
ALGEBRA

Algebra is the language of mathematics. It can be used to solve many types of problems and is one of the most fundamental tools for engineers.

One of the major differences between arithmetic and algebra is the use of variables. A variable is a letter (or symbol) that stands for a number.

Evaluating Algebraic Expressions

To evaluate an algebraic expression, simply substitute the given numbers for each variable and perform the necessary calculations in the proper order.

Examples:

If $x = 3$ and $y = -4$ evaluate the following expressions:

- $y^2 + y^3$
 $(-4)^2 + (-4)^3$
 $16 + -64 = 16 - 64 = -48$
- $-y - x$
 $-(-4) - 3 = 4 - 3 = 1$
- $\frac{y-0}{x-(-1)}$
 $\frac{-4-0}{3-(-1)} = \frac{-4}{3+1} = \frac{-4}{4} = -1$
- $x^2 - y^3$
 $3^2 - (-4)^3 = 9 - -16 = 9 + 16 = 25$

Simplifying Algebraic Expressions

Simplifying an algebraic expression is usually done by replacing one algebraic expression with another that is equivalent and simpler in form.

Examples:

- Simplify $3x + 5x$**
 $3x + 5x = 8x$
- Simplify $4(a + 3) - 2a$**
First lets expand out the brackets
 $4a + 12 - 2a$
Now collect the 'like' terms together

$$4a - 2a + 12 = \mathbf{2a + 12}$$

- 3. Simplify $2(x + 3y - z) - 3(x + y - z) + 4(x - y)$**

First lets expand out the brackets

$$2x + 6y - 2z - 3x - 3y + 3z + 4x - 4y$$

Now collect the 'like' terms together

$$2x - 3x + 4x + 6y - 3y - 4y - 2z + 3z = \mathbf{3x - y + z}$$

Expanding Brackets

Each term in the first bracket multiplies by each term in the second bracket. When multiplying two brackets with two terms each, a quick way of remembering what to do is by remembering 'FOIL'.

First, Outers, Inneres, Last

Examples:

- 1. Expand $(x + 3)(x + 5)$**

$$x^2 + 5x + 3x + 15$$

Now collect the 'like' terms together

$$x^2 + \mathbf{8x + 15}$$

- 2. Expand $(x^2 + 4x - 3)(x + 8)$**

$$x^3 + 8x^2 + 4x^2 + 32x - 3x - 24$$

Now collect the 'like' terms together

$$x^3 + \mathbf{12x^2 + 29x - 24}$$

- 3. Expand $(x - 5)^3$**

Rewrite as

$$(x - 5)(x - 5)(x - 5)$$

Multiply out the first two brackets

$$(x^2 - 5x - 5x + 25)(x - 5)$$

Now collect the 'like' terms in the first bracket together

$$(x^2 - 10x + 25)(x - 5)$$

$$x^3 - 5x^2 - 10x^2 + 50x + 25x - 125$$

Now collect the 'like' terms together

$$x^3 - \mathbf{15x^2 + 75x - 125}$$

Solving Linear Equations

These are the easiest equation you can have and such equations have just one solution.

The thing to remember is: get all of the terms with x in together on one side of the equals sign, and all of the terms which do not contain x (i.e. numbers) on the other.

- 1. Solve $3x = 21$**

Divide both sides by 3 to get x on it's own.

$$x = \frac{21}{3} = 7$$

Remember when you put the answer back into the equation, as long as the equation is satisfied then we know that we have the correct answer.

2. Solve $3x + 9 = x - 7$

First rearrange the equation to get all the x's on one side and the constants on the other.

$$3x - x = -7 - 9$$

Now collect the 'like' terms together

$$2x = -16$$

$$x = -\frac{16}{2} = -8$$

3. Solve $3(x - 4) - 2(x + 9) = 4x + 3$

First expand the brackets

$$3x - 12 - 2x - 18 = 4x + 3$$

Now rearrange the equation to get all the x's on one side and the constants on the other.

$$3x - 2x - 4x = 3 + 12 + 18$$

Now collect the 'like' terms together

$$-3x = 33$$

$$x = \frac{33}{-3} = -11$$

4. Solve $\frac{x+3}{2} = \frac{2x-1}{5}$

Here the first stage is to cross multiply.

$$5(x + 3) = 2(2x - 1)$$

Now expand out the brackets

$$5x + 15 = 4x - 2$$

Now rearrange the equation to get all the x's on one side and the constants on the other.

$$5x - 4x = -2 - 15$$

Now collect the 'like' terms together

$$x = -17$$