

Q1)

$$a) \quad A = \begin{pmatrix} 4 & 3 & 4 \\ 1 & 2 & 2 \\ 0 & 4 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 3 & -1 & 3 \\ 3 & 2 & 5 \\ 1 & 2 & 4 \end{pmatrix} \quad C = (2 \ 3 \ 4 \ 5)$$

$$(i) \quad 3C = (6 \ 9 \ 12 \ 15)$$

$$(ii) \quad AB = \begin{pmatrix} 25 & 19 & 43 \\ 11 & 13 & 21 \\ 13 & 22 & 24 \end{pmatrix}$$

(iii) Calc det

$$b) \quad (i) \quad = 5(21 - 32) - 1(0 - 8) + 3(0 - 7) \\ = -55 + 8 - 21 \\ = -68$$

$$(ii) \quad = 1(0 - 6) + 3(0 - 15) + 4(6 - 10) \\ = -6 - 45 - 16 \\ = -67$$

5

1

7 marks

1
1
1

1
1
1

6 marks

$$\begin{aligned} c) \quad & 2x + 5y + z = 15 \\ & x + y + 4z = 13 \\ & 3x + 2y + 2z = 18 \end{aligned}$$

$$A = \begin{pmatrix} 2 & 5 & 1 \\ 1 & 1 & 4 \\ 3 & 2 & 2 \end{pmatrix} \quad |A| = 37$$

2

$$A_1 = \begin{pmatrix} 15 & 5 & 1 \\ 13 & 1 & 4 \\ 18 & 2 & 2 \end{pmatrix} \quad |A_1| = 148$$

2

$$A_2 = \begin{pmatrix} 2 & 15 & 1 \\ 1 & 13 & 4 \\ 3 & 18 & 2 \end{pmatrix} \quad |A_2| = 37$$

2

$$A_3 = \begin{pmatrix} 2 & 5 & 15 \\ 1 & 1 & 13 \\ 3 & 2 & 18 \end{pmatrix} \quad |A_3| = 74$$

2

$$x = \frac{148}{37}$$

$$y = \frac{37}{37}$$

$$z = \frac{74}{37}$$

1

$$x = 4$$

$$y = 1$$

$$z = 2$$

3

12 marks

Q2)

$$a) \quad u = 2 + 5j \quad v = 3 - 4j$$

$$(i) \quad uv = (2 + 5j)(3 - 4j)$$

$$uv = 6 - 8j + 15j - 20j^2$$

$$= 6 + 7j + 20$$

$$= 26 + 7j$$

$$(ii) \quad \frac{u}{v} = \frac{(2 + 5j)}{(3 - 4j)} \times \frac{(3 + 4j)}{(3 + 4j)}$$

$$= \frac{6 + 8j + 15j + 20j^2}{9 + 12j - 12j - 16j^2}$$

$$= \frac{6 + 23j - 20}{9 + 16}$$

$$= \frac{-14 + 23j}{25} = \text{i.e.} \left(\frac{-14}{25} + \frac{23}{25}j \right)$$

$$(iii) \quad V^2 = (3-4j)(3-4j)$$

$$= 9 - 12j - 12j + 16j^2$$

$$= 9 - 24j - 16$$

$$= -7 - 24j$$

②

$$(iv) \quad u+v = 2+5j + 3-4j$$

$$= 5+j$$

$$(v) \quad u-\bar{v} = (2+5j) - (3+4j)$$

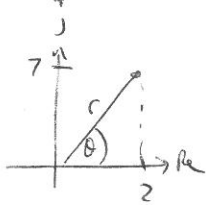
$$= 2+5j - 3-4j$$

$$= -1+j$$

②

9 MARKS

b) (i) $2+7j$ in PF:



$$\tan \theta = \frac{7}{2}$$

$$\theta = 74.05^\circ$$

$$r^2 = 7^2 + 2^2$$

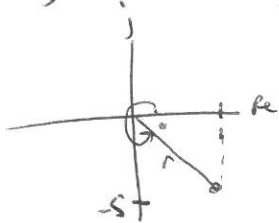
$$r = \sqrt{49+4}$$

$$r = \sqrt{53} = 7.3$$

$$\therefore \text{PF} = 7.3 (\cos 74.05^\circ + j \sin 74.05^\circ)$$

③

(ii) $1 - 5j$ in PF:



$$\tan \phi = \frac{5}{1}$$

$$\phi = 78.69^\circ$$

$$\therefore \theta = 281.31^\circ$$

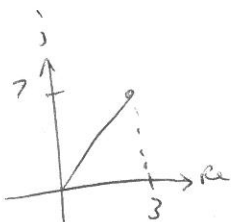
$$r^2 = 1^2 + 5^2$$

$$r = \sqrt{26} = 5.10$$

$$PF = 5.1 (\cos 281.31^\circ + j \sin 281.31^\circ)$$

(3)

c) (i) $(3 + 7j)^4$



$$r^2 = 7^2 + 3^2$$

$$r = \sqrt{58}$$

$$r = 7.62$$

$$\theta = \tan^{-1}(7/3)$$

$$\theta = 66.8^\circ$$

$$7.62 (\cos 66.8^\circ + j \sin 66.8^\circ)$$

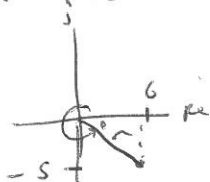
$$DMT = (7.62)^4 (\cos 267.2^\circ + j \sin 267.2^\circ)$$

$$3371.47 (-0.05 - j)$$

$$-168.57 - 3371.47j$$

(5)

(ii) $(6 - 5j)^5$



$$r = \sqrt{61} = 7.81$$

$$\phi = 39.8^\circ$$

$$\theta = 320.2^\circ$$

$$7.81 (\cos 320.2^\circ + j \sin 320.2^\circ)$$

$$DMT = (7.81)^5 (\cos 1601^\circ + j \sin 1601^\circ)$$

$$= 29057.29 (-0.95 + 0.33j)$$

$$= -27604.43 + 9588.91j$$

(5)

Q3)

$$a) \int_0^4 (4x-3)^2 dx$$

$$\text{i.e.} \int_0^4 16x^2 - 24x + 9 dx$$

$$= \left[\frac{16x^3}{3} - \frac{24x^2}{2} + 9x \right]_0^4$$

$$= \left[\frac{16(4)^3}{3} - 12(4)^2 + 9(4) \right] - [0]$$

$$= [341.33 - 192 + 36] - [0]$$

$$= 185.33$$

$$b) \int_2^4 x^{-3} + \frac{1}{x} + 1 dx$$

$$\left[\frac{x^{-2}}{-2} + \ln x + x \right]_2^4$$

$$= \left[\frac{(4)^{-2}}{-2} + \ln 4 + 4 \right] - \left[\frac{(2)^{-2}}{-2} + \ln 2 + 2 \right]$$

$$= [-0.03 + 1.39 + 4] - [-0.125 + 0.69 + 2]$$

$$= [5.36] - [2.57]$$

$$= 2.79$$

$$(iii) \int_0^3 x^{0.5} + e^{4x} dx$$

$$= \left[\frac{x^{1.5}}{1.5} + \frac{1}{4} e^{4x} \right]_0^3$$

$$= \left[\frac{(3)^{1.5}}{1.5} + \frac{1}{4} e^{12} \right] - \left[0 + \frac{1}{4} e^0 \right]$$

$$= [3.46 + 40688.70] - \left[\frac{1}{4} \right]$$

$$= 40692.16 - 0.25$$

$$= 40691.91$$

④

14 MARKS

$$b) \int 8x \sqrt{4x^2 + 5} dx \quad u = 4x^2 + 5$$

$$\frac{du}{dx} = 8x$$

$$\int 8x \sqrt{u} \frac{du}{8x}$$

$$\frac{du}{8x} = dx$$

$$= \frac{u^{1.5}}{1.5} + C$$

$$= \frac{(4x^2 + 5)^{1.5}}{1.5} + C$$

⑤

5 MARKS

$$c) \quad \frac{3x-5}{(x+3)(x-4)} = \frac{A}{(x+3)} + \frac{B}{(x-4)}$$

$$\therefore 3x - 5 = A(x-4) + B(x+3)$$

$$\underline{x=4}$$

$$7 = 7B$$

$$\textcircled{1 = B}$$

$$\underline{x=-3}$$

$$-14 = -7A$$

$$\textcircled{2 = A}$$

$$\therefore = \int \frac{2}{(x+3)} + \int \frac{1}{(x-4)} dx$$

$$= \textcircled{2 \ln(x+3) + \ln(x-4) + C}$$

⑥

6 MARKS

Q4)

(i) $\int 4x e^{2x} dx$ $= uv - \int v \frac{du}{dx} dx$

$\uparrow$ $\uparrow$
 u $\frac{dv}{dx}$

$\frac{du}{dx} = 4$ $v = \frac{1}{2} e^{2x}$

$$\begin{aligned} &= 4x \left(\frac{1}{2} e^{2x} \right) - \int \left(\frac{1}{2} e^{2x} \right) (4) dx \\ &= 2x e^{2x} - \int 2 e^{2x} dx \\ &= 2x e^{2x} - e^{2x} + C \end{aligned}$$

1 1

1

1 1 1

⑥

(ii) $\int 2x \cos 3x dx$

$\uparrow$ $\uparrow$
 u $\frac{dv}{dx}$

$\frac{du}{dx} = 2$ $v = \frac{1}{3} \sin 3x$

$$\begin{aligned} &= 2x \left(\frac{1}{3} \sin 3x \right) - \int \frac{2}{3} \sin 3x dx \\ &= \frac{2x}{3} \sin 3x - \left(-\frac{2}{6} \cos 3x \right) + C \\ &= \frac{2x}{3} \sin 3x + \frac{1}{3} \cos 3x + C \end{aligned}$$

1 1

1 1 1

⑥

12 MARKS

$$b) = \ln(5x^3 + 2x^2 - 3) + C$$

(2)

$$c) \int \frac{2}{3x+5} dx$$

$$2 \int \frac{1}{3x+5} dx$$

$$\frac{2}{3} \int \frac{3}{3x+5} dx$$

$$= \frac{2}{3} \ln(3x+5) + C$$

(3)

$$d) \int_0^2 3x(x^2 - 2x + 2) dx$$

$$\int_0^2 3x^3 - 6x^2 + 6x dx$$

$$= \left[\frac{3x^4}{4} - \frac{6x^3}{3} + \frac{6x^2}{2} \right]$$

$$= \left[\frac{3(2)^4}{4} - 2(2)^3 + 3(2)^2 \right] - [0]$$

$$= [12 - 16 + 12] - [0]$$

$$= 8$$

(8)