

Degrees, Radians & Arc Length

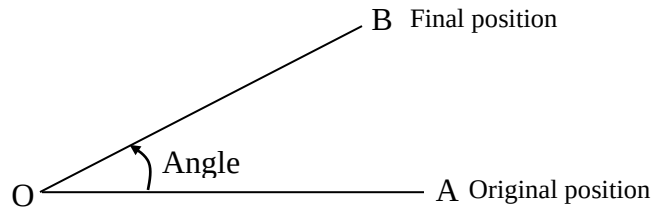


Fig.1

An angle may be looked upon as the amount of rotation or turning. In Fig.1 the line OA has been turned about O until it takes up the position OB. The angle through which the line has turned is the amount of opening between the lines OA and OB.

There are two commonly units of measurement for angles:

- **Degrees.** The symbol $^{\circ}$ is usually used for degrees.
- **Radians.** The abbreviation rad can be used for radians but usually no unit are used. So when no unit are given for the measurement of an angle, one has to assume that radians are considered.

Ⓐ Degrees

By convention if the line OA in Fig.1 rotates through a full angle and return to its original position, it is said that OA has rotated through 360° and has described one revolution.

Fig.2 shows a circle divided up into 8 equal parts.

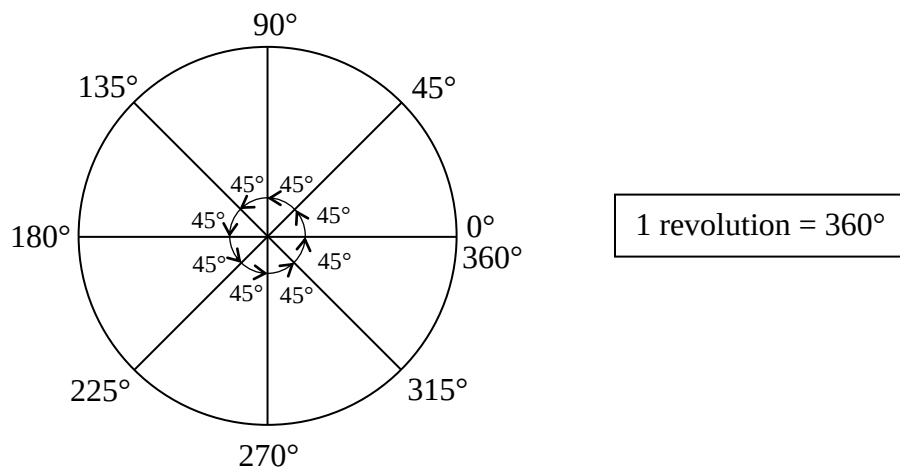


Fig.2

Each degree is subdivided into 60 minutes, written $60'$, and each minute is further divided into 60 seconds, written $60''$.

An angle that is measured in degrees, minutes and seconds can be converted to a decimal degree as follows:

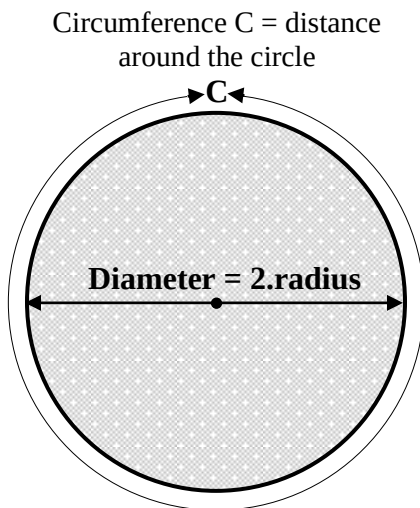
$$\begin{aligned}
 18^\circ 28' 41'' &= 18^\circ + \left(\frac{28}{60}\right)^\circ + \left(\frac{41}{60 \times 60}\right)^\circ \\
 &= 18^\circ + \left(\frac{28}{60}\right)^\circ + \left(\frac{41}{3600}\right)^\circ \\
 &= (18 + 0.4667 + 0.0114)^\circ \\
 &= 18.478^\circ \quad \text{to three decimal places}
 \end{aligned}$$

How about the other way? To convert 18.478° to degrees, minutes and seconds we proceed as follows:

$$\begin{aligned}
 18.478^\circ &= 18^\circ + (0.478 \times 60)' && \text{Multiply the fractional part of the degree by 60} \\
 &= 18^\circ + 28.68' \\
 &= 18^\circ + 28' + (0.68 \times 60)'' && \text{Multiply the fractional part of the minute by 60} \\
 &= 18^\circ + 28' + 40.8'' \\
 &= 18^\circ 28' 41'' \quad \text{to the nearest second}
 \end{aligned}$$

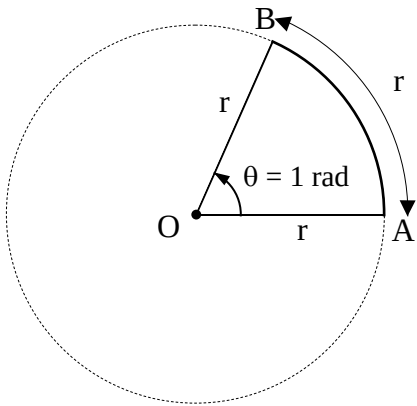
Ⓑ Radians

An alternate unit of measure of an angle is the radian.



In any circle the ratio $\frac{\text{circumference}}{\text{diameter}}$ is always constant and has a value 3.141592653...

The value of this ratio is so important that it has been given a special symbol π (the Greek letter 'pi').



If a line OA of length r rotates about O until it takes the position OB so that AB describes an arc of length r , the line is said to have rotated through 1 radian (see opposite diagram).

Because the arc described when the line OA rotates about O through a full angle is the circumference of a circle which measure $2\pi r$, the number of radians in a full angle is 2π rad.

Consequently:

$$\begin{aligned} 360^\circ &= 2\pi \text{ rad} \\ &= (2 \times 3.1416) \text{ rad} \\ &= 6.2832 \text{ rad} \quad \text{to 4 decimal places} \end{aligned}$$

Hence to convert angle θ from degrees to radians:

$$\theta^\circ = \left(\frac{\pi}{180} \times \theta \right) \text{ radians}$$

and to convert angle θ from radians to degrees :

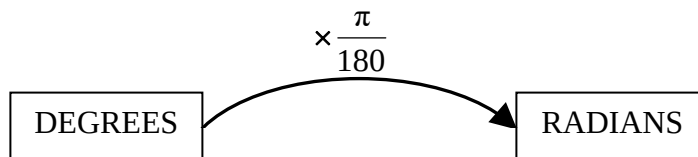
$$\theta \text{ radian} = \left(\frac{180}{\pi} \times \theta \right) \text{ degrees}$$

Examples:

1) Convert the following angles in radians.

- a) 30° b) 315°

Answer. To convert an angle from degrees to radians we have to multiply the angle value by $\frac{\pi}{180}$:



a) $30^\circ \times \frac{\pi}{180} = \frac{30.\pi}{180}$ and because 30 goes 6 times into 180 we can write $\frac{30.\pi}{180} = \frac{\pi}{6}$
so $30^\circ = \frac{\pi}{6} \text{ rad} = 0.5236 \text{ rad (to 4 decimal places)}$

b) $315^\circ \times \frac{\pi}{180} = \frac{315.\pi}{180}$ and because 45 goes 7 times into 315 and 4 times into 180, we can write

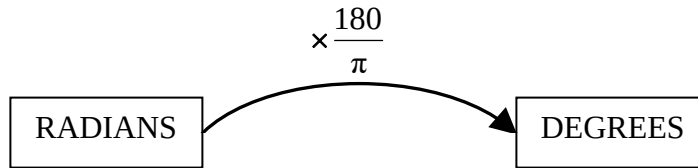
$$\frac{315.\pi}{180} = \frac{7.\pi}{4}$$

so $315^\circ = \frac{7.\pi}{4} \text{ rad} = 5.4978 \text{ rad (to 4 decimal places)}$

2) Convert the following angles in degrees.

a) $\frac{3\pi}{2}$ rad b) 0.7854 rad

Answer. To convert an angle from radians to degrees we have to multiply the angle value by $\frac{180}{\pi}$:



a) $\frac{3\pi}{2} \times \frac{180}{\pi} = \frac{540}{2} = 270^\circ$

b) $0.7854 \times \frac{180}{\pi} = (0.7854 \times 57.2958)^\circ = 45^\circ$

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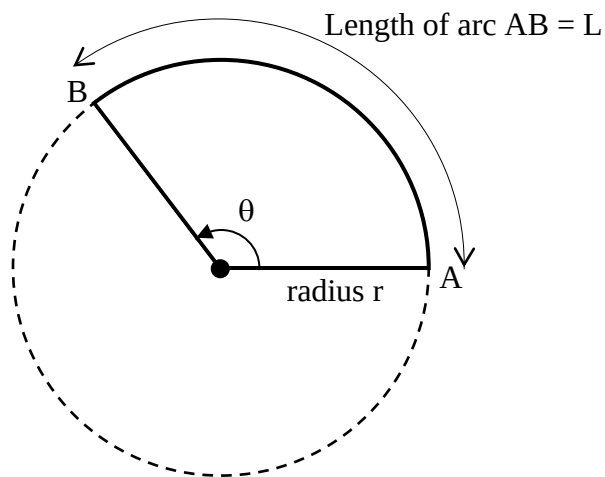


Fig.3

The length L of an arc of a circle is equal to $r.\theta$ where θ is the angle, **in radians**, subtended by the arc at the centre of the circle, with r the radius of the circle, as shown in Fig.3.

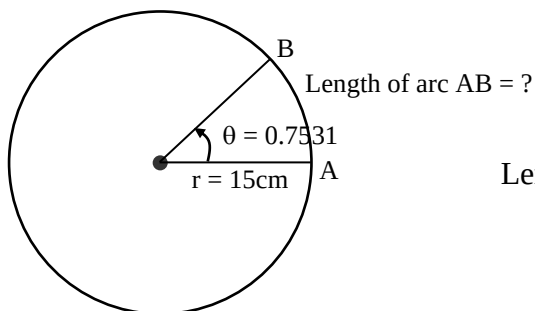
Length of arc = angle in radians $\times$ radius of circle

$$\Rightarrow L = \theta . r$$

Examples:

1) Find the length of the arc of a circle which subtends an angle of 0.7531 radians at the centre, if the radius of the circle is 15cm.

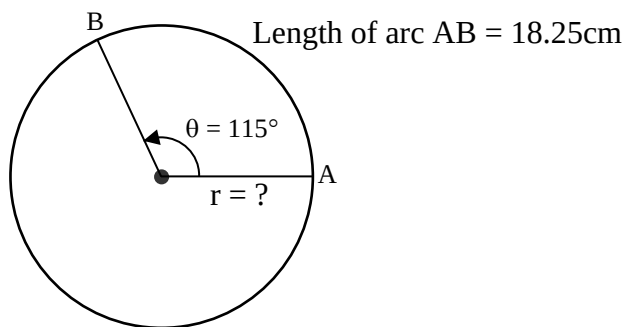
Answer



$$\begin{aligned}\text{Length of arc AB} &= \theta \cdot r \\ &= 0.7531 \times 15 \\ &= 11.297 \text{ cm to 3 decimal places}\end{aligned}$$

2).An arc 18.25cm long subtends an angle of 115° at the centre of the circle. Calculate the radius of the circle.

Answer



To use the formula:

$\text{Length of arc AB} = \theta \cdot r$
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the angle θ has to be expressed in radians.

So we convert the angle θ to radians as follows:

$$\begin{aligned}\theta = 115^\circ &= \left(115 \times \frac{\pi}{180}\right) \text{ rad} \\ &= \left(\frac{23 \cdot \pi}{36}\right) \text{ rad} && \text{because 5 goes 23 times into 115 and 36 times into 180} \\ &= 2.00713 \text{ rad}\end{aligned}$$

Now that we know the angle θ in radians, we can use the formula to find the value of the radius r :

$$\text{Length of arc AB} = \theta \cdot r$$

$$\Rightarrow 18.25 = 2.00713 \cdot r$$

$$\Rightarrow r = \frac{18.25}{2.00713} = 9.0926 \text{ cm to 3 decimal places}$$