DATE: 11 May 2020

TIME: 1:00 PM

MODULE: Advanced Stress Analysis & FEA (M6X00420)

STUDENT NAME: António Maria Paquete Januário

STUDENT NUMBER: 1805566

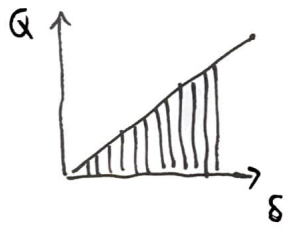
QUESTION 2

a) (i)

$$\tau = G\gamma \quad , \quad \tau = \text{SHEAR STRESS} = \frac{Q}{A}$$

$$\Longleftrightarrow \frac{Q}{A} = G\gamma \Longleftrightarrow \gamma = \frac{Q}{AG}$$

(ii)

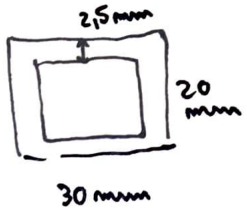


$$U = \frac{1}{2} Q \cdot \delta \quad , \quad \text{WHERE} \quad \delta = \gamma \cdot L \quad , \text{because } \gamma = \frac{\delta}{L}$$

$$U = \frac{1}{2} Q \cdot \gamma \cdot L = \frac{1}{2} Q \frac{Q}{AG} \cdot L \Longleftrightarrow$$

$$\Longleftrightarrow U = \frac{Q^2 L}{2AG}$$

b) (i)



$L = 2{,}4\,m$

$\sigma = 480\,MPa$

$E = 190\,GPa$

$U = ?$

$$U = \frac{P^2 L}{2AE} = \frac{\sigma^2 A L}{2E}$$

$$A = (0{,}03 \times 0{,}02) - (0{,}025 \times 0{,}015) = 2{,}25 \times 10^{-4}\ m^2$$

$$U = \frac{\sigma^2 A L}{2E} = \frac{(480 \times 10^6)^2 \times 2{,}25 \times 10^{-4} \times 2{,}4}{2 \times 190 \times 10^9} \simeq 327{,}4\ J$$

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QUESTION 2

b) (ii) $P.E. = mgh$ (1) , $mg = W$

$U = \frac{\sigma^2 AL}{2E}$ (2) , $AL = V$

WHEN (1) = (2)

$$mgh = \frac{\sigma^2 AL}{2E} \Leftrightarrow Wh = \frac{\sigma^2 V}{2E} \Leftrightarrow$$

$$\Leftrightarrow h = \frac{\sigma^2 V}{2EW}$$

(iii) $m = 20\,kg$ $\quad \sigma = \sqrt{\frac{h.2EW}{V}}$, $W = mg = 20 \times 9{,}81 = 196{,}2\,N$

$h = 0{,}45\,m$ $\quad V = AL = 2{,}25 \times 10^{-4} \times 2{,}4 = 5{,}4 \times 10^{-4}\,m^3$

$$\sigma = \sqrt{\frac{0{,}45 \times 2 \times 190 \times 10^9 \times 196{,}2}{5{,}4 \times 10^{-4}}} \simeq 249{,}26\,MPa$$

iv) ~~$\phi = 20mm$ $\sigma = 249{,}26\,MPa$~~

~~$L = 2{,}4\,m$~~

~~IF, $h = \frac{\sigma^2 AL}{2EW} \Leftrightarrow A = \frac{h\,2EW}{\sigma^2 L} \Leftrightarrow \pi(R^2 - r^2) = \frac{h\,2EW}{\sigma^2 L} \Leftrightarrow$~~

~~$\Leftrightarrow R^2 - r^2 = \frac{h\,2EW}{\sigma^2 L \pi} \Leftrightarrow r = \sqrt{R^2 - \frac{h\,2EW}{\sigma^2 L \pi}} \Leftrightarrow$~~

~~$\Leftrightarrow r = \sqrt{0{,}01^2 - \frac{0{,}45 \times 2 \times 190 \times 10^9 \times 196{,}2}{\ldots}} =$~~

iv) IN NEXT PAGE

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QUESTION 2

b) (iv)

$\phi = 20\ mm = 0{,}02\ m$

$L = 2{,}4\ m$

$\sigma = 249{,}26\ MPa$

$E = 190\ GPa$

$w = 196{,}2$

$$\text{IF, } h = \frac{\sigma^2 A L}{2EW} \iff A = \frac{h\,2EW}{\sigma^2 L} \iff \pi(R^2 - r^2) = \frac{h\,2EW}{\sigma^2 L} \iff$$

$$\iff R^2 - r^2 = \frac{h\,2EW}{\sigma^2 L \pi} \iff r = \sqrt{R^2 - \frac{h\,2EW}{\sigma^2 L \pi}}$$

$$r = \sqrt{0{,}01^2 - \frac{0{,}45 \times 2 \times 190 \times 10^9 \times 196{,}2}{(249{,}26 \times 10^6)^2 \times 2{,}4 \times \pi}} = 5{,}3 \times 10^{-3}\ m = 5{,}3\ mm$$

$$\text{THICKNESS} = R - r = 10 - 5{,}3 = 4{,}7\ mm$$

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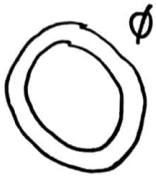
QUESTION 3



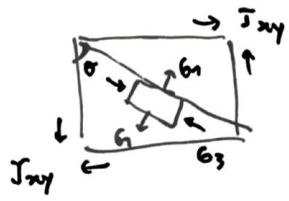
$\phi = 75\,mm = 0{,}075\,m$ $\quad \sigma = 325\ MN/m^2$ $\quad \eta = 6$

THICK $= 5\,mm = 5\times10^{-3}\,m$ $\quad \nu = 0{,}3$

$$\eta = \frac{\text{TENSILE STRENGTH}}{\text{ALLOWABLE WORKING STRESS}} \Leftrightarrow \text{ALLOWABLE STRESS} = \frac{325\times10^6}{6} = 541{,}7\ MPa$$

$\sigma_\theta = \tau_{xy}\cdot\sin 2\theta$, MAXIMUM FOR $\theta = 45°$

THEREFORE, $\sigma_{1,3} = \pm\,\tau_{xy}\sin 90°$



$\sigma_1 = \tau_{xy}$

$\sigma_3 = -\tau_{xy}$

$$T/J = \frac{\tau}{R} = \frac{G\theta}{L} \quad \therefore \quad \tau = \frac{T\cdot R}{J}\ ,\ J = \frac{\pi}{32}(D^4 - d^4) = 7{,}5\times10^{-7}\ m^4$$

$$\tau = \frac{0{,}075/2\ .T}{7{,}5\times10^{-7}} = 50000\ T$$

a) $\sigma_1 - \sigma_3 = \sigma_A$, because $\eta = \frac{\sigma_Y}{\sigma_A} \Leftrightarrow \sigma_A = \frac{\sigma_Y}{\eta}$ (PREVIOUSLY CALCULATED)

$$\tau_{xy} - \tau_{xy} = 54{,}17\times10^6 \Leftrightarrow 2\cdot(50000\,.T) = 54.17\times10^6$$

$$T = \frac{54.17\times10^6}{2\cdot50000} = 541{,}7\ Nm$$

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QUESTION 3

b) $$\left[(\sigma_1 - \cancel{\sigma_2}_{0})^2 + (\cancel{\sigma_2}_{0} - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right] = 2\sigma_A^2$$

$$\tau_{xy}^2 + (-(-\tau_{xy}))^2 + (-\tau_{xy} - \tau_{xy})^2 = 2\sigma_A^2$$

$$\tau_{xy}^2 + \tau_{xy}^2 + (2\tau_{xy})^2 = 2\sigma_A^2$$

$$2\tau_{xy}^2 + (2\tau_{xy})^2 = 2\sigma_A^2$$

$$2\tau_{xy}^2 + 4\tau_{xy}^2 = 2\sigma_A^2$$

$$6\tau_{xy}^2 = 2\sigma_A^2$$

$$\tau_{xy} = \sqrt{\frac{1}{3}\sigma_A^2}$$

$$50000T = \sqrt{\frac{1}{3}\cdot(54{,}17\times10^6)^2}$$

$$T = 625{,}5 \text{ Nm}$$

c) $\phi_e = 50$ mm

$\phi_i = 44$ mm

$\eta = 2{,}75$

$\sigma_y = 880$ MN/m^2

$$\tau = \frac{T\cdot R}{J}, \quad J = \frac{\pi}{32}(0{,}05^4 - 0{,}044^4) = 2.46\times10^{-7}\text{ m}^4$$

$$\tau = \frac{0{,}05/2\cdot T}{2{,}46\times10^{-7}} = 101626\,T$$

$$\tau_{xy} = \sqrt{\frac{1}{3}\left(\frac{\sigma_y}{\eta}\right)^2} \iff 101626\cdot T = \sqrt{\frac{1}{3}\left(\frac{880\times10^6}{2{,}75}\right)^2}$$

$$T = 1817{,}96 \text{ Nm}$$

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QUESTION 3

d)

LOW CARBON STEEL

$T_{STEEL} = 625,5 \text{ Nm}$

TITANIUM

$T_{TITA} = 1817,96 \text{ Nm}$

$$\% \text{ DIFFERENCE} = \frac{T_{TITA} - T_{STEEL}}{\frac{T_{TITA} + T_{STEEL}}{2}} \times 100 = \frac{1817,96 - 625,5}{\frac{1817,96 + 625,5}{2}} \times 100 = 97,6 \%$$

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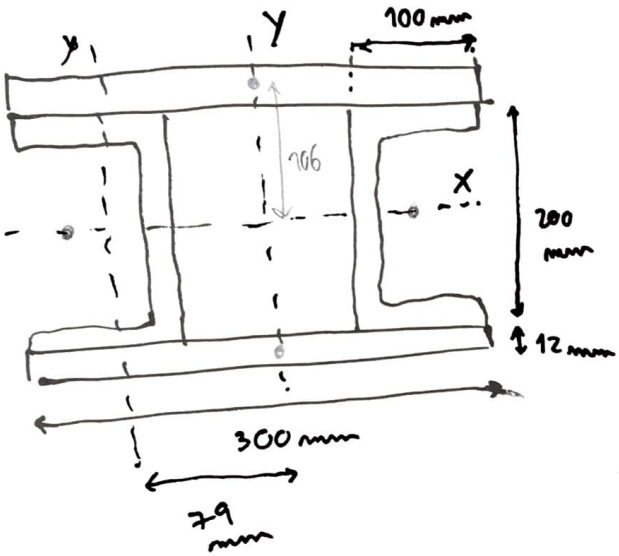
QUESTION 4



[: $I_{xx} = 26{,}2 \times 10^6 \ mm^4$

$I_{yy} = 1{,}57 \times 10^6 \ mm^4$

$A = 4400 \ mm^2$

$L = 8 \ m$ $\quad a = 1/7500$

$E = 200 \ GPa$ $\quad \sigma = 320 \ MPa$

a) $I_{TOTAL} = 2 \times I_{PLATES} + 2 \ I_{CHANNELS}$

$$I_{xx} = 2 \times \left(\frac{b d^3}{12} + A h^2 \right) + 2 \ I_{xx\,CHANNELS}$$

$$I_{xx} = 2 \times \left(\frac{300 \times 12^3}{12} + (300 \times 12) . 106^2 \right) + 2 \times (26{,}2 \times 10^6) = 133385600 \ mm^4$$

$$I_{yy} = 2 \times \left(\frac{b d^3}{12} \right) + 2 \times \left(I_{yy\,CHANNELS} + A h^2 \right)$$

$$I_{yy} = 2 \times \left(\frac{12 \times 300^3}{12} \right) + 2 \times \left(1{,}57 \times 10^6 + 4400 \times 79^2 \right) = 112060800 \ mm^4$$

$$\boxed{I_{yy} < I_{xx}}$$

IT WILL BUKLE ABOUT THE Y AXIS!

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QUESTION 4

b) $K = \sqrt{\frac{I}{A}} = \sqrt{\frac{112060800}{2\times(300\times12)+2\times4400}} = 83,69 \text{ mm}$

c) SLENDERNESS RATIO $= \frac{L}{K} = \frac{8}{83,69\times10^{-3}} = 95,59$

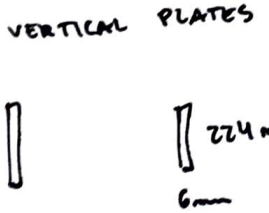
d) PINNED - PINNED

$$P_e = \frac{\pi^2 E I}{L^2} = \frac{\pi^2 \times 200\times10^9 \times (112060800\times10^{-12})}{8^2} = 3456,2 \text{ kN}$$

e)

$$P_A = \frac{\sigma_y A}{1+a(L/K)^2} = \frac{320\times10^6 \times (2\times(300\times12)+2\times4400)\times10^{-6}}{1+\frac{1}{7500}(95,59)^2} = 2308 \text{ kN}$$

f) VERTICAL PLATES



$I_{xx_{NEW}} = \frac{6\times224^3}{12} = 5619712 \text{ mm}^4$

$I_{yy_{NEW}} = \frac{224\times6^3}{12} + (224\times6).153^2 = 31465728 \text{ mm}^4$

NEW TOTAL MOMENT OF INERTIA:

$I_{xx_{TOTAL}} = I_{xx} + 2I_{xx_{NEW}} = 13338560 + 2\times5619712 = 144625024 \text{ mm}^4$

$I_{yy_{TOTAL}} = I_{yy} + 2I_{yy_{NEW}} = 112060800 + 2\times31465728 = 174992256 \text{ mm}$

· NEW STRUCTURE WILL BUCKLE ABOUT X AXIS!

$$P_e = \frac{\pi^2 . E . I}{L^2} = \frac{\pi^2 \times 200\times10^9 \times (144625024\times10^{-12})}{8^2} = 4460,5 \text{ kN}$$

∴ THERE WAS AN INCREASE OF EULER BUCKLING LOAD FROM 3456,2 kN TO 4460,5 kN!