

Power triangle / Current triangle

$$P.F = \cos \phi = \frac{P(\text{Real power})}{S(\text{appant power})}$$

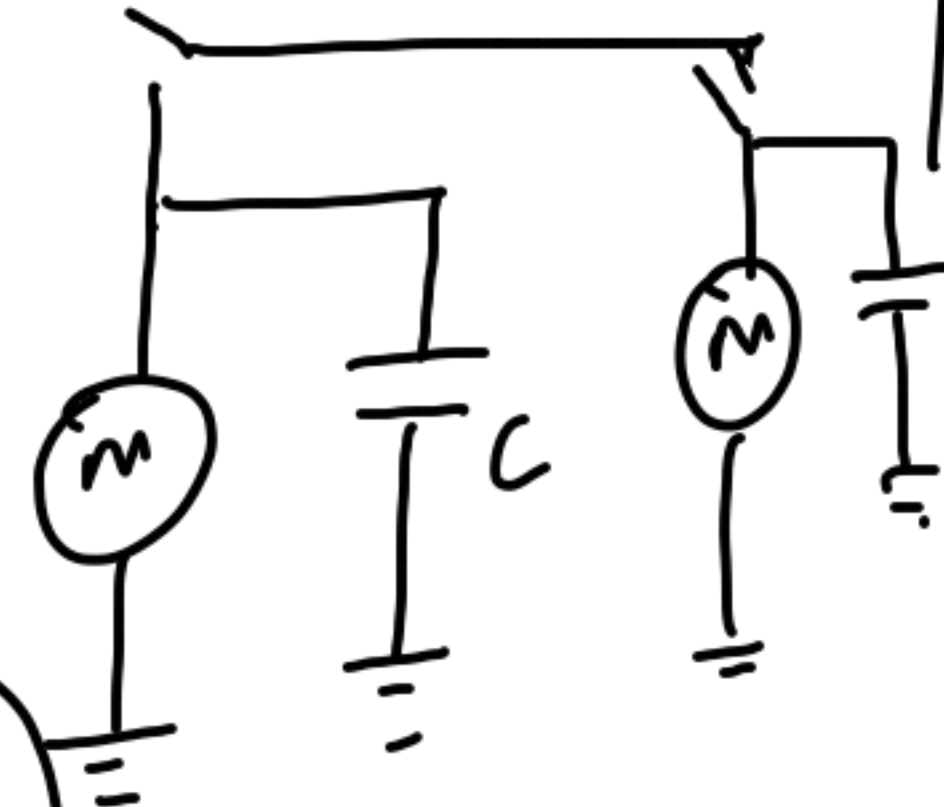
→ $P.F < 1$, fine.

→ coil, motor, generator,
transformer, heater,

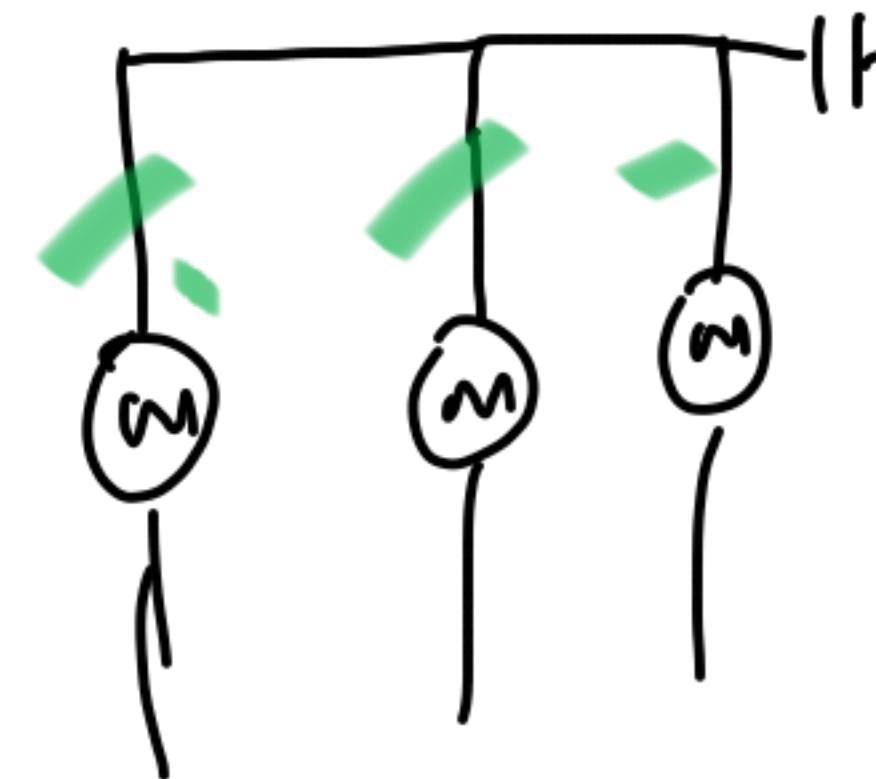
→ $V \neq I$ (phase shift)

— Using PFC — Parallel

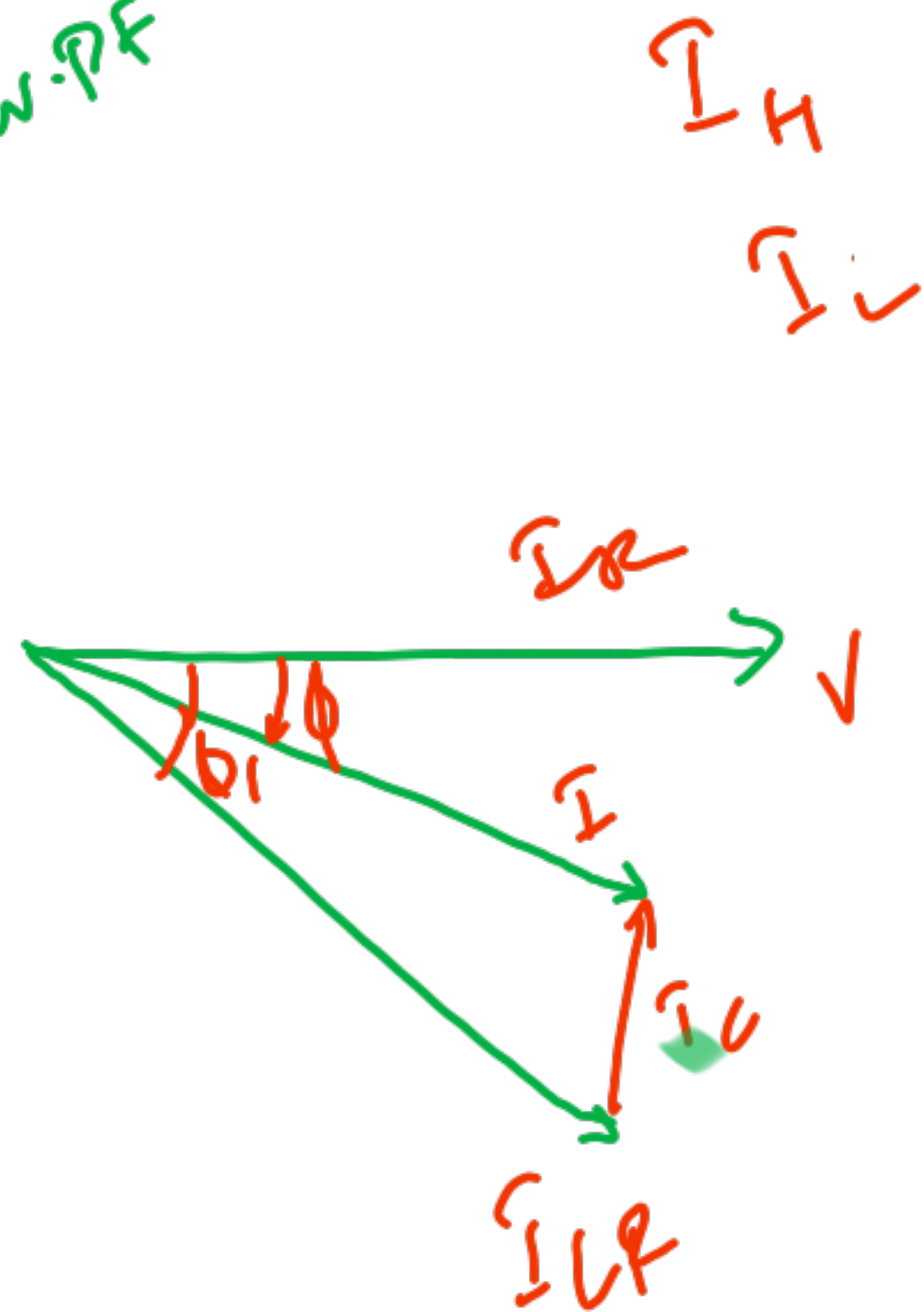
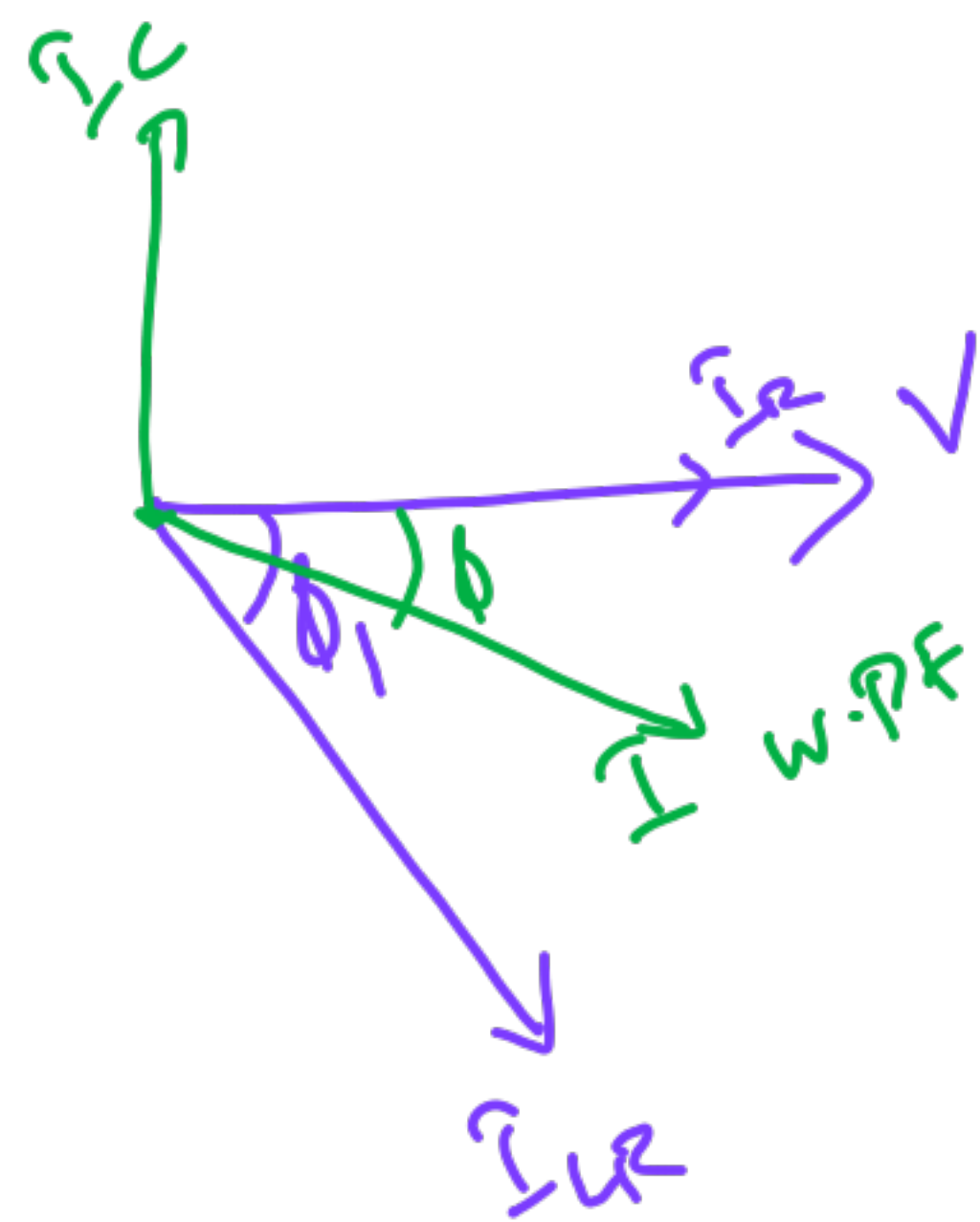
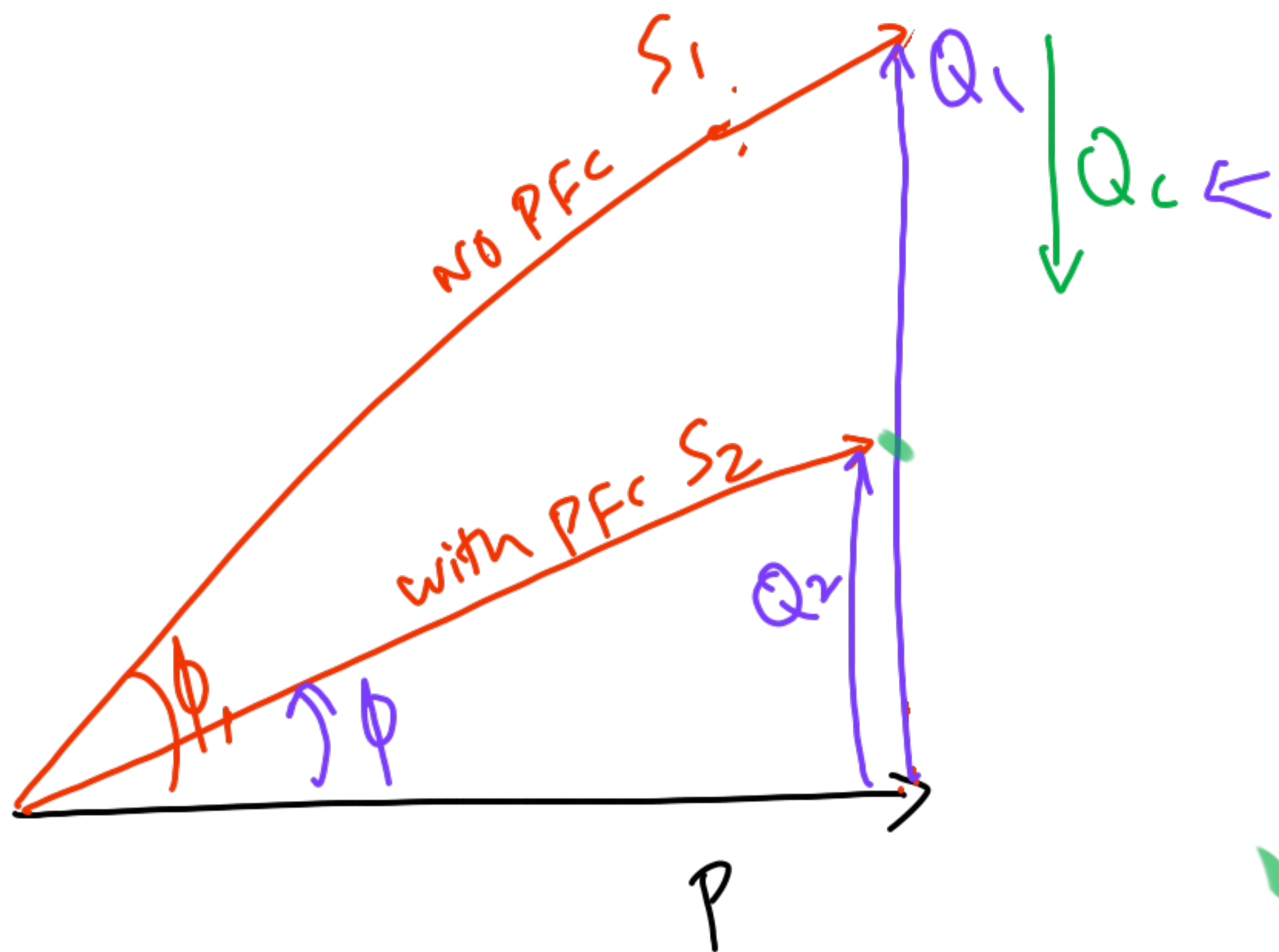
→ Individual



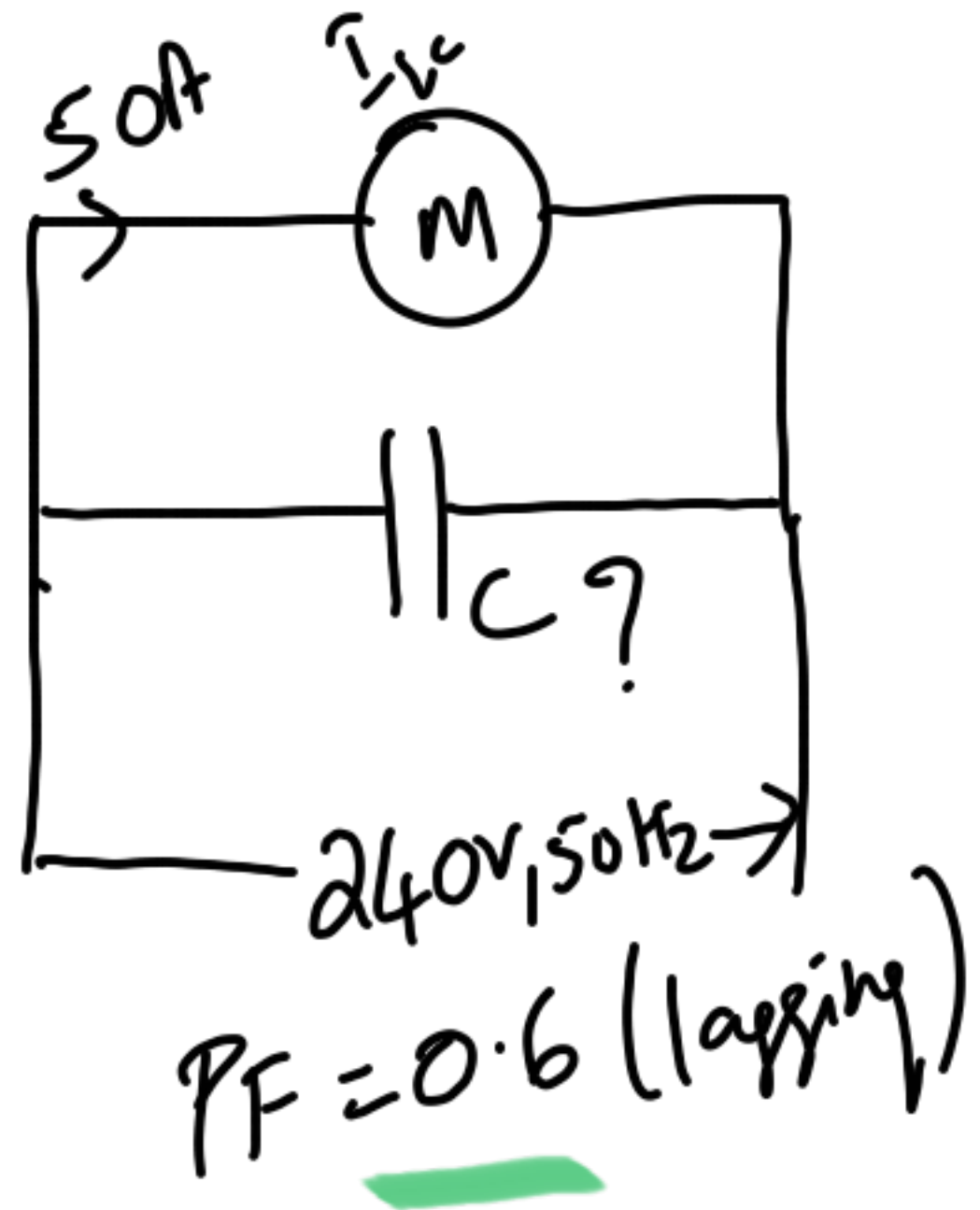
Central



$\eta < 1$



1) In the circuit shown below what value of capacitor is required to improve the power factor to one.



Soln.

$$I_m = 50 \text{ A}$$

$$V = 240 \text{ V},$$

$$f = 50 \text{ Hz}.$$

$$\text{PF} = \cos \phi = 0.6$$

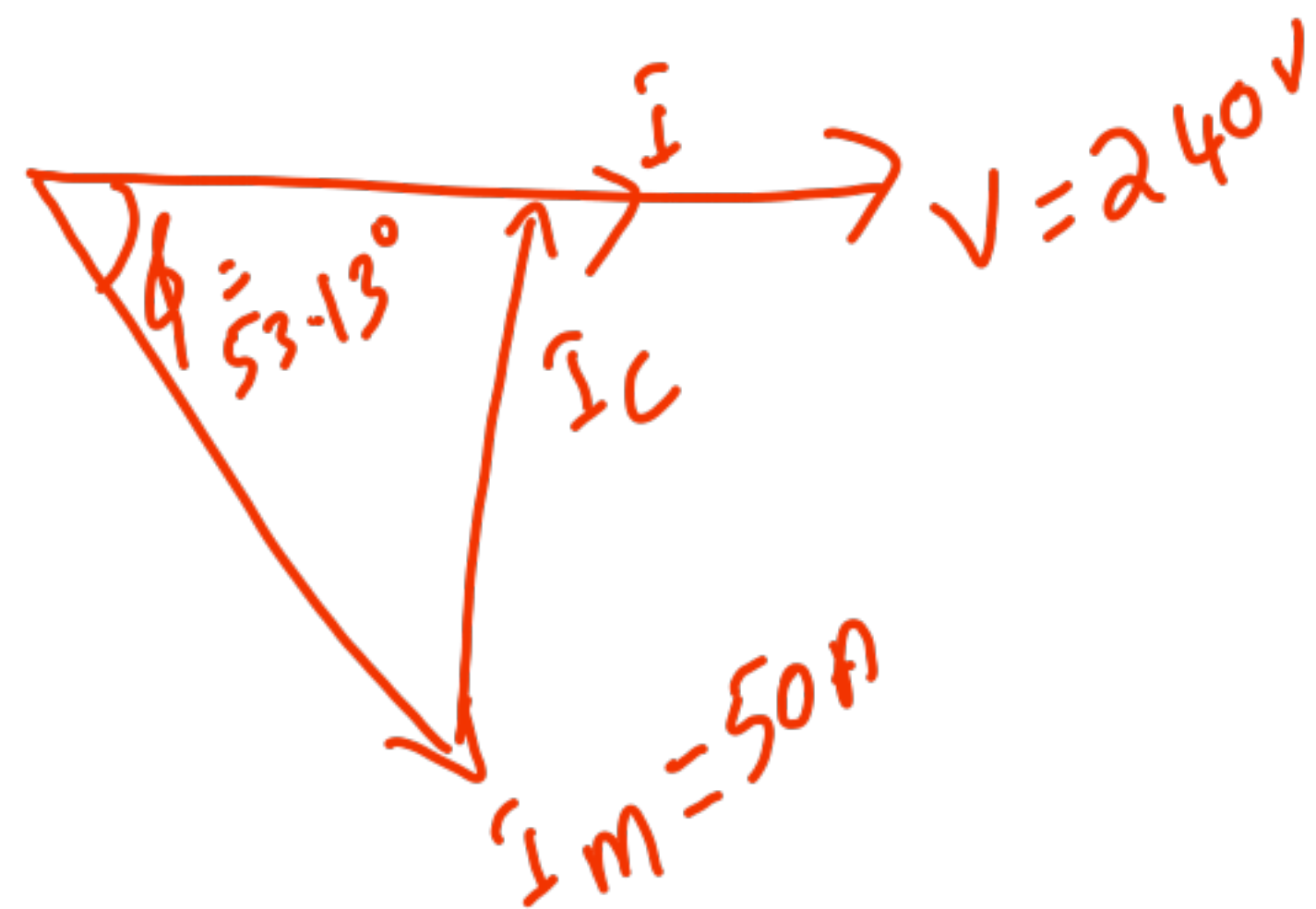
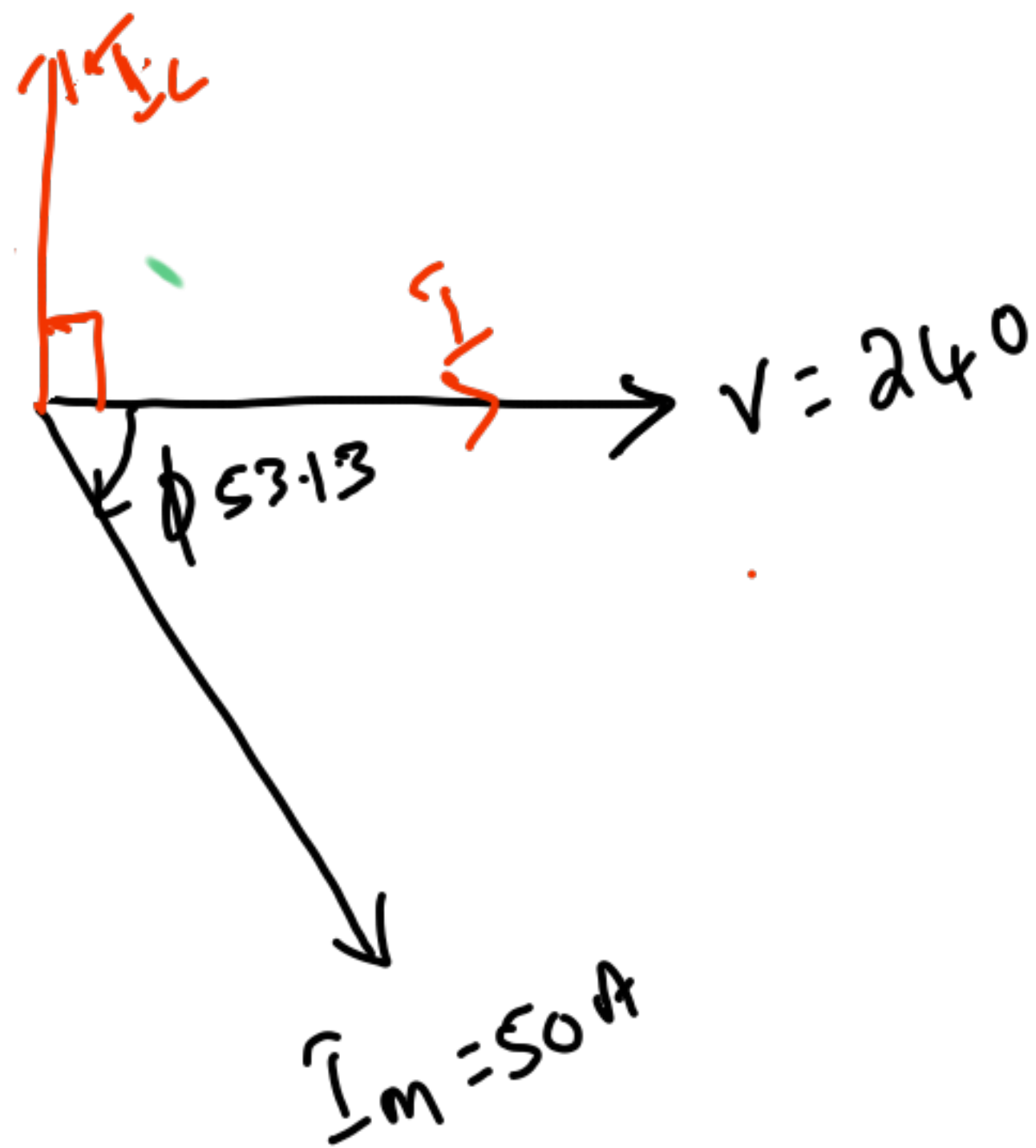
$$\phi = \cos^{-1}(0.6)$$

$$\phi = 53.13^\circ,$$

$$\text{To get, P.F.} = 1$$

$$\cos \phi = 1, \phi = \cos^{-1}(1)$$

$$\phi = 0$$



From the above triangle

$$\sin \phi = \frac{\text{OPP}}{\text{hyp}}$$

$$\sin(53.13^\circ) = \frac{I_c}{I_m}$$

$$\sin(53.13^\circ) = \frac{I_c}{50}$$

$$I_c = 50 \sin(53.13^\circ)$$

$$I_c = 40 \text{ A}$$

Current across
Capacitor.

$$I_c = \frac{V}{X_c}$$

$$40 = \frac{240}{X_c}$$

$$X_c = \frac{240}{40}$$

$$X_c = 6 \Omega$$

$$X_C = \frac{1}{2\pi fC}$$

$$b = \frac{1}{2\pi \times 50 \times C}$$

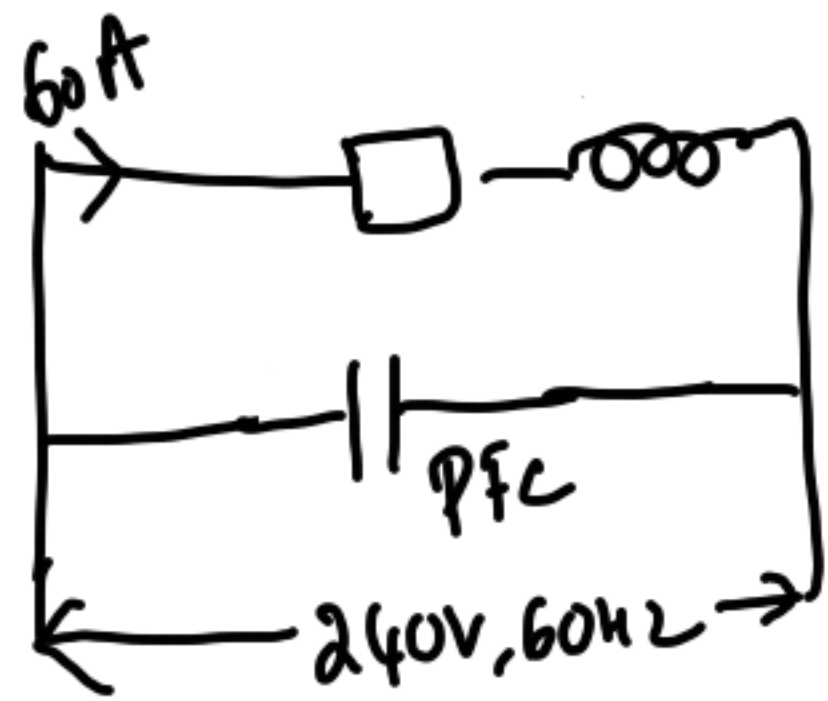
$$C = \frac{1}{2\pi \times 50 \times b}$$

$$C = 530.51 \times 10^{-6} \text{ F}$$

$$C = 530.51 \mu\text{F}$$

2) An inductive load takes a current of 60A at a power factor of 0.643 lagging when connected to a 240V, 60Hz supply. It is required to improve the power factor to 0.80 by connecting a capacitor in parallel to the load. Calculate -

- new supply current
- capacitor current
- value of PFC.



Given PF, 0.643

$$PF = \cos \phi_1 = 0.643$$

$$\phi_1 = \cos^{-1}(0.643)$$

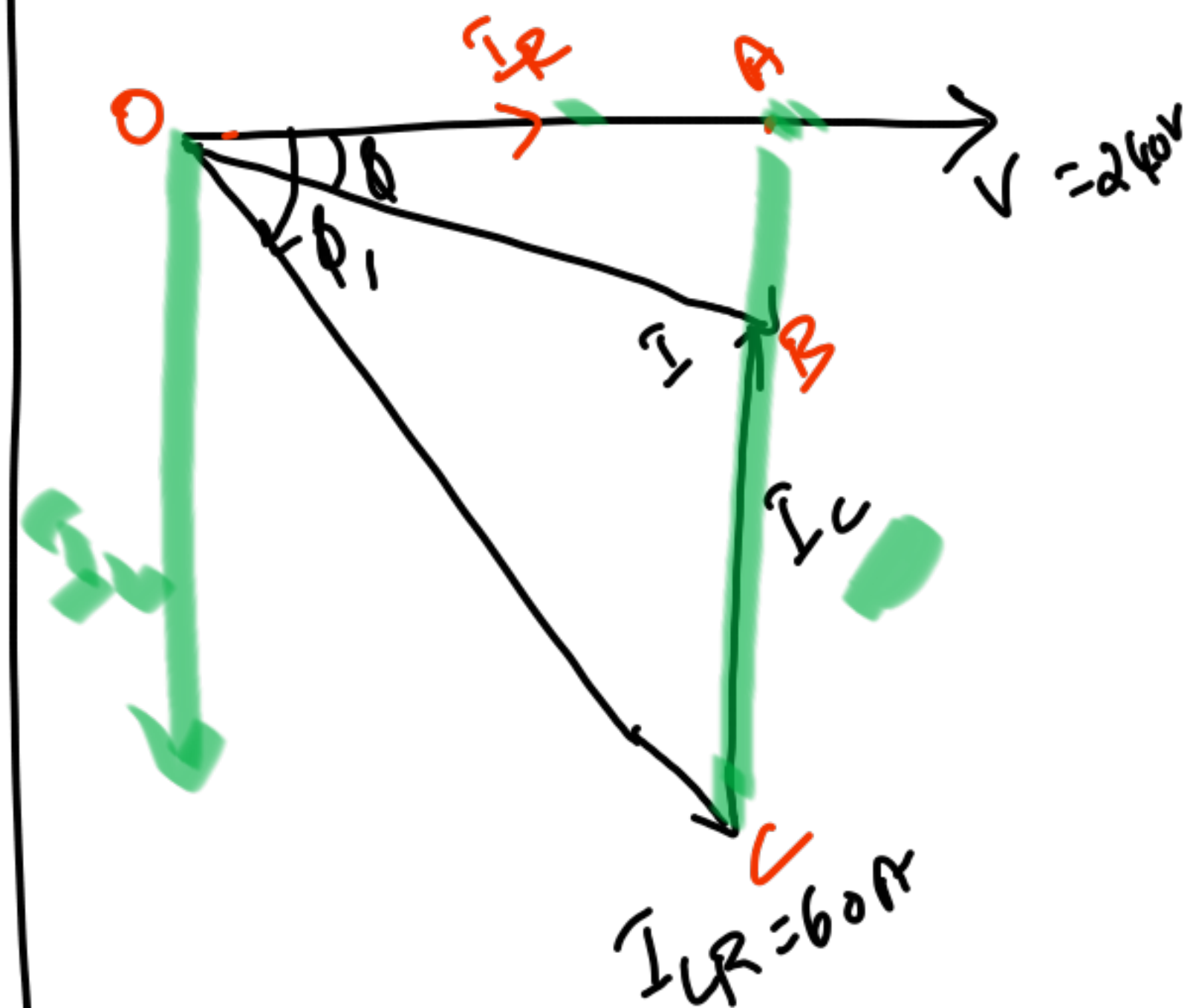
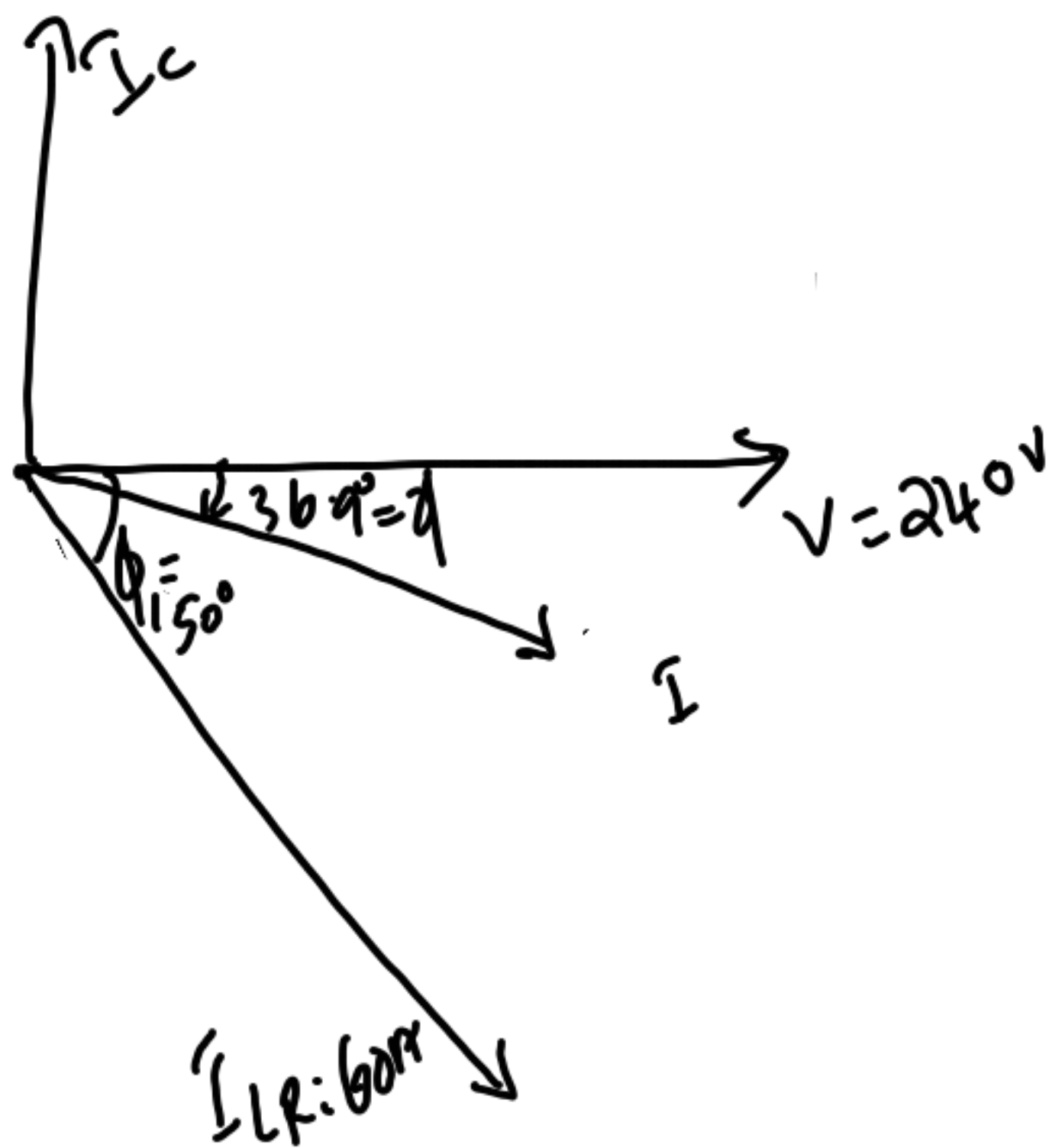
$$\phi_1 = 50^\circ$$

New power factor

$$PF = \cos \phi = 0.8$$

$$\phi = \cos^{-1}(0.8)$$

$$\phi = 36.9^\circ$$



$\Delta OAB \rightarrow \phi$
 $\Delta OAC \rightarrow \phi_1$

In $\triangle OAC$

$$\cos \phi_1 = \frac{\text{Adj}}{\text{hyp}}$$

$$\cos \phi_1 = \frac{OA}{OC}$$

$$\cos 50^\circ = \frac{OA}{I_{LR}}$$

$$\cos 50^\circ = \frac{OA}{60}$$

$$OA = 60 \times \cos 50^\circ$$

$$OA = 38.6$$

$$OA = I_R = 38.6 A$$

In $\triangle OAC$

$$\sin \phi_1 = \frac{\text{opp}}{\text{hyp}}$$

$$\sin \phi_1 = \frac{AC}{OC}$$

$$\sin 50^\circ = \frac{AC}{I_{LR}}$$

$$\sin 50^\circ = \frac{AC}{60}$$

$$AC = 60 \cdot \sin 50^\circ$$

$$AC = 45.96$$

$$I_V = I_L = 45.96 A$$

In $\triangle OAB$

$$\cos \phi = \frac{OA}{OB}$$

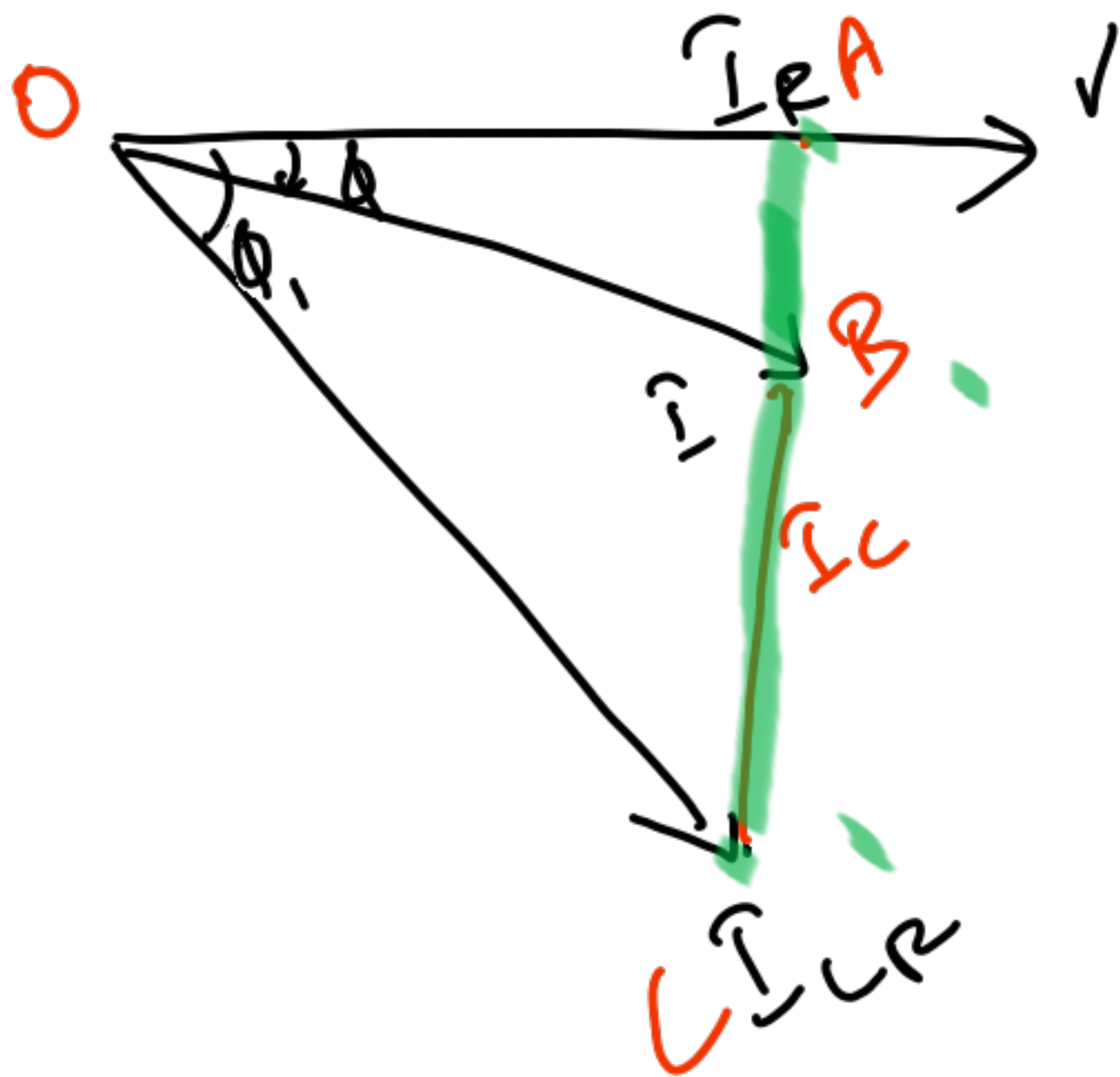
$$\cos 36.9^\circ = \frac{I_R}{I}$$

$$I = \frac{38.6}{\cos 36.9}$$

$$I = 48.3 A$$

Current after PFC

$$I = 48.3 A$$



From triangle OAB

$$\sin \phi = \frac{AB}{OB}$$

$$\sin \phi = \frac{AB}{I}$$

$$\sin(36.9) = \frac{AB}{48.3}$$

$$AB = 48.3 \sin(36.9)$$

$$AB = 29$$

$$I_c = BC$$

From ΔOAC

$$AC = AB + BC$$

$$BC = AC - AB$$

$$BC = 45.96 - 29$$

$$BC = I_c = 16.96 A$$

$$I_c = \frac{V}{X_c}$$

$$X_c = \frac{V}{I_c}$$

$$= \frac{240}{16.96}$$

$$X_c = 14.11 \Omega$$

$$X_c = \frac{1}{2\pi f C}$$

$$C = \frac{1}{2\pi f X_c}$$

$$C = \frac{1}{2\pi \times 60 \times 14.11}$$

$$C = 187 \mu F$$

PFC - Coursework

$$R_{scr} = 2.1 \Omega \cdot R = 65 \Omega \cdot L = 181 \text{ mH}$$

Simulate, RL, Circuit

$$t = 2.3 \text{ ms}$$

$$T = 20 \text{ ms}$$

$$\phi_1 = \frac{t}{T} \times 360$$

$$\phi_1 = \frac{2.3 \text{ ms}}{20 \text{ ms}} \times 360$$

$$\phi_1 = 41.4^\circ$$

$$\text{PF} : \cos \phi$$

$$\text{PF} = \cos(41.4^\circ)$$

$$\text{PF} = 0.75$$

from the graph. (I_m, V_m)

$$I_m = 4.8 \text{ V}$$

$$V_m = 415 \text{ V}$$

$$I_{\text{rms}} = \frac{I_m}{\sqrt{2}} = \frac{4.8}{\sqrt{2}} = 3.39 \text{ A}$$

$$V_{\text{rms}} = \frac{V_m}{\sqrt{2}} = \frac{415}{\sqrt{2}} = 293.45 \text{ V}$$

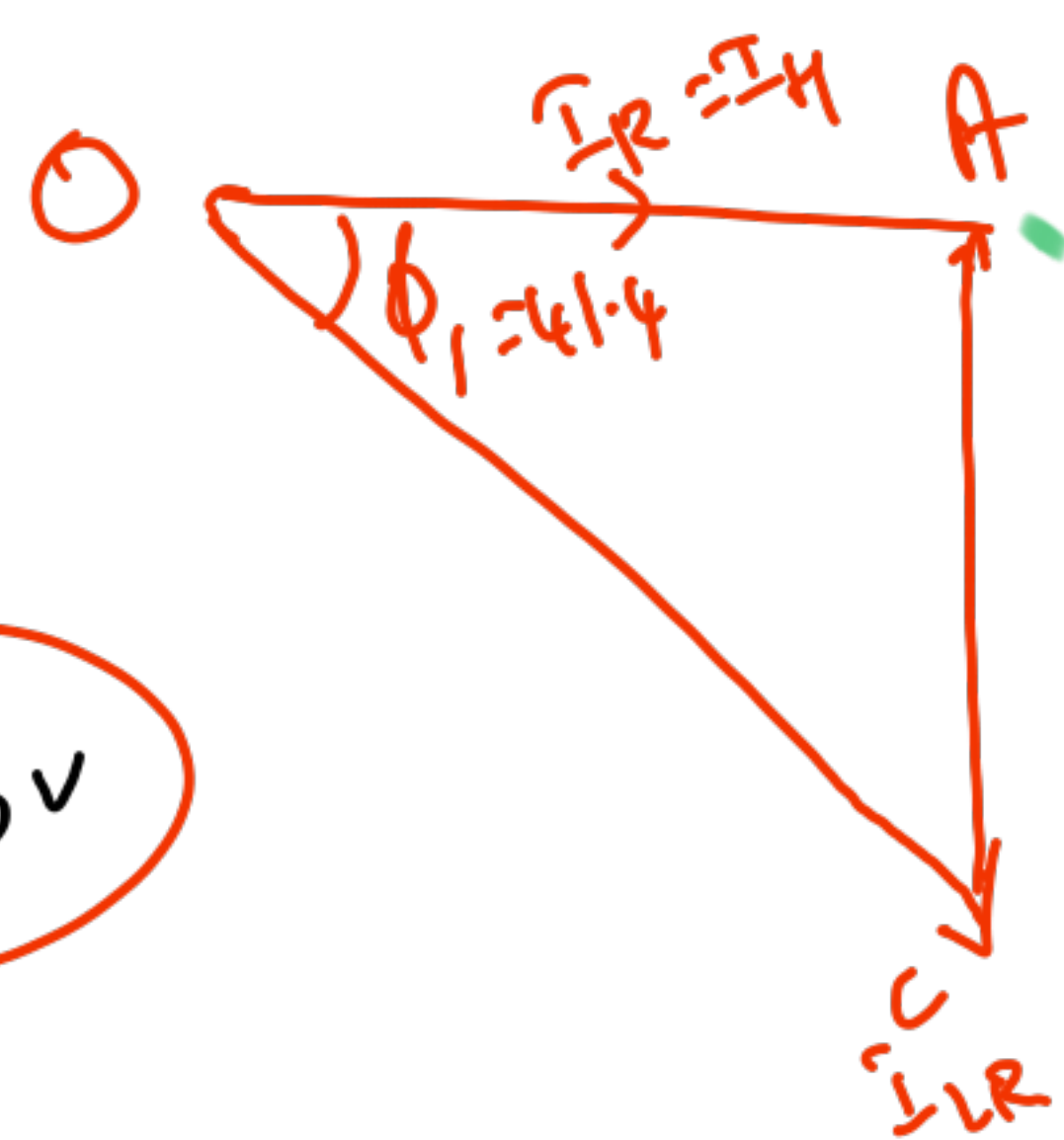
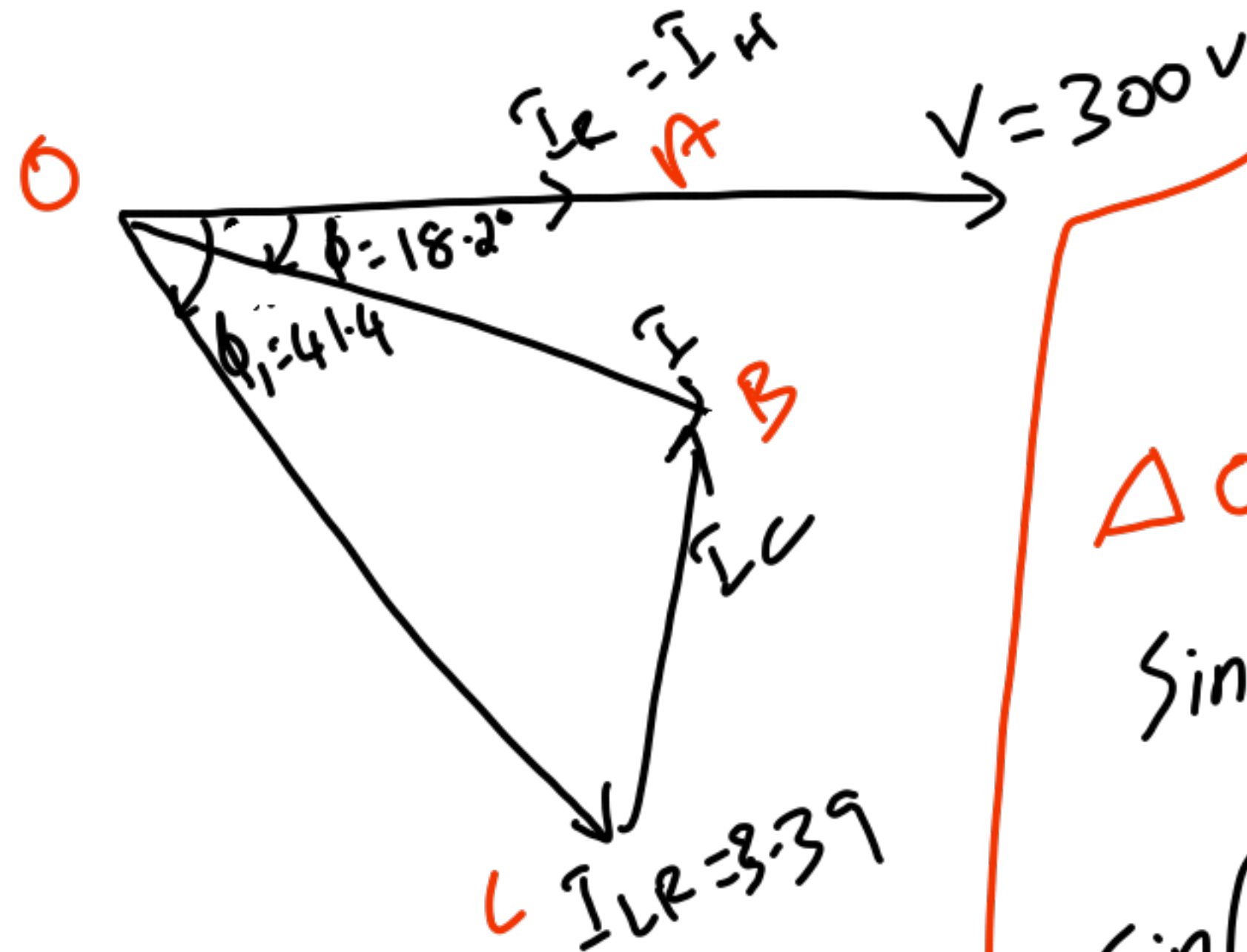
The new power factor.

$$P.F = 0.95$$

$$\cos \phi = 0.95$$

$$\phi = \cos^{-1}(0.95)$$

$$\phi = 18.2^\circ$$



$\triangle OAC$

$$\sin \phi_1 = \frac{AC}{OC}$$

$$\sin(41.4) = \frac{AC}{I_{LR}}$$

$$\sin(41.4) = \frac{AC}{3.39}$$

$$AC = 3.39 \sin(41.4)$$

$$AC = 2.24 A$$

$$AC = I_V = 2.24 A$$

(vertical)

In AOAC

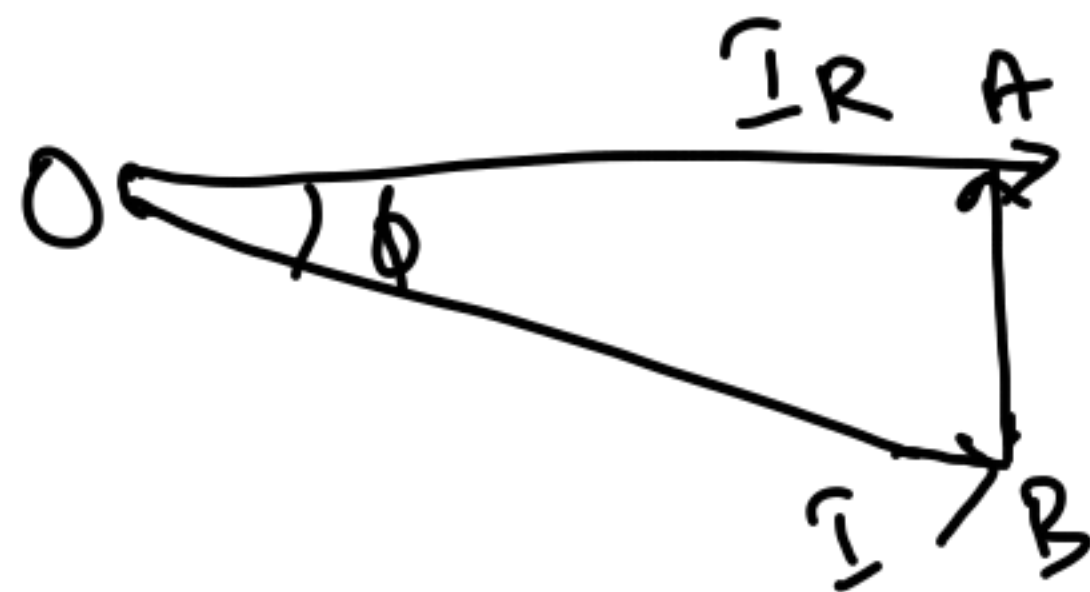
$$\cos \phi_1 = \frac{OA}{OC}$$

$$\cos(\phi_1) = \frac{OA}{I_{LR}}$$

$$OA = I_{LR} \cos \phi_1$$

$$= 3.39 \cos(41.4)$$

$$OA = 2.55 = I_R = 2.55 A$$



In AOAB

$$\cos \phi = \frac{OA}{OB}$$

$$\cos(18.2) = \frac{I_R}{I}$$

$$I = \frac{2.55}{\cos(18.2)}$$

$$I = 2.68 A$$

In AOAB

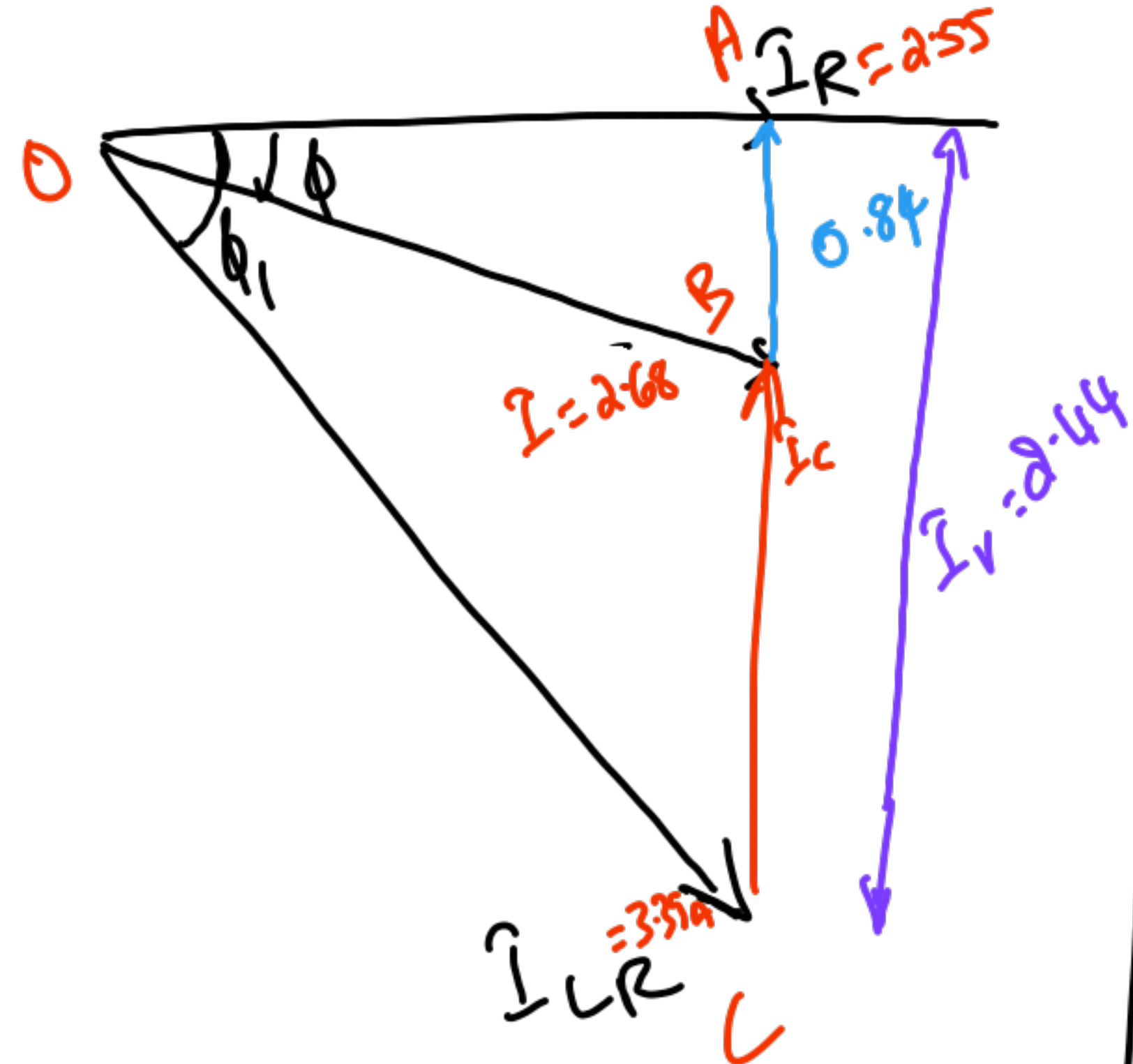
$$\sin \phi = \frac{AB}{OB}$$

$$\sin(18.2) = \frac{AB}{I}$$

$$AB = I \cdot \sin(18.2)$$

$$AB = 2.68 \cdot \sin 18.2$$

$$AB = 0.84 A$$



$$AC = AB + BC$$

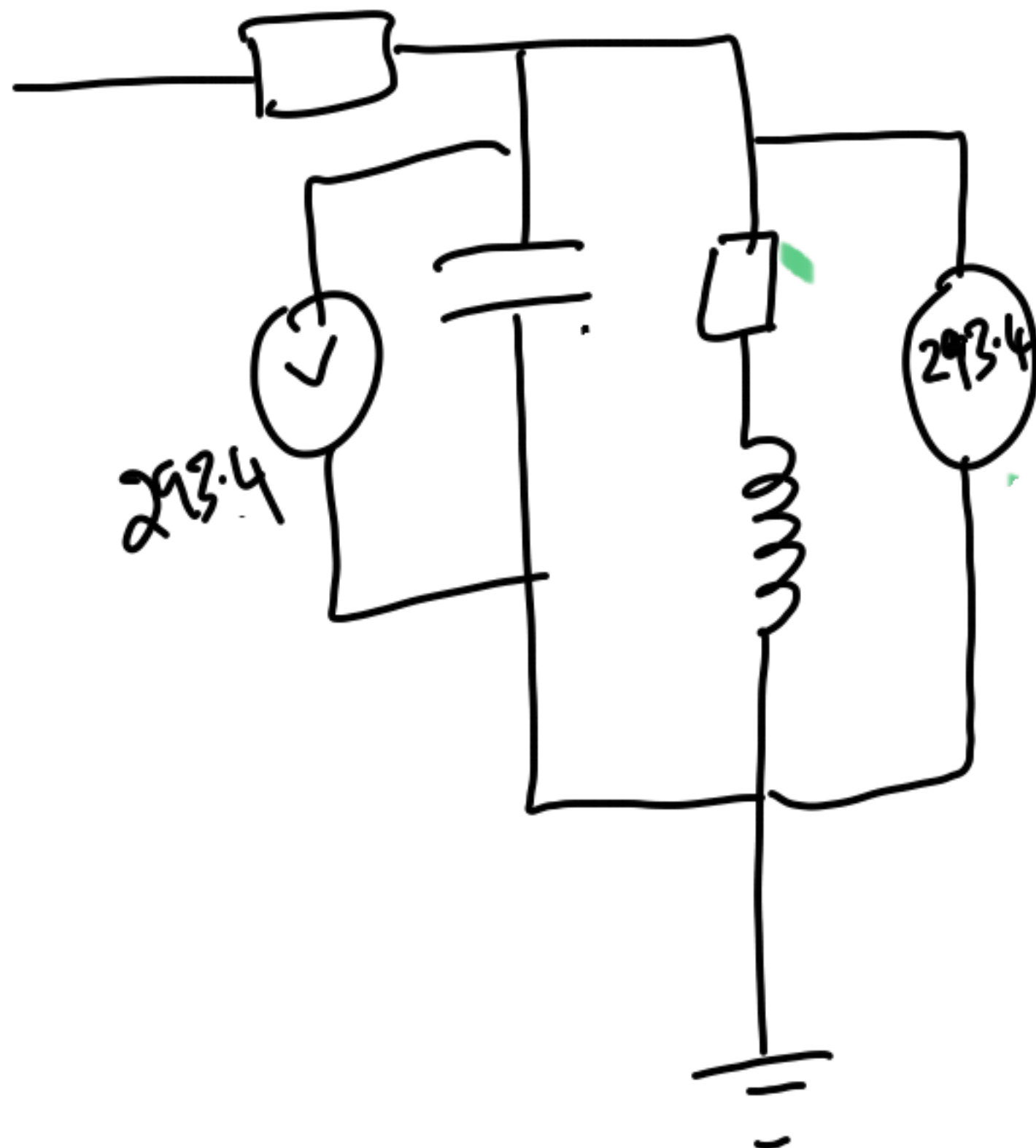
$$BC = AC - AB$$

$$BC = 2.44 - 0.84$$

$$BC = 1.4$$

$$I_C = 1.4 \text{ A}$$

$$I_C = \frac{V_C}{X_C}$$



$$1.4 = \frac{293.4}{X_C}$$

$$X_C = \frac{293.4}{1.4}$$

$$X_C = 209.5 \Omega$$

$$X_C = \frac{1}{2\pi fC}$$

$$C = \frac{1}{2\pi fX_C}$$

$$= \frac{1}{2\pi \times 50 \times 209.5}$$

$$C = 15.19 \mu F$$

Power calculation

$$P = V I \cos \phi_1 = 300 \times 3.39 \times \cos(41.4) = 762.86 \text{ W}$$

$$Q = V I \sin \phi_1 = 300 \times 3.39 \times \sin(41.4) = 672.55 \text{ VAR}$$

$$S = V I = 300 \times 3.39 = 1017 \text{ VA}$$

Resonance Frequency

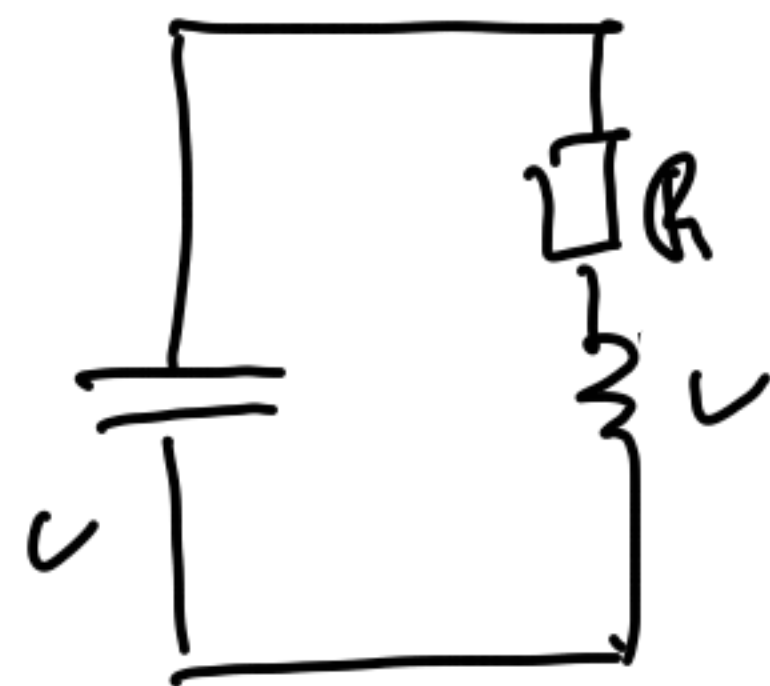
$$f_r = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$f_r = \frac{1}{2\pi} \sqrt{\frac{1}{181 \times 10^{-3} \times 15.19 \times 10^{-6}} - \frac{65^2}{181 \times 10^{-3}}}$$

$$f_r = 77.11 \text{ Hz.}$$

$$f_r - 25 \text{ Hz} = 52.11 \text{ Hz}$$

$$f_r + 25 \text{ Hz} = 102.11 \text{ Hz} \checkmark$$



$$I_{LR}, I_C, I, \phi_{LR}, \phi$$

When $f = 102.11 \text{ Hz.}$

$$X_C = \frac{1}{2\pi \times 102.11 \times 15.19 \times 10^{-6}}$$

$$X_C = 102.6 \Omega.$$

$$X_L = 2\pi \times 102.11 \times 181 \times 10^{-3}$$

$$X_L = 116.12 \Omega$$

$$Z_{LR} = \sqrt{R^2 + X_L^2}$$

$$= \sqrt{65^2 + 116.12^2}$$

$$Z_{LR} = 133.07 \Omega$$

$$I_{LR} = \frac{V_{LR}}{Z_{LR}}$$

from graph. $V_{LR}(\text{max}) = 419\text{V}$

$$V_{LR}(\text{Rms}) = \frac{419}{\sqrt{2}} = 296.27\text{V}$$

$$I_{LR} = \frac{296.27\text{V}}{133.07}$$

$$I_{LR} = 2.23\text{A}$$

$$I_C = \frac{V_C}{X_C} = \frac{296.27}{102.6}$$

$$I_C = 2.89\text{A}$$

$$\phi_{LR} = \tan^{-1}\left(\frac{X_L}{R}\right)$$

$$= \tan^{-1}\left(\frac{116.12}{65}\right)$$

$$\phi_{LR} = 60.76^\circ$$

LR-C

$$\phi = \tan^{-1} \left(\frac{I_{LR} \sin \phi_{LR} \sim I_c}{I_{LR} \cos \phi_{LR}} \right)$$

$$\phi = \tan^{-1} \left(\frac{2.23 \sin 60.76^\circ \sim 2.89}{2.23 \cos 60.76^\circ} \right)$$

$$\phi = 40.92^\circ \text{ [lead]}$$

$$I_{LR} \sin \phi_{LR} = 2.23 \sin 60.76^\circ \\ = 1.94 \text{ A}$$

$$I_c > I_{LR} \sin \phi_{LR}.$$

$$I = \sqrt{(I_{LR} \cos \phi_{LR})^2 + (I_{LR} \sin \phi_{LR} \sim I_c)^2}$$

$$I = 1.44 \text{ A}$$

