



(Gweler yr ochr arall am y fersiwn Cymraeg)

1a)

$$f(t) = \begin{cases} 2, & -1 \leq t \leq 1 \\ 1, & -2 \leq t \leq -1 \\ 1, & 1 \leq t \leq 2 \end{cases}$$

$$F(j\omega) = \int_{-1}^1 2e^{-j\omega t} dt + \int_{-2}^{-1} e^{-j\omega t} dt + \int_1^2 e^{-j\omega t} dt$$

using $\int e^{-j\omega t} dt = \frac{e^{-j\omega t}}{-j\omega}$

$$F(j\omega) = \left[\frac{2e^{-j\omega t}}{-j\omega} \right]_{-1}^1 + \left[\frac{e^{-j\omega t}}{j\omega} \right]_{-2}^{-1} + \left[\frac{e^{-j\omega t}}{j\omega} \right]_1^2$$

$$= \frac{1}{-j\omega} \left[2(e^{-j\omega(1)} - e^{-j\omega(-1)}) + (e^{-j\omega(-1)} - e^{-j\omega(-2)}) + (e^{-j\omega(2)} - e^{-j\omega(1)}) \right]$$

 $\times \frac{2}{2}$

$$= \frac{1}{-j\omega} \left[\frac{4}{2} (e^{-j\omega} - e^{j\omega}) + \frac{2}{2} (e^{j\omega} - e^{j2\omega}) + \frac{2}{2} (e^{-j2\omega} - e^{-j\omega}) \right]$$

Rearrange

$$= \frac{1}{-j\omega} \left[\frac{4}{2} (e^{-j\omega} - e^{j\omega}) + \frac{2}{2} (e^{j2\omega} - e^{j\omega}) + \frac{2}{2} (e^{j\omega} - e^{-j\omega}) \right]$$

Rearrange

$$= \frac{1}{\omega} \left[\frac{4(e^{j\omega} - e^{-j\omega})}{2j} + \frac{2(e^{j2\omega} - e^{j\omega})}{2j} - \frac{2(e^{j\omega} - e^{-j\omega})}{2j} \right]$$

using $\sin(\omega) = \frac{e^{j\omega} - e^{-j\omega}}{2j}$

$$= \frac{1}{\omega} \left[4 \sin(\omega) + 2 \sin(2\omega) - 2 \sin(\omega) \right]$$

Rearrange

$$= \frac{2 \sin \omega}{\omega} + \frac{4 \sin(2\omega)}{2\omega}$$

2

(2)

3

(3)

4

1

(4)

3

using $\text{sinc}(x) = \frac{\sin(x)}{x}$

$$F(j\omega) = 2\text{sinc}(\omega) + 4\text{sinc}(2\omega)$$

(C)

1b) ~~$F(j\omega) = \frac{1}{\omega^2 + 1} e^{\frac{-2\omega^2}{\omega^2 + 1}}$~~

using $f(t-t_0) = F(j\omega) e^{-j\omega t_0}$
and $f(t)e^{j\omega t_0} = F(j\omega - j\omega_0)$

~~$$F(j\omega) = \frac{1}{(\omega-1)^2 + 1} e^{\frac{-2(\omega-1)^2}{(\omega-1)^2 + 1}} e^{-j\omega t_0}$$~~

~~$$f(t-2)e^{j\omega t} = \frac{1}{(\omega-1)^2 + 1} e^{\frac{-2(\omega-1)^2}{(\omega-1)^2 + 1}} e^{-j2(\omega-1)}$$~~

(B)

2a) $x(n) = \begin{cases} \frac{1}{5}, & \text{for } -1 \leq n \leq 1 \\ 0, & \text{else} \end{cases} \quad N=3$

$$x(n) = (\frac{1}{5}, \frac{1}{5}, \frac{1}{5})$$

using $X(k) = \sum_{n=0}^{N-1} x(n) e^{-j(2\pi k n/N)}$

~~$$X(0) = (\frac{1}{5})(1) + (\frac{1}{5})(1) + (\frac{1}{5})(1) = \frac{3}{5}$$~~

$$X(0) = (\frac{1}{5})(1) + (\frac{1}{5})(1) + (\frac{1}{5})(1) = \frac{3}{5}$$

$$X(1) = (\frac{1}{5})(1) + (\frac{1}{5})(1) + (\frac{1}{5})(1)$$

$$X(2) =$$

~~$$X(2) =$$~~

$$x(n) = \frac{3}{5},$$

- b i) • FIR filters require a ~~long~~ amount of computation
 • FIR filters are effectively used in the receiver end of a communications line to ~~smooth~~ the data

b ii) $x(n) = 12.5, 8, 8, 12.5$

$y(n) = x(n) * w(n)$

using $w(n) = 0.54 - 0.46 \cos\left(\frac{2\pi n}{N-1}\right)$

$N = 4$

$w(0) = 0.54 - 0.46 \cos\left(\frac{2\pi(0)}{4-1}\right) = 0.08$

$w(1) = 0.54 - 0.46 \cos\left(\frac{2\pi(1)}{4-1}\right) = 0.77$

$w(2) = 0.54 - 0.46 \cos\left(\frac{2\pi(2)}{4-1}\right) = 0.77$

$w(3) = 0.54 - 0.46 \cos\left(\frac{2\pi(3)}{4-1}\right) = 0.08$

$y(0) = 1$ $y(1) = 6.16$

$y(2) = 6.16$ $y(3) = 1$

$y(n) = 1, 6.16, 6.16, 1$

$Q_2 = 2 + 2 + 4 + 5 = 13$

13
25

$$3a) \quad S(t) = 3 \cos(2\pi(1200)t + \frac{\pi}{6})$$

$$L_{O_{in-phase}} = 2 \cos(2\pi(1150)t)$$

$$L_{O_{q-phase}} = -2 \sin(2\pi(1150)t)$$

$$I(t) = S(t) \cdot L_{O_{in-phase}}$$

$$Q(t) = S(t) \cdot L_{O_{q-phase}}$$

$$I(t) = 6 \left[\cos(2\pi(1200)t + \frac{\pi}{6}) \cos(2\pi(1150)t) \right]$$
~~$$= 6 \cos(2\pi(1150)t)$$~~

$$\text{using } \cos A \cdot \cos B = \frac{1}{2} [\cos(A+B) + \cos(A-B)]$$

$$I(t) = 3 \left[\cos(2\pi(2350)t + \frac{\pi}{6}) + \cos(2\pi(50 + \frac{\pi}{6})t) \right]$$

$$I'(t) = 3 \cos(2\pi(50 + \frac{\pi}{6})t)$$

$$Q(t) = -6 \left[\cos(2\pi(1200)t + \frac{\pi}{6}) \sin(2\pi(1150)t) \right]$$

$$\text{using } \cos A \cdot \sin B = \frac{1}{2} [\sin(A+B) - \sin(A-B)]$$

$$Q(t) = -3 \left[\sin(2\pi(2350)t + \frac{\pi}{6}) - \sin(2\pi(50 + \frac{\pi}{6})t) \right]$$

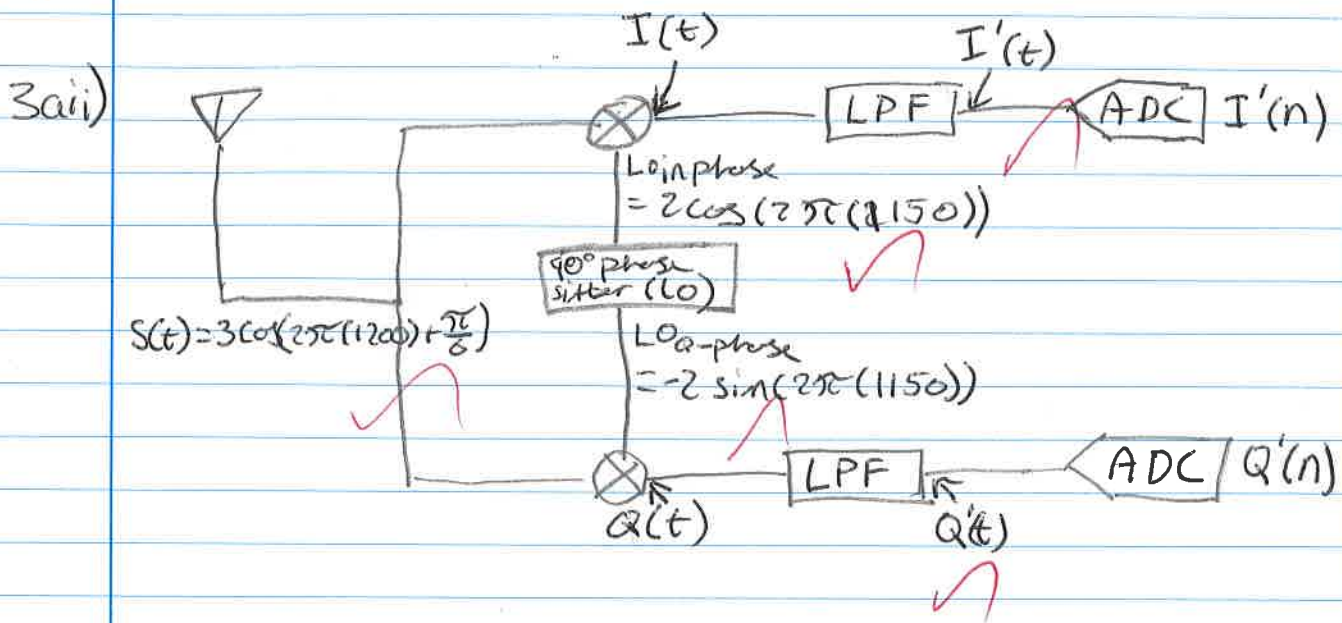
$$Q'(t) = 3 \sin(2\pi(50 + \frac{\pi}{6})t)$$

$$Z(t) = I'(t) + iQ'(t)$$

$$Z(t) = 3 \left[\cos(2\pi(50 + \frac{\pi}{6})t) + i \sin(2\pi(50 + \frac{\pi}{6})t) \right]$$

$$\text{using } e^{i\theta} = \cos(\theta) + i \sin(\theta)$$

$$Z(t) = 3e^{i(2\pi(50 + \frac{\pi}{6})t)}$$



3a iii) The system creates a sum of the waves or a sum of difference of the waves, 2350 Hz & 50 Hz then passes this through the low pass filter leaving the message signal of 50 Hz

3b i) Sum = 2380 Hz
Sum of difference = 80 Hz

this means a LPF should have a value of $55 - 60\text{ Hz}$ to remove the noise

3b ii) the addition of the 80 Hz frequency will cause a distortion on the ~~sig~~ signal where as without there will be no effect

3b iii) use of an LPF such as a FIR with a sharp cut off

$$Q_3 = 3+3+4+2+4+2+3 = 21$$

$$\frac{21}{25}$$

4ai) even = ~~0, 2, 4, 6~~ 1, 3, 4, 2 = y(0), y(2), y(4), y(6)
odd = 2, 4, 3, 1 = y(1), y(3), y(5), y(7)

even even = 1, 4 = y(0), y(4)
even odd = 3, 2 = y(2), y(6)
odd even = 2, 3 = y(1), y(5)
odd odd = 4, 1 = y(3), y(7)

4aii) $W_N^K = e^{-j\frac{2\pi}{N}K}$

$W_8^0 = 1$

$W_8^1 = 0.707 - j0.707$

$W_8^2 = -j$

$W_8^3 = -0.707 - j0.707$

4b) ~~workings~~ on sheet

$y(0) = 20$

$y(1) = -5.828 - j2.414$

$y(2) = 0$

$y(3) = -0.172 - j0.414$

$y(4) = 0$

$y(5) = 0.172 + j0.414$

$y(6) = 0$

$y(7) = -5.828 + j2.414$

$Q_4 = 2+2+16 = 20+5 = 25$

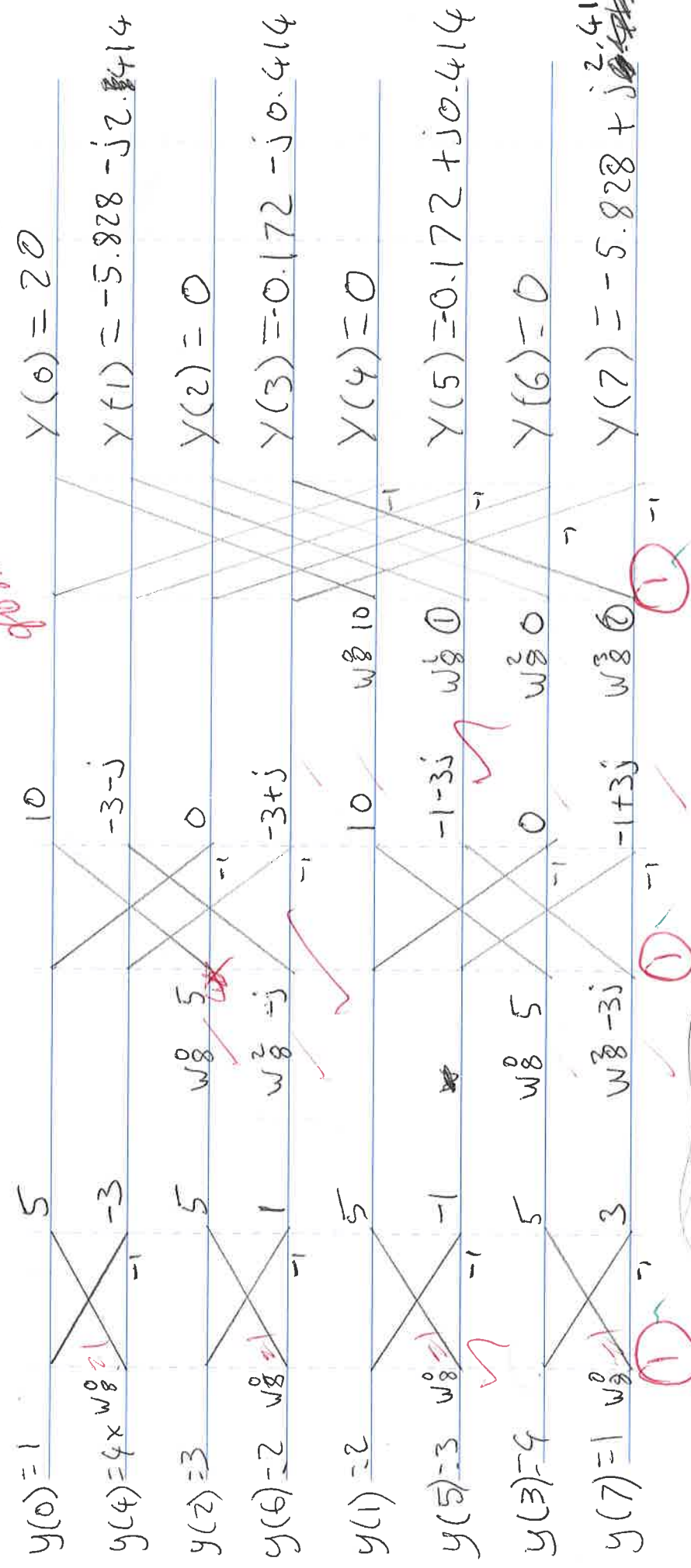
$\frac{25}{25} = 1$

~~W_N^k = e^{-j 2π k / N}~~

$W_8^0 = 1$

$W_8^2 = -1$

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① $= (-1 - 3j)(0.707 - j0.707) = -0.707 + j0.707 - j2.121 - 2.121 = -2.828 - j1.414$

② $= (-1 + 3j)(-0.707 - j0.707) = 0.707 + j0.707 - j2.121 + 2.121 = 2.828 - j1.414$