

MATHS EXAM 2 - Matthew Norman

Q1

(a)

$$(i) \quad y = \frac{x^2}{2x+1}$$

$$\left(\frac{f}{g}\right)' = \frac{f' \times g - g' \times f}{g^2}$$

$$\frac{d}{dx}(x^2) = 2x^{(2-1)}$$

$$= 2x$$

$$\frac{d}{dx}(2x+1) = 2$$

$$f = x^2, \quad f' = 2x, \quad g = 2x+1, \quad g' = 2$$

$$\left(\frac{f}{g}\right)' = \frac{2x(2x+1) - 2x^2}{(2x+1)^2}$$

expand the brackets:

$$2x(2x+1) = 4x^2 + 2x$$

$$= 4x^2 + 2x - \cancel{2x^2} = 2x^2 + 2x$$

$$= \frac{2x^2 + 2x}{(2x+1)^2}$$

$$(ii) \quad y = \sqrt{x^2 + 1}$$

Chain rule:

$$f = \sqrt{u}, \quad u = x^2 + 1$$

$$\frac{df(u)}{dx} = \frac{df}{du} \times \frac{du}{dx}$$

Q1 (a) (ii) continued

$$\frac{d}{du} (\sqrt{u})$$

$$\frac{d}{du} (u^{\frac{1}{2}})$$

$$\frac{d}{dx} (x^a) = a \times x^{a-1} \quad a = \frac{1}{2}$$

$$= \frac{1}{2} \times u^{\frac{1}{2}-1}$$

$$a^{-b} = \frac{1}{a^b}$$

$$u^{-\frac{1}{2}} = \frac{1}{\sqrt{u}}$$

$$= \frac{1}{2} \times \frac{1}{\sqrt{u}} = \frac{1 \times 1}{2\sqrt{u}}$$

$$= \frac{1}{2\sqrt{u}}$$

$$\frac{1}{2\sqrt{u}} \frac{dx}{dx} (x^2 + 1)$$

Substitute u value back in:

$$\frac{1}{2\sqrt{x^2+1}} \frac{d}{dx} (x^2 + 1)$$

$$\begin{aligned} \text{Find } \frac{d}{dx} (x^2 + 1) \\ = 2x \end{aligned}$$

$$\begin{aligned} f &= x^2, & f' &= 2x \\ g &= 1, & g' &= 0 \end{aligned}$$

Q1 (a) (ii) continued

$$= \frac{1}{2\sqrt{x^2+1}} \times 2x$$

(iii) $y = 3x^{3/2} + \frac{3}{x^2} - 6x$

$$\frac{d}{dx} (3x^{3/2})$$

$$= 3 \frac{d}{dx} (x^{3/2})$$

$$\frac{d}{dx} (x^a) = a \times x^{a-1}$$

$$= 3 \times \frac{3}{2} x^{3/2-1}$$

$$x^{3/2-1} = x^{1/2}$$

$$= 3 \times 3x^{1/2}$$

$$= \frac{9x^{1/2}}{2}$$

$$\frac{d}{dx} \left(\frac{3}{x^2} \right)$$

$$= 3 \frac{d}{dx} \left(\frac{1}{x^2} \right) = 3 \frac{d}{dx} (x^{-2})$$

$$= 3(-2x^{-2-1}) = 3(-2x^{-3})$$

$$= 3 \times 2 \times \frac{1}{x^3} = -\frac{1 \times 3 \times 2}{x^3} = -\frac{6}{x^3}$$

Q1 (a) (iii) continued

$$\frac{d}{dx}(6x) \quad f = 6x, \quad f' = 6$$
$$= 6$$

$$= \frac{9x^{1/2}}{2} - \frac{6}{x^3} - 6$$

$$-6 + -6 = -12$$

$$1/2 + -6 = 7/2$$

$$2 \times x^3 = 2x^3$$

$$= \frac{9x^{7/2} - 12}{2x^3} - 6$$

(iv) $y = \sec^3 4x$

$$\frac{d}{dx}(\sec^3(4x)) \quad f = u^3, \quad u = \sec(4x)$$

$$\frac{d}{du}(u^3) = 3u^{(3-1)} = 3u^2$$

$$= 3u^2 \frac{d}{dx}(\sec(4x))$$

$$= 3(\sec(4x))^2 \frac{d}{dx}(\sec(4x))$$

$$= \sec(4x) \tan(4x) \frac{d}{dx}(4x)$$

$$\frac{d}{dx}(4x) = 4$$

Q1 (a) (iv) continued

$$= 3(\sec(4x))^2 \sec(4x) \tan(4x) \times 4$$

~~simply~~ Simplify answer:

$$3 \times 4 = 12$$

$$12 \sec^2(4x) \sec(4x) \tan(4x)$$

$$a^b \times a^c = a^{b+c}, \text{ therefore:}$$

$$\sec^2(4x) \sec(4x) = \sec^{(2+1)} 4x$$

$$= \underline{\underline{12 \sec^3(4x) \tan(4x)}}$$

$$(v) \ y = (x + \sin 2x)$$

$$\frac{d}{dx}(x) \quad \text{for } f = x, \ f' = 1$$
$$= 1$$

$$\frac{d}{dx}(\sin(2x))$$

$$f = \sin(u), \ u = 2x$$

$$= \frac{d}{du}(\sin(u)) \frac{d}{dx}(2x)$$

$$\frac{d}{dx}(2x) = 2 \frac{d}{dx}(x) = 2 \times 1 = 2$$

Q1 (a) (v) continued

$$\frac{d}{du} (\sin(u)) = \cos(u)$$

$$= \cos(u) \frac{d}{dx} (2x)$$

$$\text{Sub } u = 2x$$

$$= \cos(2x) \overset{\times 2}{\frac{d}{dx} (2x)}$$

$$\text{Sub } \frac{d}{dx} (x)$$

$$= \underline{\underline{1 + \cos(2x) 2}}$$

$$(vi) \quad y = \frac{1 - 10x}{(2x - 1)^2}$$

$$\frac{d}{dx} ((2x - 1)^2)$$

$$f = u^2, \quad u = (2x - 1)$$

$$= \frac{d}{du} (u^2) \frac{d}{dx} (2x - 1)$$

$$\frac{d}{du} (u^2) = 2u^{(2-1)} = 2u$$

$$= 2u \frac{d}{dx} (2x - 1) = 2(2x - 1) \frac{d}{dx} (2x - 1)$$

Q1 (a) (vi) continued

$$\frac{d}{dx} (2x-1)$$

$$2x = 2$$

$$-1 = 0$$

$$\frac{d}{dx} (2x-1) = 2$$

$$= 2(2x-1) \times 2$$

$$= 2 \times 2 (2x-1)$$

$$= 4(2x-1)$$

$$= \frac{(-10)(2x-1)^2 - 4(2x-1)(1-10x)}{((2x-1)^2)^2}$$

$$= \frac{-2(2x-1)(-10x-3)}{(2x-1)^4}$$

$$= - \frac{2(-10x-3)}{(2x-1)^3}$$

(b) $y = \frac{1}{x} + \cos 2x$, find $\frac{d^5 y}{dx^5}$

$$\frac{d}{dx} (f(x))$$

$$\frac{d}{dx} \left(\cos(2x) + \frac{1}{x} \right)$$

Q1 (b) continued

$$= \frac{d}{dx} (\cos(2x)) + \frac{d}{dx} \left(\frac{1}{x} \right)$$

$$= (-\sin(2x)) + \frac{d}{dx} \left(\frac{1}{x} \right)$$

$$= -2 \times \frac{d}{dx} (x) \times \sin(2x) - \frac{1}{x^2}$$

$$= -2 \times 1 \sin(2x) - \frac{1}{x^2}$$

$$f'(x) = -2 \sin(2x) - \frac{1}{x^2}$$

$$f''(x)$$

$$= -2 \cos(2x) \times \frac{d}{dx} (2x) - (-2) x^{-3}$$

$$= \frac{2}{x^3} - 2 \cos(2x) \times 2 \times \frac{d}{dx} (x)$$

$$= \frac{2}{x^3} - 4 \cos(2x) \times 1$$

$$f''(x) = \frac{2}{x^3} - 4 \cos(2x)$$

$$\begin{aligned} f'''(x) &= 2(-3)x^{-4} - 4(-\sin(2x)) \times \frac{d}{dx} (2x) \\ &= 4 \times 2 \times \frac{d}{dx} (x) \times \sin(2x) - \frac{6}{x^4} \end{aligned}$$

$$= 8 \sin(2x) - \frac{6}{x^4}$$

Q1 (b) continued.

$$f^{(4)}(x)$$

$$= 8 \cos(2x) \times \frac{d}{dx}(2x) - 6(-4)x^{-5}$$

$$= 8 \cos(2x) \times 2 \times \frac{d}{dx}(x) + \frac{24}{x^5}$$

$$= 16 \cos(2x) \times 1 + \frac{24}{x^5}$$

$$= 16 \cos(2x) + \frac{24}{x^5}$$

$$f^{(5)}(x)$$

$$= 16(-\sin(2x)) \times \frac{d}{dx}(2x) + 24(-5)x^{-6}$$

$$= -16 \times 2 \times \frac{d}{dx}(x) \times \sin(2x) - \frac{120}{x^6}$$

$$= -32 \times 1 \sin(2x) - \frac{120}{x^6}$$

$$\text{Therefore } \frac{d^5 y}{dx^5} = \underline{\underline{-32 \sin(2x) - \frac{120}{x^6}}}$$

Q2

(a) Evaluate integrals giving exact form answer.

$$(i) \int \frac{2x}{(4+3x^2)} dx$$

$$= 2 \int \frac{x}{4+3x^2} dx$$

$$\text{if } u = 4x + 3x^2$$

$$\frac{d}{dx}(4) = 0 \quad \frac{d}{dx}(3x^2) = 6x^{(2-1)} = 6x$$

$$u = 0 + 6x \\ = 6x$$

$$du = 6x dx \quad dx = \frac{1}{6x} du$$

$$= \int \frac{x}{u} \times \frac{1}{6x} du$$

$$x \times 1 = 1$$

$$u \times 6x = 6u$$

$$= 2 \int \frac{1}{6u} du = 2 \frac{1}{6} \int \frac{1}{u} du$$

$$= 2 \frac{1}{6} \ln|u| = 2 \frac{1}{6} \ln|4+3x^2|$$

$$a \times \frac{b}{c} = \frac{a \times b}{c}$$

Q2 (a) (i) continued

$$= \frac{1 \times 2}{6} \ln |3x^2 + 4|$$

simplify

$$= \frac{1 \times 2}{6} = \frac{2}{6} = \frac{1}{3}$$

$$= \frac{1}{3} \ln |3x^2 + 4|$$

$$\text{add the constant} = \frac{1}{3} \ln |3x^2 + 4| + c$$

$$(ii) \int_0^4 e^{1/2 x} dx$$

$$\text{if } u = \frac{1}{2} x$$

$$\frac{d}{dx} \left(\frac{1}{2} x \right) = \frac{1}{2} \frac{d}{dx} (x)$$

$$= \frac{1}{2} \times 1 = \frac{1}{2} \quad \frac{d}{dx} = \frac{1}{2}$$

$$\frac{d}{du} = \frac{1}{2} dx$$

$$dx = 2 du$$

$$\text{sub } u \text{ back in} = \int e^u \times 2 du$$

Q2 (a) (ii) continued

apply boundaries!

$$u = \frac{1}{2}x = \frac{1}{2} \times 0 = 0$$

$$u = \frac{1}{2}x = \frac{1}{2} \times 4 = \frac{1}{2} \times \frac{4}{1} = \frac{4}{2} = \frac{2}{1} = 2$$

$$= \int_0^2 e^u du = 2(e^u)_0^2$$

$$\lim_{u \rightarrow 0} (e^u) \quad u=0 \quad (e^0 = 1)$$

$$\lim_{u \rightarrow 2} (e^u) \quad u=2 \quad (e^2)$$

$$= e^2 - 1$$

$$= 2(e^2 - 1)$$

$$(iii) \int_0^4 \frac{5x+13}{(2x+1)(x+4)} dx$$

$$\text{if } \frac{a_0}{2x+1}, \frac{a_1}{x+4}$$

$$\frac{(5x+13)(x+4)(2x+1)}{(x+4)(2x+1)}$$

$$= \frac{a_0(x+4)(2x+1)}{2x+1} + \frac{a_1(x+4)(2x+1)}{x+4}$$

$$= 5x+13 = a_0(x+4) + a_1(2x+1)$$

$$x = \frac{1}{2}, \quad x = -4$$

Q3 (a) (iii) continued

$$x = \frac{1}{2}$$

$$5\left(-\frac{1}{2}\right) + 13 = a_0\left(\left(-\frac{1}{2}\right) + 4\right) + a_1\left(2\left(-\frac{1}{2}\right) + 1\right)$$

$$5\left(-\frac{1}{2}\right) + 13 = \frac{21}{2}$$

$$\left(-\frac{1}{2}\right) + 4 + \left(2\left(-\frac{1}{2}\right) + 1\right) = \frac{7}{2} = \frac{7a_0}{2}$$

$$\frac{21}{2} = \frac{7a_0}{2} \quad \Rightarrow \quad 21 = 7a_0$$

$$\div \text{b.s by } 7) \quad \frac{21}{7} = \frac{7a_0}{7}$$

$$\frac{21}{7} = 3, \quad \frac{7a_0}{7} = a_0$$

$$a_0 = 7 \quad \text{Now find } a_1$$

$$5(-4) + 13 = a_0((-4) + 4) + a_1(2(-4) + 1)$$

$$= -7 = -7a_1$$

$$a_1 = 1$$

$$\frac{3}{2x+1} + \frac{1}{x+4}$$

$$\int_0^4 \frac{1}{x+4} dx$$

$$\int \frac{1}{u} du$$

$$(x=0) (u=4) \\ u = 0+4 \\ u = 4$$

$$(x=4) (u=8)$$

$$u = x+4 = 4+4 = 8$$

Q2 (a) (iii) continued

$$\int_4^8 \frac{1}{u} du = \ln(|u|)$$
$$= [\ln|u|]_4^8$$

$$\lim_{u \rightarrow 4^+} (\ln|u|)$$

$$= \ln|4| = \ln(2^2) = 2\ln(2)$$

$$\lim_{u \rightarrow 8^-} (\ln|u|)$$

$$= \ln|8| = 3\ln(2)$$

$$3\ln(2) - 2\ln(2) = \ln(2)$$

$$= \underline{\underline{3\ln(3) + \ln(2) \text{ or } 3.988984047}}$$

(b) find integral using suitable method.

$$f(x) = \frac{4}{x(1+4\ln x)^2} dx$$

$$f(x) = \frac{4}{x(1+4\ln x)^2}$$

$$(1+4\ln(x))^2 \quad a=1, \quad b=4\ln(x)$$

$$= 1^2 + 2 \times 1 \times 4\ln(x) + (4\ln(x))^2$$

$$= 1 + 2 \times 1 \times 4\ln(x) + (4\ln(x))^2$$

$$2 \times 1 \times 4\ln(x) = 8\ln(x)$$

$$(4\ln(x))^2 = 4^2 \ln^2(x), \quad 4^2 = 16 \quad \therefore = 16\ln^2(x)$$

Q2 (b) continued

$$= 1 + 8\ln(x) + 16\ln^2(x)$$

$$= \frac{4}{x(16\ln^2(x) + 8\ln(x) + 1)}$$

$$= x \times 1 + x \times 8\ln(x) + x \times 16\ln^2(x)$$

$$= 1 \times x + 8\ln(x) + 16\ln^2(x)$$

$$= x + 8x\ln(x) + 16\ln^2(x)$$

$$= \frac{4}{x + 8x\ln(x) + 16\ln^2(x)} dx$$

$$f(-x) = \frac{4d - x}{-x - 8\ln(-x) - 16\ln^2(-x)}$$

$$f(x) \neq f(-x) \text{ (therefore not even)}$$

Q2(c)

$$(i) \int_{\ln 2}^{\ln 5} \frac{3e^{2x}}{\sqrt{e^x - 1}} dx$$

$$= 3 \int_{\ln 2}^{\ln 5} \frac{e^{2x}}{\sqrt{e^x - 1}} dx \quad \frac{d}{dx}(e^x) = e^x$$

$$= \int \frac{e^{2x}}{\sqrt{u-1}} \times \frac{1}{e^x} du \quad u = e^x$$

$$= \int \frac{e^{2x}}{\sqrt{u-1}} \times \frac{1}{e^x} du$$

Q2 (c) (i) continued

$$\frac{e^{2x}}{\sqrt{u-1}} \times \frac{1}{u} = \frac{e^{2x}}{\sqrt{u-1}} \times \frac{1}{u} = \frac{e^{2x}}{u\sqrt{u-1}}$$

$$= \int \frac{u^x}{u\sqrt{u-1}} du \quad u = e^x \therefore e^{2x} = u^2$$

$$= \int \frac{u}{\sqrt{u-1}} du$$

$$u = e^x \therefore e^{\ln(2)} = 2$$

$$\therefore e^{\ln(5)} = 5$$

$$= \int_2^5 \frac{u}{\sqrt{u-1}} du = 3 \int_2^5 \frac{u}{\sqrt{u-1}} du$$

$$(v = \sqrt{u-1})$$

$$dv = \frac{1}{2\sqrt{u-1}} du$$

$$du = 2\sqrt{u-1} dv = \int \frac{u}{v} \times 2\sqrt{u-1} dv$$

$$= \int \frac{u}{v} \times 2v dv \quad \frac{u}{v} \times 2v = \frac{u \times 2v}{\cancel{v}} = u \times 2$$

$$= \int 2u dv$$

$$v = \sqrt{u-1} \text{ square b.s) } v^2 = u-1$$

$$+1 \text{ b.s) } v^2 + 1 = u - \cancel{1} + \cancel{1} = v^2 + 1 = u$$

Q2(c)(i) continued

$$\int 2(v^2 + 1) dv$$

$$\begin{aligned} v &= \sqrt{u-1} & (u=2) \\ &= \sqrt{2-1} \\ &= \sqrt{1} \\ &= 1 \end{aligned}$$

$$\begin{aligned} v &= \sqrt{u-1} & (u=5) \\ &= \sqrt{5-1} \\ &= \sqrt{4} = \sqrt{2^2} = 2 \end{aligned}$$

$$= \int_1^2 2(v^2 + 1) dv = 3 \times \int_1^2 2(v^2 + 1) dv$$

$$= 3 \times 2 \left(\int_1^2 v^2 dv + \int_1^2 1 dv \right)$$

$$\int_1^2 v^2 dv = \left(\frac{v^{2+1}}{2+1} \right)_1^2 = \left(\frac{v^3}{3} \right)_1^2$$

$$\lim_{v \rightarrow 1} + \left(\frac{v^3}{3} \right) = \frac{1}{3}$$

$$\lim_{v \rightarrow 2} - \left(\frac{v^3}{3} \right) = \frac{2^3}{3} = \frac{8}{3} \quad \frac{8}{3} - \frac{1}{3} = \frac{7}{3}$$

$$\int_1^2 1 dv = (1 \times v)_1^2 = (v)_1^2 = 1$$

$$= 3 \times 2 \left(\frac{7}{3} + 1 \right) = 20 = \int 20 dx$$

$$= 20x + C$$

Q2 (c) (ii)

$$\int_1^e \ln x \, dx$$

$$u = \ln(x), \quad v' = 1$$

$$\frac{d}{dx} (\ln(x)) = \frac{1}{x}$$

~~$$\int \frac{1}{x} dx = \ln(x) + C$$~~

$$\int a \, dx = ax = 1 \times x = x + C$$

$$= \left(\ln(x) x - \int \frac{1}{x} x \, dx \right)_1^e$$

$$\left(\frac{1}{x} x \right) = \left(x \ln(x) - \int 1 \, dx \right)_1^e$$

$$\int a \, dx = ax = 1 \times x = x$$
$$= (x \ln(x) - x)_1^e$$

$$\lim_{x \rightarrow 1^+} (x \ln(x) - x) = 1 \times \ln(1) - 1 = -1$$

$$\lim_{x \rightarrow e} (x \ln(x) - x) = e \ln(e) - e = e - e = 0$$

$$= 0 - (-1)$$

$$= 1$$

Therefore the value of the integral = 1

Q3

(a) Express the following in the form $a+ib$ where a and b are real numbers.

(i) $\left(\frac{1+i}{1-i}\right)^3$

$$\frac{a+bi}{c+di} = \frac{(c-di)(a+bi)}{(c-di)(c+di)} = \frac{(ac+bd)+(bc-ad)i}{c^2+d^2}$$

let, $a=1, b=1, c=1, d=-1$

$$= \frac{(1 \times 1 + 1 \times (-1)) + (1 \times 1 - 1 \times (-1))i}{1^2 + (-1)^2}$$

$$= \frac{\cancel{2}i}{\cancel{2}} = \underline{i} = \underline{i^3}$$

$$i^3 = i^2 i$$

imaginary rule makes $i = -1$ so,

$$i^2 = -1$$

$$-i \text{ b.s) } = -1i$$

multiply $1i = i$ gives $-i$.

Therefore the answer = $-i$

$$(ii) \quad \frac{3}{(1+i)(2+i)}$$

$$(a+bi)(c+di) = (ac-bd) + (ad+bc)i$$

$$\text{let, } a=1, b=1, c=2, d=1$$

~~$$= \frac{3}{(1+i)(2+i)}$$~~

$$= (2-1) + (1+2)i$$

$$= \frac{1+3i}{1+3i} = \frac{3}{1+3i}$$

multiply conjugate:

$$\frac{3}{1+3i} \cdot \frac{(1-3i)}{(1-3i)}$$

Find value for $(1+3i)(1-3i)$

$$(a+bi)(a-bi) = a^2 + b^2$$

$$\text{let, } a=1, b=3$$

$$\begin{aligned} \text{Therefore } &= 1^2 + 3^2 \\ &= 1 + 9 \\ &= 10 \end{aligned}$$

Substitute back in 10:

$$= \frac{3(1-3i)}{10} \quad \text{expand back into complex form:}$$

$$3(1-3i)$$

Q3 (ii) continued

$$3(1-3i)$$

$$a(b-c) = ab - ac$$

$$\text{let, } a=3, b=1, c=3i$$

$$\begin{aligned}\text{Therefore} &= (3 \times 1) - (3 \times 3i) \\ &= 3 - 9i\end{aligned}$$

$$= \frac{3-9i}{10}$$

$$\frac{a \pm b}{c} = \frac{a}{c} \pm \frac{b}{c}$$

$$\text{a, } a=3, b=-9i, c=10$$

$$\frac{3-9i}{10} = \frac{3}{10} - \frac{9i}{10}$$

$$\text{Therefore the answer} = \underline{\underline{\frac{3}{10} - \frac{9i}{10}}}$$

$$\text{(iii)} \quad \frac{1+i}{1-i} - (1+2i)(2+2i) + \frac{3-i}{1+i}$$

$$\frac{1+i}{1-i} \quad \frac{(ac+bd) + (bc-ad)i}{c^2+d^2}$$

$$\text{let, } a=1, b=1, c=1, d=-1$$

$$= \frac{(1 \times 1 + 1 \times (-1)) + (1 \times 1 - 1 \times (-1))i}{1^2 + (-1)^2}$$

$$= \frac{2i}{2} = i$$

Q3 (iii) continued

$$(1+2i)(2+2i) \quad (a+bi)(c+di) = (ac-bd) + (ad+bc)i$$
$$= (2-4) + (2+4)i$$
$$= -2 + 6i$$

$$\frac{3-i}{1+i} \quad \frac{(ac+bd) + (bc-ad)i}{c^2+d^2}$$

$$\text{let, } a=3, b=-1, c=1, d=1$$

$$\frac{(3+(-1)) + (-1)(-3)i}{1^2+1^2}$$

$$= \frac{2-4i}{2}$$

$$= 1-2i$$

$$\text{Therefore: } i+2-6i+1-2i$$

$$2+1=3$$

$$-6i-2i+i=-7i$$

Therefore the answer is $3-7i$

$$\text{(iv) } 2i(i-1) + (\sqrt{3}-i) + (1+i)(1+i)$$

$$2i(i-1) \quad (ai)(b+ci) = -ac + abi$$

$$\text{let, } a=2, b=-1, c=1$$

$$= -2 \times 1 + 2 \times -1i$$

$$= -2-2i$$

Q3 (iv) continued

$$(1+i)(1+i) \quad (a+bi)(c+di) = (ac-bd) + (ad+bc)i$$

$$\text{let, } a=1, b=1, c=1, d=1$$

$$= (1 \times 1 - 1 \times 1) + (1 \times 1 + 1 \times 1)i$$

$$= 0 + 2i$$

$$= 2i$$

$$= -2 - 2i + \sqrt{3} - i + 2i$$

$$-2i - i + 2i = -i$$

$$= -i - 2 + \sqrt{3}$$

$$\underline{\underline{= (-2 + \sqrt{3}) - i}}$$

Q3

(b) express the following complex number in polar form.

$$Z = (4 + 7i)(3 - 2i)$$

$$(a + bi)(c + di) = (ac - bd) + (ad + bc)i$$

From the complex number let:

$$a = 4, b = 7, c = 3, d = -2$$

$$\begin{aligned}(ac - bd) &= (4 \times 3) - (7 \times -2) \\ &= 12 - (-14) \\ &= 26\end{aligned}$$

$$\begin{aligned}(ad + bc)i &= (4 \times -2) + (7 \times 3)i \\ &= (-8) + (21)i \\ &= 13i\end{aligned}$$

$$Z = 26 + 13i \quad (\text{Now convert to polar form})$$

$$\begin{aligned}r &= \sqrt{x^2 + y^2} \\ &= \sqrt{26^2 + 13^2} \\ &= \sqrt{676 + 169} \\ &= \sqrt{845} \\ &= 13\sqrt{5} \text{ or } 29.07\end{aligned}$$

$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{13}{26}\right) \\ \theta &= 26.56505118^\circ \\ \theta &= 90 - 26.56505118 \\ \theta &= 63.43494882^\circ \\ \theta &= 63.43^\circ\end{aligned}$$

Therefore in polar form $Z = 29.07 \angle 63.43^\circ$

Q3

(c)

Find the quotient of $1+3i$ and $1+i$

$$\frac{1+3i}{1+i} \cdot \frac{1-i}{1-i} = \frac{1-i+3i-3i^2}{1-i^2}$$

$$= \frac{1+2i-3(-1)}{1-1(-1)}$$

$$= \frac{4+2i}{2}$$

Now divide into 2:

$$4 \div 2 = 2, \quad 2i \div 2 = i$$

Therefore the answer is $2+i$