

Student Name: Odysseas Androutsos

Student ID number: 2005263

Module Title: Analytical Methods

Module Code: ACHA4000

Date: 16th May 2022

Time: 3 pm

Q1)

$$a) p(x) = \sqrt{\frac{1}{2x^3} + (2x)^3 - \frac{6}{7}}$$

$$= \left| \frac{1^{-3}}{2} + 8x^3 - \frac{6}{7} \right|^{\frac{1}{2}}$$

$$e'(x) = \frac{1}{2} \left(\frac{x^{-3}}{2} - 8x^3 - \frac{6}{7} \right)^{-\frac{1}{2}} \times \left(-\frac{3}{2} x^{-4} + 24x^2 \right)$$

$$= \frac{-\frac{3}{2} x^{-4} + 24x^2}{\sqrt{\frac{x^{-3}}{2} + 8x^3 - \frac{6}{7}}}$$

$$e'(x) = \frac{-\frac{3}{2} x^{-4} + 24x^2}{\sqrt{\frac{1}{2x^3} + 8x^3 - \frac{6}{7}}}$$

b) $g(x) = \frac{5x+3}{2x+1}$ $\rightarrow u$

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$\frac{du}{dx} = 5$$

$$\frac{dv}{dx} = 2$$

$$g'(x) = \frac{(2x+1) \cdot 5 - (5x+3) \cdot 2}{(2x+1)^2}$$

$$= \frac{10x+5-10x-6}{(2x+1)^2}$$

$$\boxed{g'(x) = \frac{-1}{(2x+1)^2}} = -(2x+1)^{-2}$$

$$g''(x) = \underbrace{(-1)(-2)} (2x+1)^{-3} \cdot \frac{d}{dx}(2x+1)$$

$$= 2 \cdot (2x+1)^{-3} \cdot 2$$

$$\boxed{g''(x) = \frac{4}{(2x+1)^3}}$$

Q2)

$$y = \frac{(3x+1) \sin^3(x)}{e^{2x}}$$

Take the logarithm of both sides

$$\ln y = \ln \left[\frac{(3x+1) \sin^3(x)}{e^{2x}} \right]$$

$$\ln y = \ln(3x+1) + \ln \sin^3(x) - \ln e^{2x}$$

$$\ln y = \ln(3x+1) + 3 \ln \sin(x) - 2x \quad \text{since } \ln e^x = x$$

$$\ln y = \ln(3x+1) + 3 \ln \sin(x) - 2x$$

Derive both sides

$$\frac{1}{y} \frac{dy}{dx} = \frac{3}{(3x+1)} + \frac{3 \cos x}{\sin x} - 2$$

$$\frac{dy}{dx} = y \left[\frac{3}{3x+1} + \frac{3 \cos x}{\sin x} - 2 \right]$$

substitute y

$$\frac{dy}{dx} = \left[\frac{(3x+1) \sin^3(x)}{e^{2x}} \right] \cdot \left[\frac{3}{3x+1} + \frac{3 \cos x}{\sin x} - 2 \right]$$

Q3)

$$F(x, y) = x^2 \sin(2x + 3y)$$

$$a) \frac{dF(x, y)}{dx} = \cancel{2x} \sin(2x + 3y) + x^2 (2) \cos(2x + 3y)$$

$$\frac{dF(x, y)}{dx} = 2x \sin(2x + 3y) + 2x^2 \cos(2x + 3y)$$

$$b) \frac{d^2 F(x, y)}{dy dx} = \frac{d}{dy} \left[\frac{d}{dx} (x^2 \sin(2x + 3y)) \right]$$

← This is the first derivative

$$\frac{d^2 F(x, y)}{dy dx} = \frac{d}{dy} [2x \sin(2x + 3y) + 2x^2 \cos(2x + 3y)]$$

$$= 2x (3) \cos(2x + 3y) + 2x^2 (3) [-\sin(2x + 3y)]$$

$$\boxed{\frac{d^2 F(x, y)}{dy dx} = 6x \cos(2x + 3y) - 6x^2 \sin(2x + 3y)}$$

Q4

$$f(x) = -2x^3 + x^2 + 8x - 10 \quad \text{along } x \text{ interval } [-4, 4]$$

For $x = -4$

$$\begin{aligned} f(x) &= -2(-4)^3 + (-4)^2 - 10 \\ &= -2(-64) + 16 - 32 - 10 \\ &= 128 + 16 - 32 - 10 \\ &= 102 \end{aligned}$$

Point A $(-4, 102)$

For $x = 4$

$$\begin{aligned} f(x) &= -2(4)^3 + (4)^2 + 8(4) - 10 \\ &= -128 + 16 + 32 - 10 \\ &= -90 \end{aligned}$$

Point B $(4, -90)$

Step 1

$$\begin{aligned} f'(x) &= (-2)(3)x^2 + 2x + 8 \\ f'(x) &= -6x^2 + 2x + 8 \end{aligned}$$

Step 2

$$f'(x) = 0 \quad -6x^2 + 2x + 8 = 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{2^2 - 4(-6)(8)}}{2(-6)}$$

$$x_{1,2} = \frac{-2 \pm \sqrt{4+192}}{-12} = \frac{-2 \pm \sqrt{196}}{-12}$$

$$= \frac{-2 \pm 14}{-12}$$

$$x_1 = \frac{-2+14}{-12} = \frac{12}{-12} \Rightarrow x_1 = -1$$

$$\begin{aligned} \text{So } y_1 &= (-2)(-1)^3 + (-1)^2 + 8(-1) - 10 = \\ &= +2 + 1 - 8 - 10 = -15 \end{aligned}$$

$$\text{Critical Point (A)} = (-1, -15)$$

$$x_2 = \frac{-2-14}{-12} = \frac{-16}{-12} = \frac{4}{3} \approx 1.33$$

$$\begin{aligned} \text{So } y_2 &= (-2)\left(\frac{4}{3}\right)^3 + \left(\frac{4}{3}\right)^2 + 8\left(\frac{4}{3}\right) - 10 = \\ &= -4.74 + 1.78 + 10.67 - 10 \\ &= -2.30 \end{aligned}$$

$$\text{Critical Point (B)} = (1.13, -2.30)$$

Step 3

$$f'(x) < 0$$

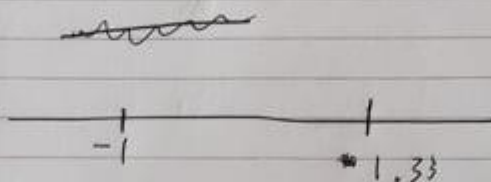
Therefore
 $f(x) \downarrow$

$$f'(x) > 0$$

Therefore
 $f(x) \uparrow$

$$f'(x) < 0$$

Therefore
 $f(x) \downarrow$



Step 4

$$f(x) = (-6)(2)x + 2$$

$$f''(x) = -12x + 2$$

Step 5

$$f''(x) = 0 \Rightarrow -12x + 2 = 0$$

$$-12x = -2$$

$$x = \frac{-2}{-12}$$

$$x = \frac{1}{6}$$

$$x = 0.167$$

$$f(x) = (-2)\left(\frac{1}{6}\right)^2 + 8\left(\frac{1}{6}\right) - 10$$

$$y = -0.0093 + 0.028 + 1.33 - 10$$

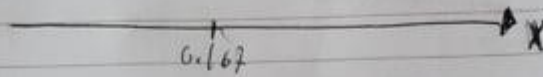
$$y = -8.65$$

The function has one inflection point
 $(0.167, -8.65)$

Step 6

$\angle^\circ > 0$
Therefore
 f is $\uparrow$

$f'' < 0$
Therefore
 f is $\cap$



Step 7

