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L4 Mathematics Exam

ACDA4008

Electronic Engineering Year 1

11/01/2022

1.) a.)

i.)

$$y = \frac{x^2}{2x+1}$$

$$u = x^2$$

$$u' = 2x$$

$$v = 2x+1$$

$$v' = 2$$

$$\frac{dy}{dx} = \frac{(2x+1)(2x) - x^2(2)}{(2x+1)^2}$$

$$= \frac{4x^2 + 2x - 2x^2}{(2x+1)^2}$$

$$= \frac{2x^2 + 2x}{(2x+1)^2}$$

$$\frac{dy}{dx} = \frac{2x(x+1)}{(2x+1)^2}$$

$$ii.) y = \sqrt{x^2+1}$$

$$y = (x^2+1)^{\frac{1}{2}}$$

$$\text{let } u = x^2+1$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{1}{2} u^{-\frac{1}{2}} \times 2x$$

$$= \frac{x}{u^{\frac{1}{2}}}$$

$$\frac{dy}{dx} = \frac{x}{\sqrt{x^2+1}}$$

$$1.) a) iii.) \quad y = 3x^{\frac{3}{2}} + \frac{3}{x^2} - 6x = 3x^{\frac{3}{2}} + 3x^{-2} - 6x$$

$$\frac{dy}{dx} = \frac{3}{2} \cdot 3 \cdot x^{\frac{1}{2}} - 6x^{-3} - 6$$

$$= \frac{9}{2}\sqrt{x} - \frac{6}{x^3} - 6$$

$$iv.) \quad y = \sec^3(4x)$$

$$y = \frac{1}{\cos^3(4x)}$$

$$\text{let } u = \cos(4x) \quad \therefore y = \frac{1}{u^3} = u^{-3}$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= -3u^{-4} \cdot -\sin(4x) \cdot 4$$

$$= 12u^{-4} \sin(4x)$$

$$\frac{dy}{dx} = \frac{12 \sin(4x)}{\cos^4(4x)}$$

$$v.) \quad y = (x + \sin(2x))$$

$$\frac{dy}{dx} = 1 - \cos(2x) \cdot 2$$

$$\frac{dy}{dx} = 1 - 2\cos(2x)$$

1) a) vi.) $y = \frac{1-10x}{(2x-1)^2}$ $u = 1-10x \quad u' = -10$
 $v = (2x-1)^2 \quad v' = 2(2x-1)(2)$
 $= 4(2x-1)$

$$\frac{dy}{dx} = \frac{(2x-1)^2(-10) - (1-10x) \cdot 4(2x-1)}{(2x-1)^4}$$

$$= \frac{-10(2x-1) - 4(1-10x)}{(2x-1)^3}$$

$$= \frac{-20x + 10 - 4 + 40x}{(2x-1)^3}$$

$$= \frac{20x + 6}{(2x-1)^3}$$

$$= \frac{2(10x+3)}{(2x-1)^3}$$

b) $y = \frac{1}{x} + \cos(2x)$

$\nearrow \sin \searrow$
 $-\cos \quad \cos$
 $\nwarrow -\sin \nearrow$

$$\frac{dy}{dx} = -120x^{-6} + 32\cos(2x)$$

$$= 4(8\cos(2x) - 30x^{-6})$$

$$= 8(4\cos(2x) - 15x^{-6})$$

$$= 8 \left[4\cos(2x) - \frac{15}{x^6} \right]$$

$$y = \cos(2x)$$

$$y' = -\sin(2x) \cdot 2$$

$$y'' = -\cos(2x) \cdot 4$$

$$y''' = \sin(2x) \cdot 8$$

$$y^{(4)} = \cos(2x) \cdot 16$$

$$y^{(5)} = \cos(2x) \cdot 32$$

$$y = \frac{1}{x} = x^{-1}$$

$$y' = -x^{-2}$$

$$y'' = 2x^{-3}$$

$$y''' = -6x^{-4}$$

$$y^{(4)} = 24x^{-5}$$

$$y^{(5)} = -120x^{-6}$$

1) c)

$$f(x) = \ln(\operatorname{cosec}(x) - \cot(x))$$

$$1 = \cos^2(x) + \sin^2(x)$$

$$f'(x) = \frac{1}{\operatorname{cosec}(x) - \cot(x)}$$

$$-\sin^2(x) = \cos^2(x) - 1$$

$$\sin^2(x) = 1 - \cos^2(x)$$

$$= \frac{1}{\frac{1}{\sin(x)} - \frac{\cos(x)}{\sin(x)}}$$

$$= \frac{1}{\left(\frac{1 - \cos(x)}{\sin(x)}\right)}$$

$$= \frac{\sin(x)}{1 - \cos(x)} \times \frac{1 + \cos(x)}{1 + \cos(x)}$$

$$= \frac{\sin(x) + \sin(x)\cos(x)}{1 - \cos^2(x)}$$

$$= \frac{\sin(x) + \sin(x)\cos(x)}{\sin^2(x)}$$

$$= \frac{1 + \cos(x)}{\sin(x)}$$

Q2.) a.) i) $\int \frac{2x}{(4+3x^2)} dx$

let $u = 4+3x^2$, $du = 6x dx$

$$\int \frac{2x}{u} \cdot \frac{du}{6x}$$

$$\frac{1}{3} \int \frac{1}{u} du$$

$$\frac{1}{3} \ln|u| + C$$

$$\underline{\underline{\frac{1}{3} \ln|4+3x^2| + C}}$$

ii.) $\int_0^4 e^{\frac{1}{2}x} dx$

$$\left[\frac{e^{\frac{1}{2}x}}{\frac{1}{2}} \right]_0^4$$

$$\left[2e^{\frac{1}{2}x} \right]_0^4$$

$$2e^2 - 2$$

$$\underline{\underline{2(e^2 - 1)}}$$

2.) a.) iii.)

$$\int_0^4 \frac{5x+13}{(2x+1)(x+4)} dx$$

$$\int_0^4 \frac{5x+13}{2x^2+9x+4} dx$$

$$\text{let } u = 5x+13 \quad du = 5 dx$$

$$dv = (2x^2+9x+4) dx \quad v = 2x^2+9x+4$$

$$= \left[(5x+13)(2x^2+9x+4) \right]_0^4 - 5 \int_0^4 2x^2+9x+4 dx$$

$$= \left[10x^3 + 45x^2 + 20x + 6x^2 + 27x + 12 \right]_0^4 - 5 \left[\frac{2x^3}{3} + \frac{9x^2}{2} + 4x \right]_0^4$$

$$= \left[10x^3 + 51x^2 + 47x + 12 \right]_0^4 - 5 \left[\frac{2x^3}{3} + \frac{9x^2}{2} + 4x \right]_0^4$$

$$= \left[640 + 51(16) + 188 + 12 - 12 \right] - 5 \left[\frac{2}{3}(64) + \frac{9}{2}(16) + 16 \right]$$

$$= 1644 - 5 \left[\frac{392}{3} \right]$$

$$= \underline{\underline{\frac{2972}{3}}}$$

$$2b) \int \frac{4}{x(1+4\ln x)^2} dx$$

$$4 \int [x(1+4\ln x)^2]^{-1} dx$$

$$4 \left[\ln [x(1+4\ln x)^2] \right] + C$$

$$\underline{4 \ln (x + 4 \ln x)}$$

$$4 \ln (x(1+4\ln x)^2) + C$$

$$4 \left[\ln(x) + \ln((1+4\ln x)^2) \right] + C$$

$$4 \left[\ln(x) + 2\ln(1+4\ln x) \right] + C$$

$$\underline{4 \left[\ln(x) + 2\ln \right]}$$

$$\underline{4 \ln(x) + 8 \ln(1+4\ln(x)) + C}$$

$$2.) c.) i) \int_{\ln(2)}^{\ln(5)} \frac{3e^{2x}}{(e^x-1)^{\frac{1}{2}}} dx$$

$$u = 3e^{2x} \quad du = 6e^{2x} dx$$

$$dv = (e^x-1)^{-\frac{1}{2}} dx \quad v = \frac{1}{\frac{1}{2}} \ln(e^x-1) = \frac{1}{2} \ln(e^x-1)$$

$$= uv - \int v du$$

$$= \left[3e^{2x} \cdot \frac{1}{2} \ln(e^x-1) \right]_{\ln(2)}^{\ln(5)} - \int_{\ln(2)}^{\ln(5)} \frac{1}{2} \ln(e^x-1) \cdot 6e^{2x} dx$$

$$= 3e^{2x} \left(\frac{2(e^x-1)^{\frac{1}{2}}}{e^x} \right) - \int_{\ln(2)}^{\ln(5)} \frac{2(e^x-1)^{\frac{1}{2}}}{e^x} 6e^{2x} dx$$

$$= \left[6e^x (e^x-1)^{\frac{1}{2}} \right]_{\ln(2)}^{\ln(5)} - 12 \int_{\ln(2)}^{\ln(5)} e^x (e^x-1)^{\frac{1}{2}} dx$$

Ans =

2) c) ii)

$$\int_1^e \ln(x) dx$$

$$\left[x \ln(x) - x \right]_1^e$$

$$\left[e \ln(e) - e \right] - \left[\ln(1) - 1 \right]$$

$$\cancel{0 - 1} - [0 - 1]$$

$$\underline{\underline{1}}$$

$$\left[e(1) - e \right] - \left[(0) - 1 \right]$$

$$0 + 1$$

$$\underline{\underline{1}}$$

$$\begin{matrix} \swarrow & \searrow \\ = \left(\frac{2i}{2} \right)^3 & \downarrow \end{matrix}$$

$$3) a) i) \left(\frac{1+i}{1-i} \right)^3 = \left(\frac{(1+i)(1+i)}{(1-i)(1+i)} \right)^3 = \left(\frac{1+2i-1}{1+1} \right)^3 = (i)^3 = \underline{\underline{0-i}}$$

$$ii) \frac{3}{(1+i)(2+i)} = \frac{3}{2+3i-1} = \frac{3}{1+3i} \times \frac{1-3i}{1-3i} = \frac{3-9i}{1+9} = \underline{\underline{\frac{3}{10} - \frac{9}{10}i}}$$

$$iii) \frac{1+i}{1-i} - (1+2i)(2+2i) + \frac{3-i}{1+i}$$

$$= \frac{(1+i)^2}{2} - (2+6i-4) + \frac{3-i}{1+i} \times \frac{1-i}{1-i}$$

$$= \frac{1+2i-1}{2} + 2 - 6i + \frac{3-3i-i-1}{1+1}$$

$$= i + 2 - 6i + \frac{2-4i}{2}$$

$$= 2 - 5i + 1 - 2i$$

$$= \underline{\underline{3-7i}}$$

$$\begin{aligned}
 3.) \text{ a.) iv.) } & 2i(i-1) + (\sqrt{3}-i) + (1+i)(\overline{1+i}) \\
 & = (-2-2i) + (\sqrt{3}-i) + (1+i)(1-i) \\
 & = (-2+\sqrt{3}-3i) + (1+1) \\
 & = \underline{\underline{-4+2\sqrt{3}-2i}}
 \end{aligned}$$

$$\begin{aligned}
 6.) \quad z &= (4+7i)(3-2i) \\
 &= 12 - 8i + 21i + 14 \\
 z &= 26 + 13i \\
 &= 13(2+i)
 \end{aligned}$$

$$\begin{aligned}
 |z| &= \sqrt{(26)^2 + (13)^2} & \arg(z) &= \arctan\left(\frac{13}{26}\right) \\
 &= \sqrt{845} & &= 1.11^c \text{ (3sf)}
 \end{aligned}$$

$$|z| = 13\sqrt{5}$$

$$= \underline{\underline{13\sqrt{5}(\cos(1.11) + i\sin(1.11))}} \quad (3sf)$$

$$c.) = \frac{1+3i}{1+i} = \frac{(1+3i)(1-i)}{(1+i)(1-i)} = \frac{1-i+3i+3}{2} = \frac{4+2i}{2} = \underline{\underline{2+i}}$$