
Transient Response Modelling of a Vehicle

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Abstract - This paper presents a 3D vehicle body and suspension model, created using MATLAB and Simulink. The model is created as a 7-DOF system and simulates the system response of a Formula Ford race car as it performs a double lane change manoeuvre. The model utilised data and validated against a simulator run of the car and manoeuvre, performed by a BPS simulator and recorded through MoTec. Similarly, looking at previous research allowed for further validation of the mathematical theory and Simulink model. Overall, the paper suggests that the Simulink model provided an accurate representation of the car during the manoeuvre, accurately simulating the pitch and roll of the sprung mass. This model now provides a foundation for further research when looking at different track or weather conditions.

1. INTRODUCTION

The field of vehicle dynamics is constantly evolving with modern technologies and advancements in research. One area that has seen considerable progress in recent years is the development of full car models using simulation software such as Simulink. A full car model, as the name suggests, is a computer simulation that models all aspects of a vehicle's behaviour, including its kinematics and dynamics. These models can be used for a variety of purposes, such as vehicle design, testing and validation, and control system development.

The purpose of this paper is to present a detailed analysis of a Formula Ford Race Car represented as a seven degree of freedom (7-DOF) Simulink model as it performs a double lane change manoeuvre. The model will be developed using the Simulink software and will include subsystems such as the suspension and tires. The model will be validated against experimental data found in previous research to ensure its accuracy and credibility. The paper will also discuss the applications and benefits of using a full car model in the vehicle dynamics field. Additionally, the paper will also compare the simulation results with the experimental data to validate the simulation model. As such, this paper will provide insight into the capabilities of Simulink as a simulation tool for vehicle dynamics, and it will demonstrate how a full car model can be used to improve vehicle design, testing, and control system development. It will also provide a valuable resource for researchers and engineers working in the field of vehicle dynamics, providing them with a comprehensive understanding of the process of creating a full car model using Simulink. For the model, the different variable and springs were provided by the simulator, however spring rate and tire information was provided by Avon, who manufacture the slick tyres used in the Formula Ford racing category (Avon Tyres, 2007).

During a double lane change manoeuvre, a vehicle's suspension dynamics play a critical role in maintaining stability and providing a comfortable ride for the occupants. The suspension system is responsible for absorbing and damping the road inputs caused by the vehicle's movement. As the vehicle performs the double lane change manoeuvre, the suspension system must respond quickly and effectively to the lateral forces generated by the steering inputs, as well as the vertical forces caused by the vehicle's weight transfer. The suspension must also control the roll, pitch and yaw of the vehicle.

A seven degree of freedom vehicle suspension model is produced by analysing the movement of the vehicle in all seven degrees of freedom, which include pitch, roll, sprung mass and each of the four unsprung wheels. The model is typically created using a combination of analytical, and numerical methods. Analytical methods involve deriving mathematical equations, Figure 1, that describe the dynamics of the suspension

system. These equations are typically based on the principles of mechanics and consider factors such as the suspension's kinematics, the forces acting on it, and the moments of inertia for each degree of freedom. Experimental methods involve evaluating the suspension system on a vehicle test rig or in a laboratory setting. Numerical methods, such as simulation, are used to integrate the analytical and experimental data and create a complete model of the suspension system. The simulation uses the mathematical equations as input and solves them using a computer program to create a virtual model of the suspension.

This model can be used to predict the suspension's behaviour under different operating conditions and validate the analytical and experimental results. When creating a model for a vehicle's suspension dynamics, using MATLAB code without Simulink is a common method. However, from a paper published in 2019 assessing a full car model response to various road frequencies, it was found that the two methods were close in agreement and either method could be accurately employed (Sanjay, Manoj, Vinod, & Pranay, 2020).

2. MATHEMATICAL MODEL

The Simulink model is based on equations deriving from Newton's second law of motion ($F=ma$) where a vehicle body reacts to a lateral and longitudinal force acting on it as it performs a double lane change. The vehicle body is a sprung mass, where it has been supported by a spring and damper at each corner. At each corner, an unsprung mass is located, representing the wheels and tires for the vehicle. Each of the unsprung masses possess their own spring and damping rate which effects the control of the body. The Equations used within this model can be seen below in Figure 1:

$$\begin{aligned}
 -m\ddot{z} &= k_{f1}(z-z_{f1}-a\theta-\beta\dot{\theta}) + C_{f1}(\dot{z}-\dot{z}_{f1}-a\dot{\theta}-\beta\dot{\theta}) \\
 &\quad + k_{r1}(z-z_{r1}-a\theta+\beta\dot{\theta}) + C_{r1}(\dot{z}-\dot{z}_{r1}-a\dot{\theta}+\beta\dot{\theta}) \\
 &\quad + k_{f2}(z-z_{f2}-a\theta-\beta\dot{\theta}) + C_{f2}(\dot{z}-\dot{z}_{f2}-a\dot{\theta}-\beta\dot{\theta}) \\
 &\quad + k_{r2}(z-z_{r2}-a\theta+\beta\dot{\theta}) + C_{r2}(\dot{z}-\dot{z}_{r2}-a\dot{\theta}+\beta\dot{\theta}) \\
 -I_m\ddot{\theta} &= b k_{r1}(z-z_{r1}+b\theta-\beta\dot{\theta}) + b C_{r1}(\dot{z}-\dot{z}_{r1}+b\dot{\theta}-\beta\dot{\theta}) \\
 &\quad + b k_{r2}(z-z_{r2}+b\theta+\beta\dot{\theta}) + b C_{r2}(\dot{z}-\dot{z}_{r2}+b\dot{\theta}+\beta\dot{\theta}) \\
 &\quad - a k_{f1}(z-z_{f1}-a\theta-\beta\dot{\theta}) - a C_{f1}(\dot{z}-\dot{z}_{f1}-a\dot{\theta}-\beta\dot{\theta}) \\
 &\quad - a k_{f2}(z-z_{f2}-a\theta+\beta\dot{\theta}) - a C_{f2}(\dot{z}-\dot{z}_{f2}-a\dot{\theta}+\beta\dot{\theta}) \\
 -I_{f1}\ddot{\theta} &= \alpha k_{f1}(z-z_{f1}-a\theta+\alpha\dot{\theta}) + \alpha C_{f1}(\dot{z}-\dot{z}_{f1}-a\dot{\theta}+\alpha\dot{\theta}) \\
 &\quad + \alpha k_{r1}(z-z_{r1}+b\theta+\alpha\dot{\theta}) + \alpha C_{r1}(\dot{z}-\dot{z}_{r1}+b\dot{\theta}+\alpha\dot{\theta}) \\
 &\quad - \beta k_{f1}(z-z_{f1}-a\theta-\beta\dot{\theta}) - \beta C_{f1}(\dot{z}-\dot{z}_{f1}-a\dot{\theta}-\beta\dot{\theta}) \\
 &\quad - \beta k_{r1}(z-z_{r1}+b\theta-\beta\dot{\theta}) - \beta C_{r1}(\dot{z}-\dot{z}_{r1}+b\dot{\theta}-\beta\dot{\theta}) \\
 -m_{f1}\ddot{z}_{f1} &= k_{f1}(z_{f1}-y_{f1}) - k_{f1}(z-z_{f1}-a\theta-\beta\dot{\theta}) - C_{f1}(\dot{z}-\dot{z}_{f1}-a\dot{\theta}-\beta\dot{\theta}) + C_{f1}(\dot{z}_{f1}-\dot{y}_{f1}) \\
 m_{f2}\ddot{z}_{f2} &= k_{f2}(z_{f2}-y_{f2}) - k_{f2}(z-z_{f2}-a\theta+\beta\dot{\theta}) - C_{f2}(\dot{z}-\dot{z}_{f2}-a\dot{\theta}+\beta\dot{\theta}) + C_{f2}(\dot{z}_{f2}-\dot{y}_{f2}) \\
 m_{r1}\ddot{z}_{r1} &= k_{r1}(z_{r1}-y_{r1}) - k_{r1}(z-z_{r1}+b\theta-\beta\dot{\theta}) - C_{r1}(\dot{z}-\dot{z}_{r1}+b\dot{\theta}-\beta\dot{\theta}) + C_{r1}(\dot{z}_{r1}-\dot{y}_{r1}) \\
 m_{r2}\ddot{z}_{r2} &= k_{r2}(z_{r2}-y_{r2}) - k_{r2}(z-z_{r2}+b\theta+\beta\dot{\theta}) - C_{r2}(\dot{z}-\dot{z}_{r2}+b\dot{\theta}+\beta\dot{\theta}) + C_{r2}(\dot{z}_{r2}-\dot{y}_{r2})
 \end{aligned}$$

Figure 1: Equations of Motion

These equations then relate to the free body diagram of the vehicle, seen at Figure 2, which can highlight what each term in the equations represent on the vehicle model. The values for the individual variables have can then be seen in Appendix 1.

When creating a mathematical model for this this simulation, it is common to convert the equations into a matrix, which can then used to understand or explain the theory further. These matrices can then be solved within MATLAB to validate the Simulink model. Looking at previous research which uses the same mathematical model as within this report, it can be seen where they create a matrix from the equations and run it within MATLAB. The authors concluded that their Simulink Model matched the MATLAB solve, therefore it is a reasonable suggestion that the theory behind our model is correct and accurate (Mitra, et

al., 2013). The limitations within the mathematical model generally come from whether a quarter-car, half-car or full-car model is used, as this reduces the number of degrees of freedom modelled and therefore the accuracy; a half-car model allows for pitch, but not roll when comparing to a full car model (Raj, Anirban, & P, 2021)

3. SIMULINK MODEL AND MOTEC

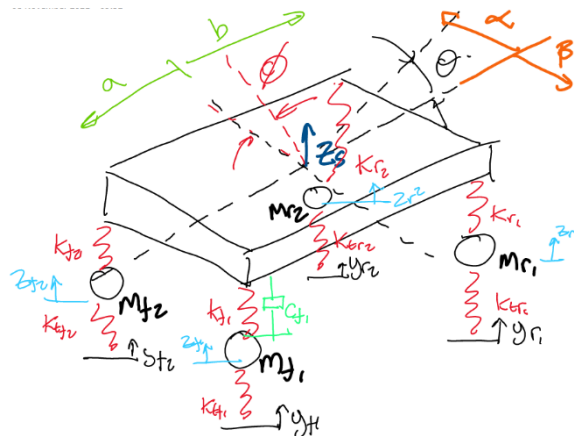


Figure 2: Drawing of Vehicle

3.1. 1-DOF Model

To begin building a Simulink Model with 7-DOF, a 1-DOF model must be built. A 1-DOF model represents a singular mass, spring and damper, where the only degree of freedom is the position of the sprung mass (Balkwill, 2018). As a signal is input into the system from a Signal Builder, such as a road bump, the sprung mass will move upwards, in the positive Z direction. An opposing force ($-ma$) is then applied to reduce this motion, which is produced by the springs and dampers. The model then uses the springs extension position and dampers position velocity to calculate the sprung mass' movement. Figure 3 then shows a simple 1DOF model made in Simulink.

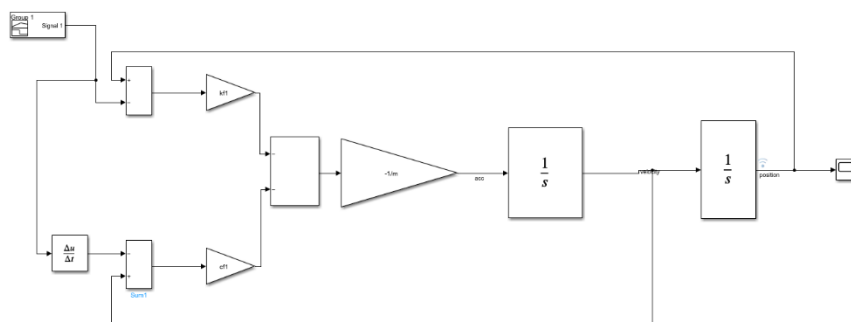


Figure 3: 1DOF Simulink Model

Looking at figure 3, the basics of Simulink models can be seen, a signal is created from the Signal Builder, this is then sent to a sum block, this then is sent to a gain block, defined at Kf1 (spring rate), then into another sum block and a gain block, $-1/m$. Now the model can calculate acceleration, then integrated once gives velocity and integrated a second time gives position. The model now uses position data in the initial spring rate sum block. Damping within the system is calculated from velocity rather than position, therefore signal from the Signal Generator needs to be derived before use within the sum block and damping coefficient gain block. Similarly, information within the systems comes from the first integration

within the calculation and is fed back into the sum block. By relaying the information calculated back into the system, the model can calculate the position and velocity of the sprung mass as it travels over the bump, and how the model damps the system.

The model is based on the motion equations seen in equation 1.

Equation 1: 1DOF Motion Equation

$$-m\ddot{z} = K(z-y) + C(\dot{z}-\dot{y})$$

3.2. 4-DOF Model

After creating a suitable 1-DOF model, the next step would either be creating a 2 or 4-DOF system. Within a 4DOF, the same principles apply, however there is now 2 unsprung masses (front and rear) which also introduces another degree of freedom, Pitch. Since a 2 DOF model is similar to a 1-DOF system, this report will focus on the progression to a 4-DOF model. The equations for the model can be seen in figure 4.

$$\begin{aligned}
 -M\ddot{z}_s &= k_f(z_s - a\theta - z_f) + c_f(\dot{z}_s - a\dot{\theta} - \dot{z}_f) \\
 &\quad + k_r(z_s + b\theta - z_r) + c_r(\dot{z}_s + b\dot{\theta} - \dot{z}_r) \\
 -I\ddot{\theta} &= b k_r(z_s + b\theta - z_r) + b c_r(\dot{z}_s + b\dot{\theta} - \dot{z}_r) \\
 &\quad - a k_f(z_s - a\theta - z_f) - a c_f(\dot{z}_s - a\dot{\theta} - \dot{z}_f) \\
 -M_f\ddot{z}_f &= k_{tf}(z_f - y_f) + c_{tf}(\dot{z}_f - \dot{y}_f) \\
 &\quad - k_f(z_s - a\theta - z_f) - c_f(\dot{z}_s - a\dot{\theta} - \dot{z}_f) \\
 -M_r\ddot{z}_r &= k_{tr}(z_r - y_r) + c_{tr}(\dot{z}_r - \dot{y}_r) \\
 &\quad - k_r(z_s + b\theta - z_r) - c_r(\dot{z}_s + b\dot{\theta} - \dot{z}_r)
 \end{aligned}$$

Figure 4: Equations for 4DOF Model

A 4-DOF, also known as a Half-Car model, includes the movement of 2 unsprung masses, the sprung mass and pitch angle of the sprung mass between the two models. When building the model, it is easier to categorise the different masses into subsystems. For example, creating the two unsprung masses as each subsystems, seen in figure 5.

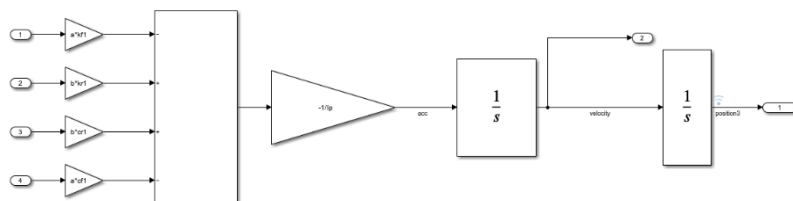


Figure 5: Unsprung Mass for 4DOF Model

Figure 5 shows the subsystem for 1 unsprung mass, subsystems now require the use of 'in' and 'out' blocks to transmit information into the system. The model uses the same principles as the 1DOF, however different variables, K_{tf} and C_{tf} to represent tyre spring and damping rates. The system is now multiplied by $-1/M_f$, (mass of unsprung wheel). Looking at figure 5, the pitch subsystem can be seen, which now incorporates 'a' and 'b' (as seen in figure 2) within the spring and damping rate gain blocks.

Figure 4 shows the equations used within the 4DOF model, these equations are then used at the inputs for each subsystem. For example, with Pitch (Figure 6) the second equation, $-I\theta$, is used, therefore the equations within the brackets will be input into the corresponding gain block. These smaller equations are simple to make with the use of sum blocks, inputting $+Zs$, $+b\theta$ and $-Zr$.

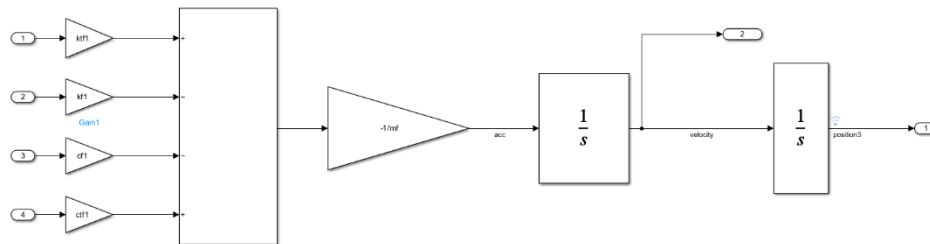


Figure 6: Pitch for 4-DOF

Previous research such as (Bhushan D. Mahajan, 2016) has created a quarter car model with Simulink in a similar manner, verifying that the research within this paper is accurate.

3.3 7-DOF Model

When creating the 7-DOF model, 2 additional unsprung masses are introduced, this simulated the left and right side from both front and rear of the car. With the additional masses, Roll is now created, as each corner acts independently based on the conditions applied to the system. With roll, an additional subsystem is created; the subsystem looks similar to the pitch and sprung mass systems, however utilising different variables. Looking at figure 1. The complete set of motion equations can be seen, where roll is now indicated by Φ (φ) with additional terms such as α and β (α, β) which represents the distance from roll centre to edge of the model (right or left).

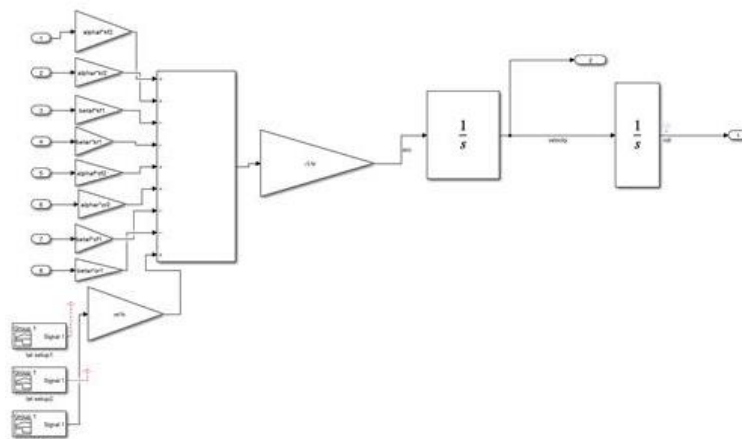


Figure 7: 7-DOF Roll Subsystem

Figure 7 shows the subsystem used for roll within the 7-DOF model. Due to the additional unsprung masses, there is 4 extra gain inputs to the first sum block; 2 for spring rates and 2 for the damping coefficients. There is also an additional torque input, which uses a signal generated from the lateral and longitudinal forces applied to the model during the lane change maneuver, the data has been recorded and output as a MATLAB file using MoTec. The three signals represent Setup 1, 2 and 3. When applying roll or pitch to the model from the MoTec data, this is done by a torque vector: $\text{torque} = m \cdot a \cdot h$.

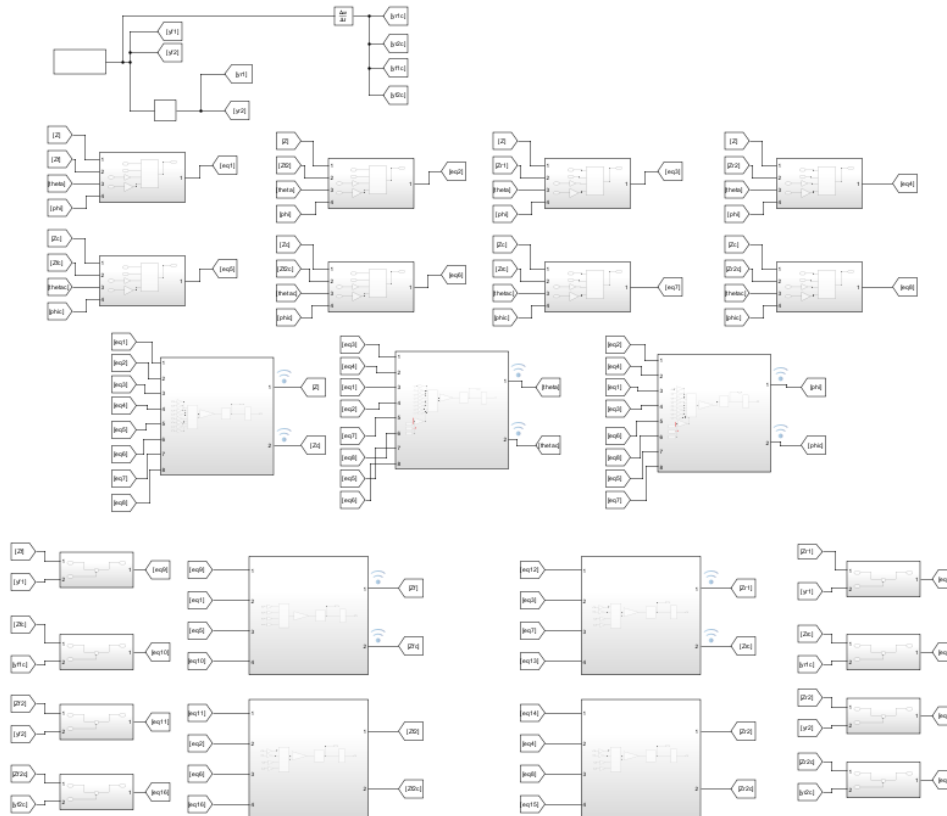


Figure 8: Complete Simulink Model

Figure 8 shows the complete 7 Degree of Freedom Simulink model. The signal generator to run the model can be seen at the top, where the outputs vary, 2 outputs are delayed using a 'Delay' block, to simulate the front and rear of a car. The additional 4 outputs are used for damping, hence the derivate block used to convert the signal to a velocity. Overall, there are 16 equation subsystems used; 8 for Sprung Mass, Pitch and Roll and 8 for the 4 Unsprung masses. Each subsystem outputs a sprung position and damping velocity. A Simulink model is a useful way of modelling a vehicles dynamic, however there are some limitations such as, the scalability where the model cannot easily be changed for other vehicles or configurations. Similarly, a full car model can have over 14 degrees of freedom, which during extensive testing with road conditions and maneuvers, can become computationally intensive.

4. RESULTS AND DISCUSSION

To understand the accuracy of the Simulink model, there needs to be comparison against similar research paper which have been written, regarding the same type of model or a vehicle recording the same data during the same manoeuvre. Similarly, the easiest step for comparison is to compare against the exact data within MoTec which had been produced during the simulator run. To analyse the data, Lateral and Longitudinal G-Force Acceleration from the simulator is compared against Roll and Pitch data from Simulink. Key features within the graphs include the two positive peaks, where a lane change has occurred and the negative troughs and peaks where the suspension attempts to reduce the body roll occurring as a result of the movement. The different spring rates used in each of the setups are found within Appendix 1.1. For each setup, the damping coefficient for the sprung mass or the tyres does not change, however

correct damping needs to be setup correctly for a certain spring rate, meaning if the spring rate changes, it can appear that the system becomes over or under damped (Shim ReStackor).

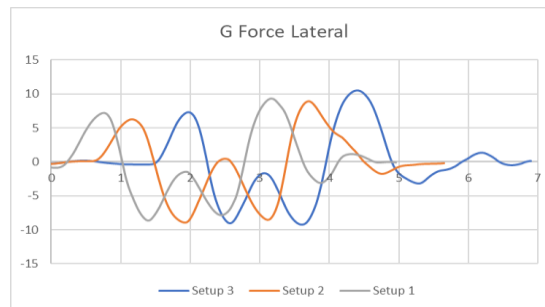


Figure 9: Lateral G-force from MoTec

Looking at Figure 9 shows the lateral G-force Data produced by MoTec, on the x-axis, is time and y-axis is g-force acceleration. The difference in peaks and troughs between each of the setups is due to a difference in timing. However, looking at Setup 3, the return towards normal at the end of the manoeuvre, around 6 seconds, takes longer to return to normal than the other setups. This could be down to a damping issue, allowing the spring and therefore vehicle body to oscillate for longer than optimum. From returning to normal after the second peak it takes approximately 2 seconds for setup 3, contrast this with setup 1 and 2 which take around 1.5 and 1 second respectively, highlights the issue with setup 3.

For setup 2, the return to normal after the second lane change appears to be slow, which could suggest the system is overdamped, as the dampers limit the rate at which the spring can extend. Compare this to setup 1, which return to 0 quickly after overshooting slightly. This appears to be a typical vehicle response and therefore the ideal setup out of the three.

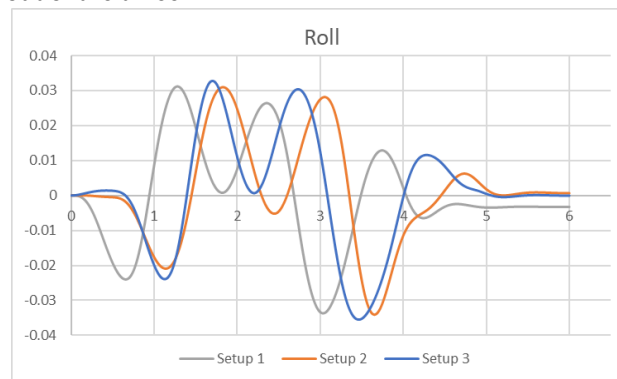


Figure 10: Roll Data from Simulink Model

Figure 10 shows the roll system response from the same lane change, similar to Figure 9, the time point for each of the setups differ, which is again due to a difference in time recordings between each setup and not a reflection of the vehicle response. Comparing figure 10 to figure 9, the graph appears to be flipped, however this is expected as MoTec records the g-force acting on the car as an outwards acceleration force, hence a positive value. Within Simulink, Roll is measured as position of the vehicle mass, and this decreases during a lane change as the system reacts, therefore a negative value. This also describes the difference in y-axis values, as Simulink is a position and MoTec is an force value; due to this difference, it is important to look at the shape of the graph and how the system responds, rather than the orientation and specific values.

In Figure 10, setup 3 does not show the same damping issue around the 6 second point, instead accurate damping and a typical response. However, looking at setup 2 from Simulink, shows a similar response to

setup 2 from MoTec, indicating a slow return to normal, suggesting overdamping has occurred. For setup 1, a similar response is seen, however, the vehicle affectively returns to the normal position, suggesting little overdamping, allowing for a quicker return and preparation for the next manoeuvre or road bump.

Longitudinal G-force values were also used as data for the Simulink model. Pitch is the Simulink equivalent to longitudinal g-force, and generally occurs due to a change in road quality, simulated by a dip or bump in the signal generator. This also can allow for a better understanding of spring stiffness due to the initial acceleration of the vehicle.

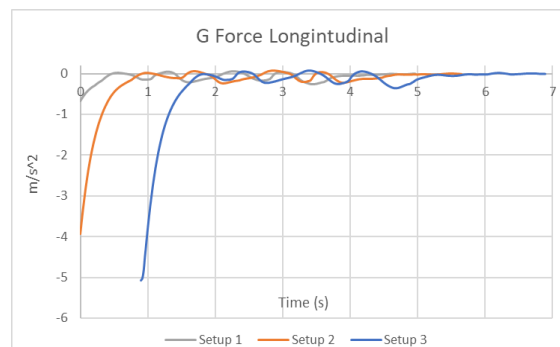


Figure 11: Longitudinal G-force from MoTec

Figure 11 shows the longitudinal g-force values produced in MoTec from the simulator run. As the test had taken place on a flat racetrack, the movements are extremely small. Setup 3 appears to have the least control, as the oscillations become greater before becoming damped. Similarly, after the final lane change, setup 3 takes longest to return to 0, after oscillating approximately 6 times over 1.5 seconds. Comparing this to setup 2 for example, where the oscillations remains a similar size. Comparably, the time to return to normal for setup 2 requires more time then recommended and there is little overshooting. This shows how pitch and roll are equally related and furthers the suggestion that setup 2 is overdamped. The initial negative starting point for the test for each of the setups is caused by the acceleration of the vehicle. For the softest rear setup (setup 3), we can see the greatest acceleration, which indicates the largest amount of movement. The opposite can be seen for the stiffest rear setup (setup1).

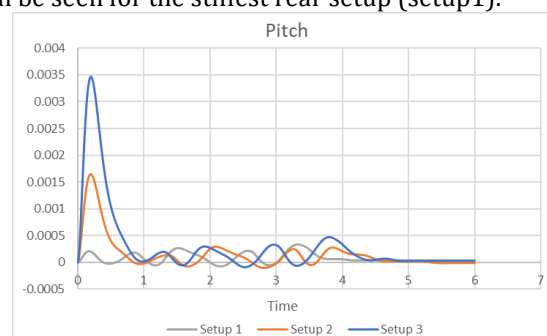


Figure 12: Pitch Data from Simulink

Pitch data produced from the Simulink model can be seen in figure 12. Like figure 11, setup 3 looks the most uncontrolled, the first peak is due to the acceleration of the vehicle and then holding a constant velocity. The peak is a positive value for the same reason as the roll data; as the car accelerates, the vehicles mass is pushed towards the rear, lifting up the front, causing the increase in pitch. This shows a softer spring rate, particularly in the rear (12 N/mm) for Setup 3 compared to 1 and 2, allowing for that increase in body movement. Little movement is then seen during the lane change for any of the setups, accurately representing the system response when comparing to the Simulator data.

Previous research within the field can also be a useful comparison for validating the accuracy of the model. For example, there have been various studies analysing the lateral accelerations of physical cars performing a double lane change; it has previously been found that an 2009 Audi A6 Avant 3.0 TDI produced lateral acceleration with a minimum of 4.47 and a maximum 4.87 m/s². Whilst this information isn't strictly relevant because of difference like all-wheel drive (AWD), weight, track condition and tyre compound, the graphs produced can help verify our model (Grabski, et al., 2022). Figure 13 shows the lateral acceleration produced by the Audi during a double lane change on wet asphalt, there are clear similarities to the roll produced by our Simulink, highlighting the model is calculating roll in an expected manner.

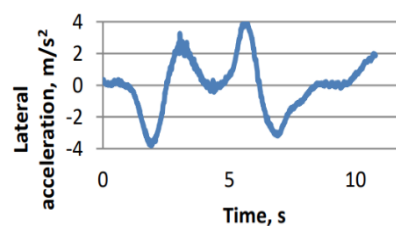


Figure 13: Lateral Accelerations of Wet Asphalt (Grabski, et al., 2022)

Similar tests have been done on larger vehicles, to assess the evasive manoeuvre characteristics, compared against simulated and real world data. A paper from a 2022 conference compared these results, where they used a simulation package called TruckSim. In a similar manner to this report, they used VBOX data from the physical test, to run the simulation and compare the accuracy (Irina-Maria, Scott, Petru, Cristina, & Tiberiu, 2022). The results between simulation and real world were similar when looking at Roll angle and Yaw rate for pavement testing, however when comparing on gravel, the results were very different, where the simulator calculated more roll and less slip then what was physically recorded, therefore highlighting errors within simulation models, and potential errors with the model created here if further analysis was to be carried out.

5. CONCLUSION

Overall, the Simulink model created within this report has been show to accurately represent a Formula Ford Race car as it performs a double lane change manoeuvre. The model was validated against simulation data produced by Base Performance Simulators, and data recorded and exported within Motec. Looking at this data, it was found that little differences between them when comparing lateral and longitudinal g-force accelerations to pitch and roll respectively. Similarly, the different effects with the setups regarding spring rates was noticeable, suggesting the data produced was reliable. Looking at the shape of the roll graphs and comparing to real world data which has been recorded was another method of validation, and aligned with our data. The mathematical model and equations used have also been proven by various research papers to be accurate, further ensuring accuracy of the model. However, no model can ever ensure completely accuracy, hence further testing would need to be conducted, for example looking at different road conditions or weather activities and then comparing them against a real world run. A further degree of accuracy could also come from more degrees of freedom; 14-DOF models have been produced and have been shown to be accurate, it should be noted that this then produces another set of limitations (TAEHYUN & CHINAR, 2007).

7. REFERENCES

- Avon Tyres. (2007, September 13). Formula Ford Slicks Spring Rate Data (Excel File). Wiltshire, England.
- Balkwill, J. (2018). CHAPTER 8 Dynamic Modelling of Vehicle Suspension. In J. Balkwill, *Performance Vehicle Dynamics* (pp. 244-250). Elsevier Inc.
- Bhushan D. Mahajan, A. A. (2016). Modeling and System Identification of a Quarter car Suspension using Simulink. *IEEE International Conference On Recent Trends In Electronics Information Communication Technology*. India: IEEE.
- Grabski, P., Jaśkiewicz, M., Jurecki, R., Kurczyński, D., Łagowski, P., Stańczyk, T., . . . Zuska, A. (2022). Lateral acceleration tests during circular driving maneuvers. *The 13th International conference on Automotive Safety (Automotive Safety 2022)*. Kielce, Poland: IOP Conf. Series: Materials Science and Engineering.
- Irina-Maria, B., Scott, B., Petru, R., Cristina, P., & Tiberiu, G. (2022). Theoretical and experimental research on the double lane change maneuver . *UNIVERSITARIA SIMPRO 2022* (p. 373). Petroșani: MATEC Web of Conferences 373, 00054 (2022).
- Mitra, A., Benerjee, N., Khalane, H., M.A, S., Joshi, D., & G.R, B. (2013). Simulation and Analysis of Full Car Model for various Road profile on a analytically validated MATLAB/SIMULINK model. *IOSR Journal of Mechanical and Civil Engineering (IOSR-JMCE)*, 22-33.
- Raj, D., Anirban, G., & P, S. (2021). A Comparison of Quarter, Half and Full Car Models for Predicting Vibration Attenuation of an Occupant in a Vehicle. *Journal of Vibration Engineering & Technologies*, 983-1001.
- Sanjay, S., Manoj, C., Vinod, P., & Pranay, J. (2020). Modal and frequency response characteristics of vehicle suspension system using full car model. *ICEMEM-2019*. Indore, India: IOP Conf. Series: Materials Science and Engineering 810 (2020) 012056.
- Shim ReStackor. (n.d.). *Damping Correction Needed for a Spring Rate Change*. Retrieved from ReStackor Sample Applications : https://www.shimrestackor.com/Code/Sample_Applications/Spring_Change/spring-change.htm
- TAEHYUN, S., & CHINAR, G. (2007). Understanding the limitations of different vehicle models for roll dynamics studies. *Vehicle System Dynamics*, 191-216.

8. Appendix

Appendix 1.0

Table 1: Values for the System

Vehicle Mass (kg)	M_T	514
Front Corner Unsprung Mass (kg)	M_{usf}	22
Rear Corner Unsprung Mass (kg)	M_{usr}	22
Vehicle Sprung Mass (kg)	M_s	426
Centre of Gravity Height (m)	CoG	0.277
Weight Split F:R (%)	-	45:55
Wheelbase (m)	W	2.443
Front Track (m)	T_f	1.62
Rear Track (m)	T_r	1.57
Front Tyre Rolling Radius (m)	r_f	0.2667
Rear Tyre Rolling Radius (m)	r_r	0.2794
Front Motion Ratio	MR_f	1
Rear Motion Ratio	MR_r	1
Front Default Spring Rate (N/mm)	K_{sf}	18
Rear Default Spring Rate (N/mm)	K_{sr}	16
Front Roll Centre Height (mm)	h_f	0.9
Rear Roll Centre Height (mm)	h_r	59
Front Slow Bump	N/m/s	1660
Front Fast Bump	N/m/s	511
Front Slow Rebound	N/m/s	1915
Front Fast Rebound	N/m/s	511
Rear Slow Bump	N/m/s	1477
Rear Fast Bump	N/m/s	537
Rear Slow Rebound	N/m/s	1612
Rear Fast Rebound	N/m/s	537

Appendix 1.1

Table 2: Spring Rates for Different Setups

Springs Investigation		Setup 1	Setup 2	Setup 3
Front Spring Rate (N/mm)	K_{sf}	14	18	23
Rear Spring Rate (N/mm)	K_{sr}	20	16	12