

1) a. differentiate with respect to x

i $y = \frac{x^2}{2x+1}$

$$y = (x^2)(2x+1)^{-1}$$

apply Product rule

$$\frac{d}{dx}(fg) = f \frac{dg}{dx} + \frac{df}{dx} g$$

$$f = x^2 \quad \frac{df}{dx} = 2x$$

$$g = (2x+1)^{-1} \quad \frac{dg}{dx} =$$

apply Chain rule

$$u = 2x+1 \quad \frac{du}{dx} = 2$$

$$g = u^{-1} \quad \frac{dg}{du} = -u^{-2}$$

$$\begin{aligned} \frac{dg}{dx} &= \frac{du}{dx} \times \frac{dg}{du} \\ &= 2x - (2x+1)^{-2} \end{aligned}$$

$$\frac{d}{dx}(fg) = x^2 \times -2(2x+1)^{-2} + 2x(2x+1)^{-1}$$

1) a. ii $y = \sqrt{x^2 + 1}$

$$= (x^2 + 1)^{1/2}$$

$$u = x^2 + 1$$

$$\frac{du}{dx} = 2x$$

$$y = u^{1/2}$$

$$\frac{dy}{du} = \frac{1}{2} u^{-1/2}$$

$$\frac{dy}{dx} = \frac{du}{dx} \times \frac{dy}{du}$$

$$= 2x \times \frac{x^2 + 1^{-1/2}}{2}$$

$$= x (x^2 + 1)^{-1/2}$$

iii $y = 3x^{3/2} + \frac{3}{x^2} - 6x$
 $= 3x^{3/2} + 3x^{-2} - 6x$

$$\frac{dy}{dx} = \frac{9}{2} x^{1/2} + 6x^{-3} - 6$$

$$= 4.5x^{1/2} - 6x^{-3} - 6$$

iv $y = \sec^3 4x$

$$\frac{dy}{dx} = (3 \sec^3 4x \tan 4x)(4)$$

$$= 12 \sec^3 4x \tan 4x$$

$$f = 4x$$

$$\frac{df}{dx} = 4$$

v $y = x + \sin 2x$

$$= \frac{d}{dx}(x) + \frac{d}{dx}(\sin 2x)$$

$$= 1 + (\cos 2x)(2)$$

$$= 1 + 2 \cos 2x$$

$$u = 2x$$

$$\frac{du}{dx} = 2$$

vi $y = \frac{1 - 10x}{(2x - 1)^2}$

$$= (1 - 10x)(2x - 1)^{-2}$$

$$u = 2x - 1$$

$$\frac{du}{dx} = 2$$

$$\frac{dy}{dx} = 40(2x - 1)^{-3}$$

$$f = 1 - 10x$$

$$\frac{df}{dx} = -10$$

$$\frac{df}{dx} = -10$$

b) given $y = \frac{1}{x} + \cos 2x$

find $\frac{d^5 y}{dx^5}$

~~dy~~ $y = x^{-1} + \cos 2x$

$$\frac{dy}{dx} = \frac{d}{dx}(x^{-1}) + \frac{d}{dx}(\cos 2x)$$

$$1 \quad -x^{-2} \quad -\sin 2x \times 2$$

$$2 \quad +2x^{-3} \quad -2\sin 2x$$

$$3 \quad -6x^{-4} \quad -4\cos 2x$$

$$4 \quad 24x^{-5} \quad 8\sin 2x$$

$$5 \quad -120x^{-6} \quad 16\cos 2x$$

$$\frac{d^5 y}{dx^5} = -120x^{-6} - 32\sin 2x$$

c) if $f(x) = \ln(\operatorname{cosec}(x) - \cot(x))$

show $f'(x) = \operatorname{cosec}(x)$

$$f(u) = \ln(u)$$

$$u = \operatorname{cosec}(x) - \cot(x)$$

$$f'(u) = \frac{1}{u}$$

$$f(x) =$$

$$\begin{aligned} f'(x) &= f'(\operatorname{cosec} x) - f'(\cot x) \\ &= -\cot x \operatorname{cosec} x - f'(\cot x) \\ &= -\cot x \operatorname{cosec} x + \operatorname{cosec}^2 x \end{aligned}$$

$$\begin{aligned} f'(x) &= \frac{1}{\ln(\operatorname{cosec} x - \cot x)} \times -\cot x \operatorname{cosec} x + \operatorname{cosec}^2 x \\ &= (\ln(\operatorname{cosec} x - \cot x))' \times -\cot x \operatorname{cosec} x + \operatorname{cosec}^2 x \end{aligned}$$

$$2a) i \int \frac{2x}{(4+3x^2)} dx$$

$$= (2x)(4+3x^2)^{-1} dx$$

$$u = 4+3x^2$$

$$\int u = 4x + \frac{3}{3}x^3 \\ = 4x + x^3$$

$$f(x)g(x) = \int f(x)g(x) - \int f(x)g'(x) dx$$

$$= x^2(4+3x^2)^{-1} - \int x^2 \cdot (6x(4+3x^2)^{-2}) \frac{du}{dx} = 6x$$

$$g = u^{-1}$$

$$\int g = \frac{1}{-1} \ln|4+3x^2|$$

$$f = 2x$$

$$\frac{dg}{du} = 6x \cdot -(4+3x^2)^{-2}$$

$$ii) \int_0^4 e^{\frac{1}{2}x} dx$$

$$= \left. \frac{1}{\frac{1}{2}} e^{\frac{1}{2}x} \right|_0^4 = 12.778 \text{ 3dp}$$

iii $\int_4^4 \frac{5x+13}{(2x+1)(x+4)} dx$

~~Quotient Rule~~

$$\frac{g(x)}{h(x)} = g$$

$$f(x)g(x) = F(x)g(x) - \int f(x)g'(x) dx$$

$$(2x+1)(x+4)$$

$$f(x) = 2x+1 \quad g(x) = x+4$$

$$f(x) = x^2 + x + c \quad g'(x) = 1$$

$$(x^2 + x + c)(x+4) - \int (x^2 + x + c)(1) dx$$

$$x^3 + x^2 + xc + 4x^2 + 4x + 4c - \frac{x^3}{3} + \frac{x^2}{2} + c$$

$$= \frac{2}{3}x^3 + 5x^2 + 4x + xc + 4c - \frac{x^3}{3} + \frac{x^2}{2} + c$$

$$b) \int f(x) = \frac{4}{x(1+4\ln x)^2} dx$$

$$= 4 \left(x(1+4\ln x)^2 \right)^{-1}$$

$$= 4 \left(x^{-1} (1+4\ln x)^{-2} \right)$$

$$u = x^{-1} (1+4\ln x)^{-2}$$

$$p = 1+4\ln x$$

$$\frac{dp}{dx} = \frac{4}{x}$$

$$\begin{aligned} dp &= \frac{4}{x} dx \\ &= 4 x^{-1} dx \end{aligned}$$

$$\begin{aligned} \frac{du}{dx} &= p^{-2} dp \\ &= -2p^{-3} dp \end{aligned}$$

$$4 \left(2x(1+4\ln x) \right)$$

c) i

Integration by parts

$$\int f(x)g(x) = Fxgx - \int Fx g'x dx$$

$$Fx = \int fx$$

$$g'x = \frac{d}{dx}(gx)$$

$$\int_{\ln 2}^{\ln 5} \frac{3e^{2x}}{\sqrt{e^x - 1}} dx$$

$$= \int (3e^{2x}) (e^x - 1)^{-1/2}$$

$$f(x) = 3e^{2x}$$

$$g(x) = (e^x - 1)^{-1/2}$$

$$Fx = 3 \frac{e^{2x}}{2} + C$$

$$g'x = \frac{d}{dx} (e^{-\frac{x}{2}} - 1)^{-1/2}$$

$$= \frac{1}{2} e^{-x/2} F_{\frac{1}{2}}$$

$$= (3e^{2x}) (e^x - 1)^{-1/2} - \int 3 \frac{e^{2x}}{2} + \frac{1}{2} e^{-x/2}$$

$$\int_1^e \ln x = x \ln x - x$$

$$= \frac{e}{2} - \frac{1}{2}$$

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$$3ai \quad \left(\frac{1+i}{1-i} \right)^3 = -i$$

$$\begin{aligned} \text{ii} \quad \frac{3}{(1+i)(2+i)} &= \frac{3}{10} - \frac{9i}{10} \\ &= \frac{3-9i}{10} \end{aligned}$$

$$\begin{aligned} \text{iii} \quad \frac{1+i}{1-i} - (1+2i)(2+2i) + \frac{3-i}{1+i} \\ = 3-7i \end{aligned}$$

$$\begin{aligned} \text{iv) } 2i(i-1) + (\sqrt{3}-i) + (1+i)(1+i) \\ = \sqrt{3}-3i \end{aligned}$$

$$\begin{aligned} \text{b) } z &= (4+7i)(3-2i) \\ &= 26+13i \end{aligned}$$