amplified by a low noise amplifier. The signal is then mixed with the Local oscillator frequency, ~~then~~ the down converted signal is then allowed to pass using a low pass filter while the higher frequency component of the mixed signal is filtered out.

2-
(2)

Q3b: i) Int Signal

$$S(t) = \cos(1230)$$

$$L_{in} = 2\cos(1150)$$

$$L_{quad} = -2\sin(1150)$$

$$I(t) = 2(\cos(2380) + \cos(80))$$

$$Q(t) = -2(\sin(2380) + \sin(80))$$

$$I'(t) = 2\cos(80)$$

$$Q'(t) = -2\sin(80)$$

AB

After mixing with the LO and passing through the low pass filter only the 80Hz signal should remain

ii) The added interference ~~will~~ will add distortion to the desired signal making it more difficult to receive clearly and potentially making parts of the signal ~~unrecoverable~~.

iii) Using a low pass filter closer to the intended signal will exclude more received signals, a low pass filter of 60Hz cutoff frequency will ensure the interference found at 80Hz after mixing will not be able to pass through the filter.

$$Q3 = 2+3+4+2+4 = 17$$

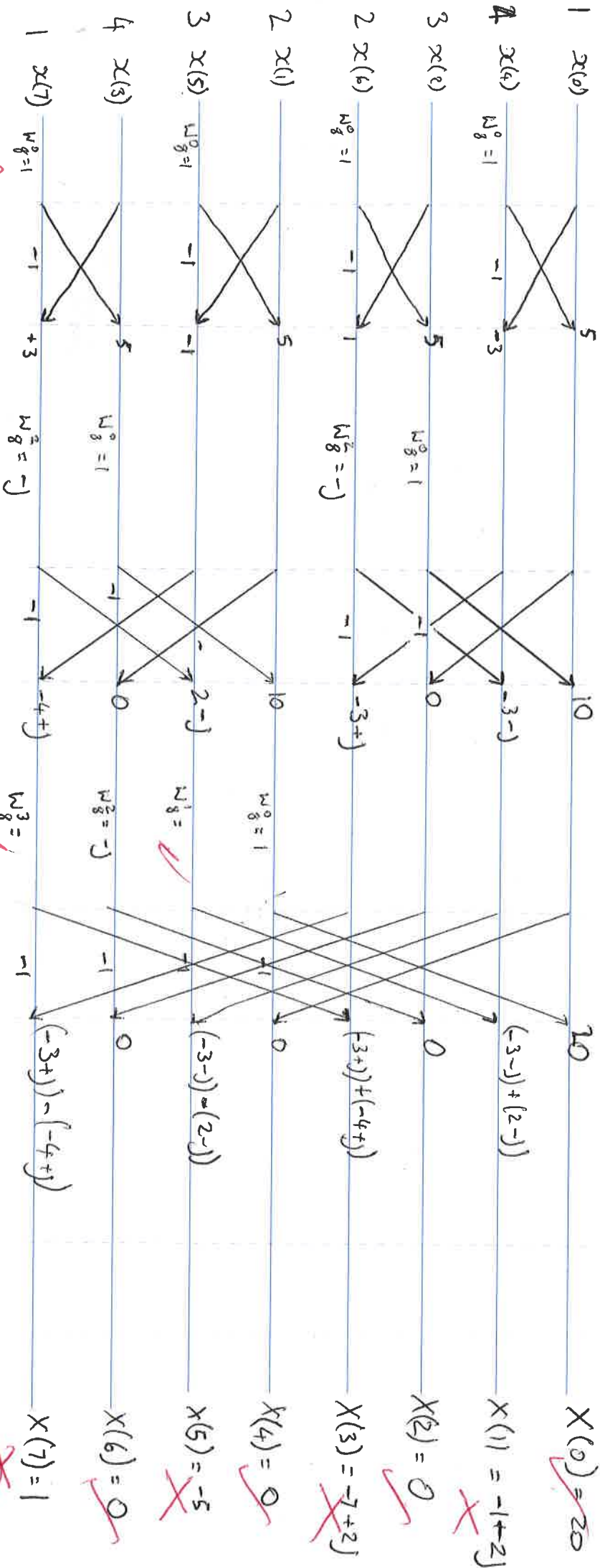
(4)

17
25

Q4b

format ①

✓



①

①

⑧