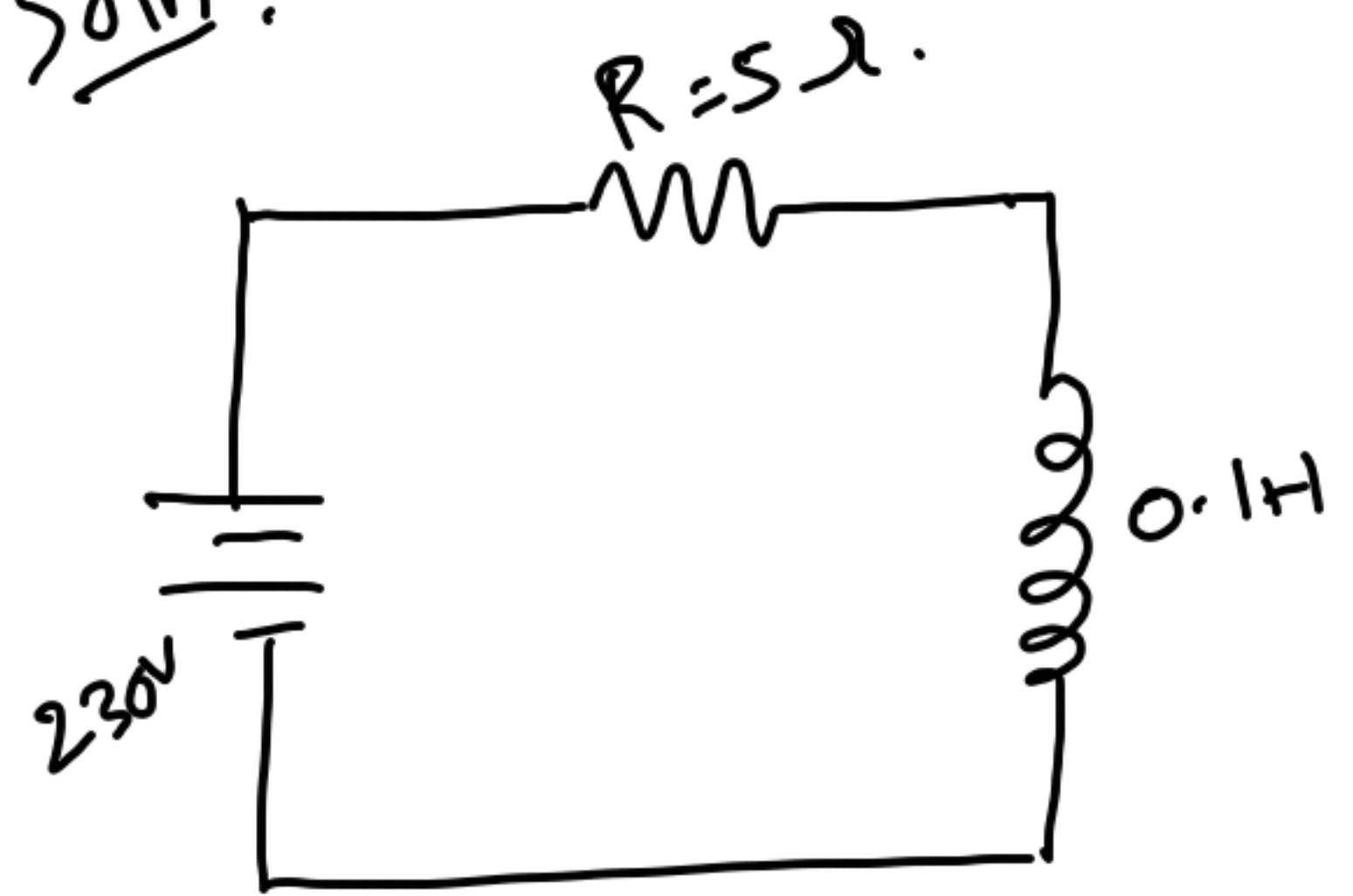


1) A coil of Resistance 5Ω and inductance of $0.1H$ is switched onto a $230V, 50Hz$ supply. Calculate the rate of rise of current at $t=0$ and $t=2\tau$. Where $\tau = \frac{L}{R}$. What is the Steady State Value of the Circuit current.

$\checkmark$ RL / RC
 $\checkmark$ Charging }
 discharging }

Soln::



To find:

Rate of Rise of current at $t=0$

Rate of Rise of current $t=2\tau$

Steady state current I_0 $\checkmark$

$$I_0 = \frac{V}{R} = \frac{230}{5} = 46A$$

$$i = I_0(1 - e^{-t/\tau})$$

$$i = 46(1 - e^{-t/\tau})$$

$$\tau = \frac{L}{R} = \frac{0.1}{5} = 0.02$$

$$i = 46(1 - e^{-t/0.02})$$

$$i = 46 - 46 e^{-t/0.02}$$

$$i = 46 - 46 e^{-50t}$$

$$\left\{ \frac{1}{0.02} = 50 \right.$$

Differentiate both the side with Respect to t.

$$\frac{di}{dt} = 0 - 46(-50)e^{-50t}$$

$$\frac{di}{dt} = 2300 e^{-50t} \text{ --- ①}$$

At $t = 0$

$$\frac{di}{dt} = 2300 e^{-50 \times 0}$$

$$\frac{di}{dt} = 2300 e^0 \quad [e^0 = 1]$$

$$\frac{di}{dt} = 2300 \times 1 = 2300 \text{ A/sec}$$

$$\left[\begin{array}{l} \text{Diff.} \\ e^{-xt} = -x e^{-xt} \\ e^{-50t} = -50 e^{-50t} \end{array} \right.$$

$$\frac{di}{dt} = \frac{A}{\text{sec}}$$

When $t = 2\tau$.

$$\text{eq ①} \Rightarrow \frac{di}{dt} = 2300 e^{-50 \times 2\tau}$$

$$\left[\tau = 0.02 \right]$$

$$\frac{di}{dt} = 2300 e^{-50 \times 2 \times 0.02}$$

$$\frac{di}{dt} = 2300 e^{-2}$$

$$\frac{di}{dt} = 2300 \times 0.135$$

$$\frac{di}{dt} = 311.27 \text{ A/Sec}$$

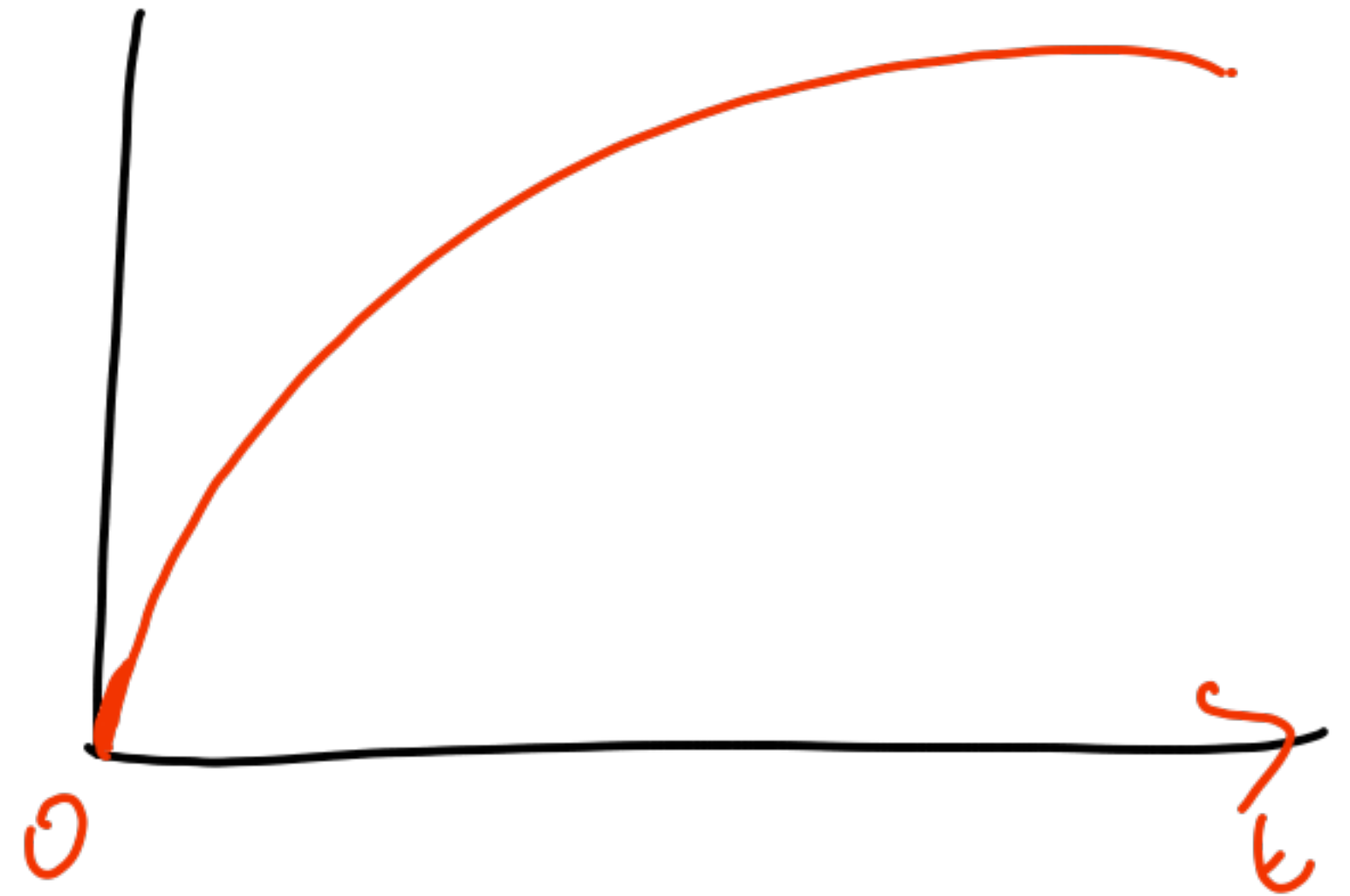
$$[e^{-2} = 0.135]$$

At $t = 0$

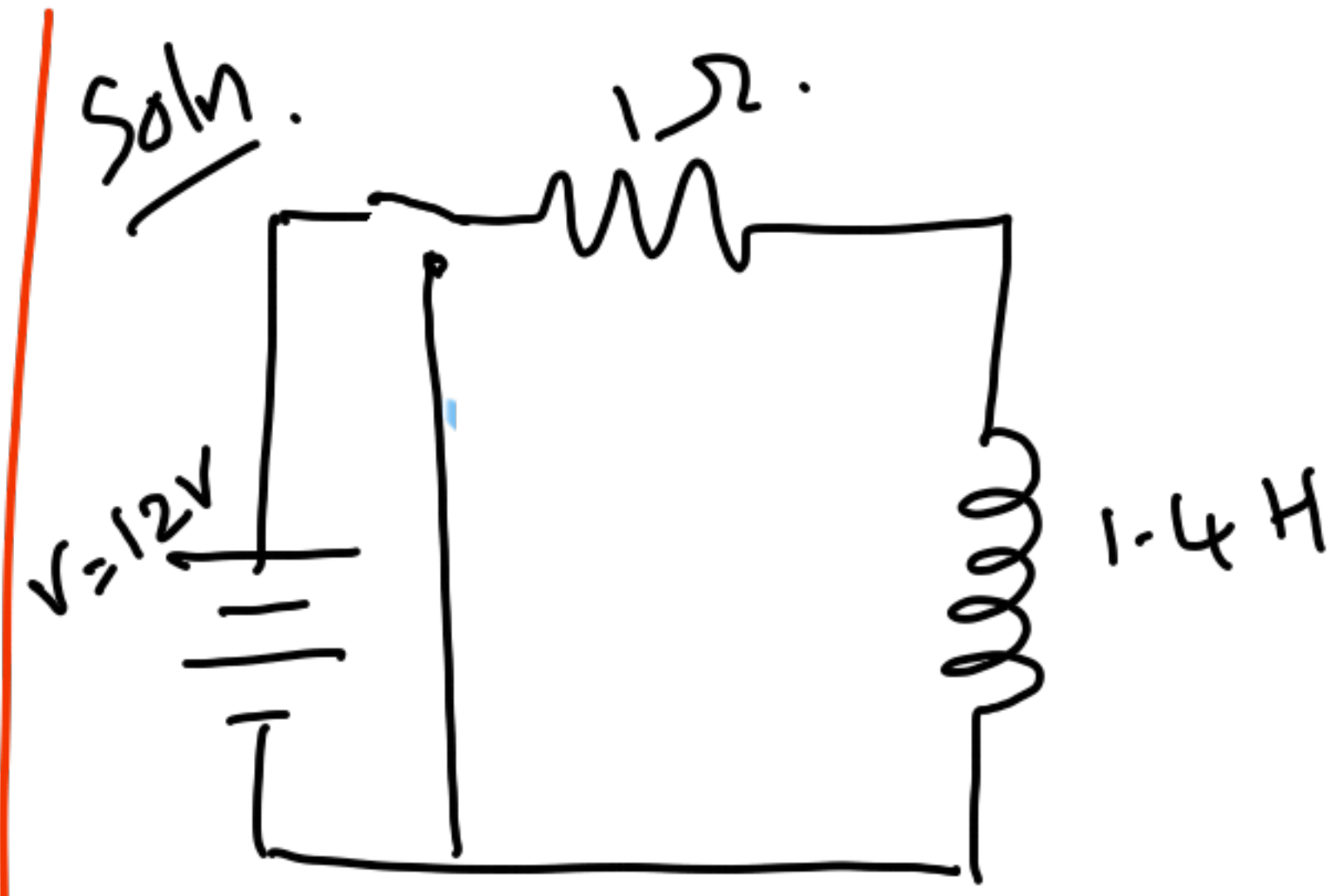
Rate of Rise of Current = 2300 A/s

At $t = 2\tau$.

Rate of Rise of Current = 310 A/s



2) A Coil having an inductance of 1.4 H and a Resistance $1\ \Omega$ is connected to a 12 V Supply through a switch. What will be the Volume of current after 400 ms of Switching ON the supply? How much time will it take for the steady state current to drop to half its value after the switch is turned ON.



To find.

$\Rightarrow i = ?$, when $t = 400\text{ ms}$.
(Charging)

$\Rightarrow t = ?$, $\frac{I_0}{2}$, (discharging)

When the inductor is charging.

$$i = I_0 (1 - e^{-t/\tau})$$

We know that $I_0 = \frac{V}{R} = \frac{12}{1} = 12 \text{ A}$

$$\tau = \frac{L}{R} = \frac{1.4}{1} = 1.4$$

$$i = 12 (1 - e^{-t/1.4})$$

To find current when $t = 400 \text{ ms} = 400 \times 10^{-3} \text{ s}$.

$$i = 12 (1 - e^{-400 \times 10^{-3} / 1.4})$$

$$i = 12 (1 - e^{-0.286})$$

$$i = 2.99 \text{ A}$$

When the inductor is discharging.

$$i = I_0 e^{-t/\tau}$$

$$2.99 = I_0 e^{-t/1.4}$$

To find time when I_0 drops half its value.

$$2.99 = \frac{I_0}{2} e^{-t/1.4}$$

$$2.99 = \frac{12}{2} e^{-t/1.4}$$

$$2.99 = 6 e^{-t/1.4}$$

$$\frac{2.99}{6} = e^{-t/1.4}$$

$$0.5 = e^{-t/1.4}$$

take log on both the side.

$$\log(0.5) = \frac{-t}{1.4}$$

$$-0.3 = \frac{-t}{1.4}$$

$$\because \log(0.5) = -0.3$$

$$0.3 \times 1.4 = t$$

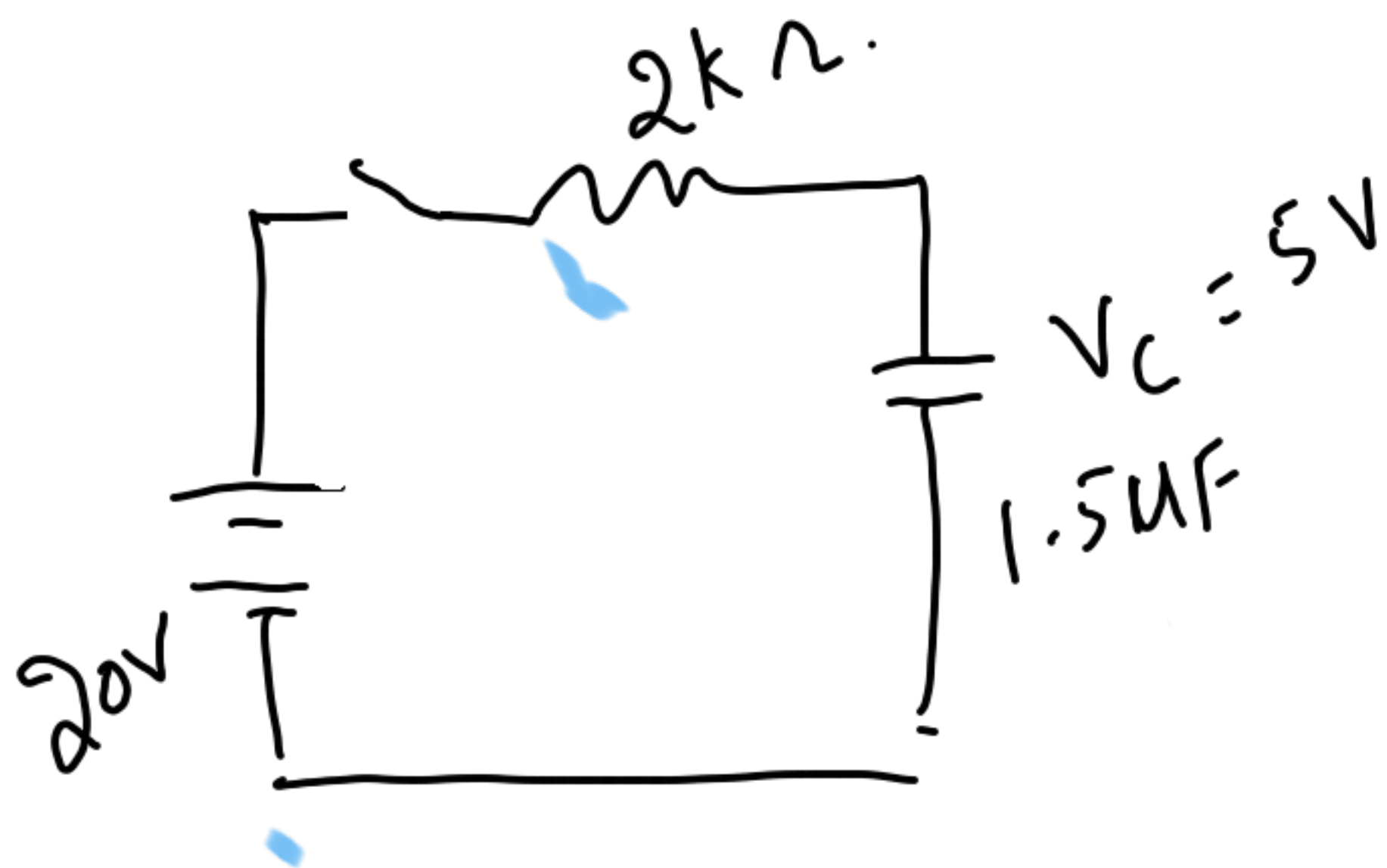
$$t = 0.42 \text{ Sec}$$

The taken for the Steady State current to drop half its value is 0.42 sec.

3) A capacitor of $1.5\mu\text{F}$ is connected to a $2\text{k}\Omega$ Resistor in Series via a switch to the 20V power supply.

Calculate the time taken by the Capacitor to attain $V_C = 5\text{V}$.

Soln



To find,
time taken to attain full charge

$$V_C = V(1 - e^{-t/\tau})$$

We know that $\tau = RC$

$$\tau = 2 \times 10^3 \times 1.5 \times 10^{-6}$$

$$\tau = 0.003$$

$$5 = 20(1 - e^{-t/0.003})$$

$$\frac{5}{20} = 1 - e^{-t/0.003}$$

$$0.25 = 1 - e^{-t/0.003}$$

$$e^{-t/0.003} = 1 - 0.25$$

$$e^{-t/0.003} = 0.75$$

taking log on both the sides

$$\frac{-t}{0.003} = \log(0.75)$$

$$\frac{-t}{0.003} = -0.125$$

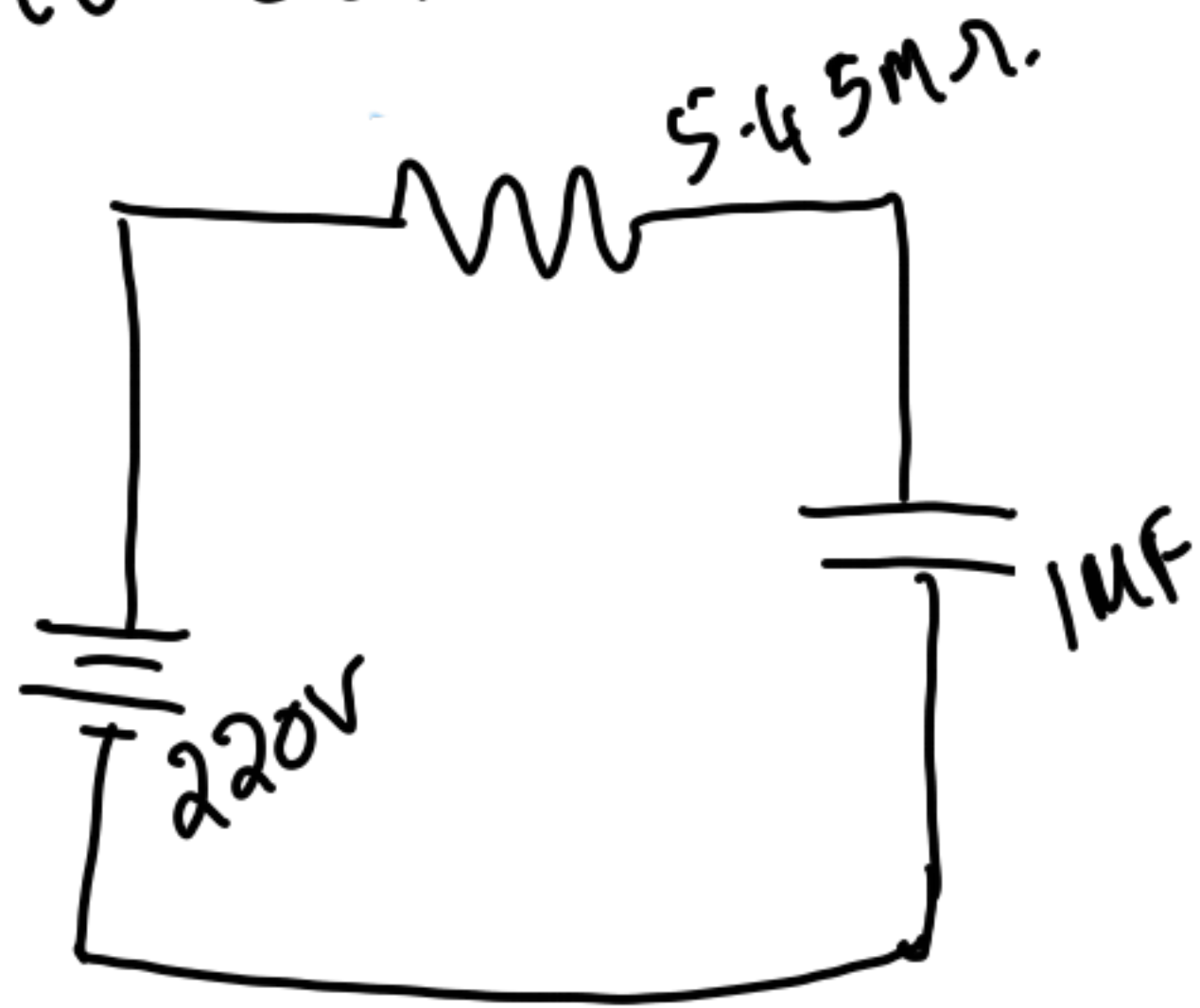
$$t = 0.125 \times 0.003$$

$$t = 3.75 \times 10^{-4}$$

$$t = 0.375 \times 10^{-3} \text{ s}$$

$$t = 0.375 \text{ ms}$$

4) A Capacitor of value $1\mu\text{F}$ and a Resistor of $5.45\text{M}\Omega$ are connected in series across a 220V power supply through a switch. Calculate the time by which the capacitor will be charged to 60% of the supply voltage.



$$V = 220\text{V}$$

$$V_C = ?$$

$$R = 5.45\text{M}\Omega.$$

$$C = 1\mu\text{F}$$

$$V_C = \frac{220}{100} \times 60\% = 132\text{V}.$$

$$V_C = 132.$$

$$V_C = V(1 - e^{-t/\tau})$$

$$\tau = RC = 5.45 \times 10^6 \times 1 \times 10^{-6}$$

$$\tau = 5.45$$

$$132 = 220 \left(1 - e^{-t/5.45} \right)$$

$$\frac{132}{220} = 1 - e^{-t/5.45}$$

$$0.6 = 1 - e^{-t/5.45}$$

$$e^{-t/5.45} = 1 - 0.6$$

$$e^{-t/5.45} = 0.4$$

taking log on both the side.

$$\frac{-t}{5.45} = \log(0.4)$$

$$\frac{-t}{5.45} = -0.398$$

$$t = 0.398 \times 5.45$$

$$\boxed{t = 2.175}$$