



HND/C PHASE TEST SPECIFICATION

Programme:	HND Automotive Engineering / Motorsport Engineering				
Unit:	Mechanical Principles				
Phase Test No:	1 R1	Lecturer:	T Tudor	Internal Verifier:	A Summerfield

Time and Date:	Wednesday 15 th February 2012	Duration:	90 mins	Contribution to Overall Unit Assessment (%):	25%
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Outcomes being Assessed (from Unit Syllabus):

Number	Description
Learning Outcome 1:	Investigate complex loading systems
Learning Outcome 2:	{Deflection of beams ('standard cases') { 'Stresses in pressure vessels'

For Attention of Students:

Answer **ALL** questions.

The mark allocation is as indicated – total 40 marks.

A 'formula sheet' is provided for reference.

Assume 'g' = 9.81 m/s^2 .

and $1 \text{ bar} = 10^5 \text{ N/m}^2 (\text{Pa})$

Model Answer

1. Mutually perpendicular stresses of 40MPa (tensile) and 30 MPa (compressive) act at a point in a material. Calculate the magnitude and sense of the strains in the directions of the stresses, given that Poisson's ratio is 0.25 and the Modulus of Elasticity is 200 GPa.

(6 marks)

2. At a certain point in a material there is a strain in the 'x' direction of 0.56×10^{-6} and a strain in the 'y' direction of -0.8×10^{-6} . Calculate the corresponding stresses in each direction and state whether they are compressive or tensile. Take Poisson's ratio to be 0.25 and the Modulus of Elasticity as 205 GPa.

(6 marks)

1. A cube of side length 200mm is subjected to an external fluid pressure of 50MPa. If the modulus of elasticity of the cube material is 163.2 GPa and the Poissons ratio is 0.3, calculate the bulk modulus and change in volume.

(8 marks)

2. A 4 m long simply supported beam carries a uniformly distributed load of 400N/m along its entire length. A point load of 1.5 kN also acts at the midpoint of the beams span. The beam has a flexural stiffness of 8MPa.

Calculate

- i) The corresponding deflection at the midpoint of the beam
- ii) How much the point load may be increased before the deflection exceeds 1.0mm

(10 marks)

3. A thin walled cylinder acts as an air receiver. It has a diameter of 0.65m, is 1.2m in length and is made from sheet steel with a thickness of 8mm. The longitudinal and circumferential joint efficiencies are 75% and 40% respectively, and the maximum allowable stress is 75MPa. What is the corresponding maximum allowable air pressure?

(10 marks)

Mechanical Principles Formula Sheet 1

Two-dimensional stress and strain equations:

$$\begin{aligned}\varepsilon_1 &= \frac{1}{E}(\sigma_1 - \nu\sigma_2) & \sigma_1 &= \frac{E}{1-\nu^2}(\varepsilon_1 + \nu\varepsilon_2) \\ \varepsilon_2 &= \frac{1}{E}(\sigma_2 - \nu\sigma_1) & \sigma_2 &= \frac{E}{1-\nu^2}(\varepsilon_2 + \nu\varepsilon_1)\end{aligned}$$

Three-dimensional strain equations:

$$\begin{aligned}\varepsilon_x &= \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)] \\ \varepsilon_y &= \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)] \\ \varepsilon_z &= \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)]\end{aligned}$$

Volumetric Strain:

$$\begin{aligned}\varepsilon_v &= \frac{\delta V}{V} = \varepsilon_x + \varepsilon_y + \varepsilon_z \\ \varepsilon_v &= \frac{\sigma_x + \sigma_y + \sigma_z}{E}(1 - 2\nu)\end{aligned}$$

Relationships between elastic constants:

$$E = \frac{\sigma}{\varepsilon} \quad K = \frac{\sigma}{\varepsilon_v} = \frac{E}{3(1-2\nu)} \quad G = \frac{E}{2(1+\nu)} \quad K = \frac{2G(1+\nu)}{3(1-2\nu)}$$

Stress and strains in thin walled pressure vessels:

Cylinders		Spheres
$\sigma_c = \frac{pD}{2t\eta_L}$		$\sigma = \frac{pD}{4t\eta}$
$\varepsilon_c = \varepsilon_D = \frac{pD}{4tE}(2 - \nu)$		
$\varepsilon_v = \frac{pD}{4tE}(5 - 4\nu)$		$\varepsilon_v = \frac{3pD}{4tE}(1 - \nu)$

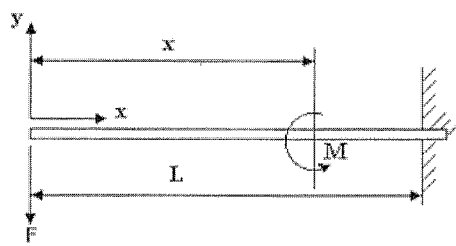
Stresses in thick walled cylinders: (Lame's Equations)

$$\sigma_R = a - \frac{b}{r^2} \quad \sigma_C = a + \frac{b}{r^2}$$

Where a and b are found
by applying boundary
conditions

Loaded beams : Standard Cases

Cantilever Beams:

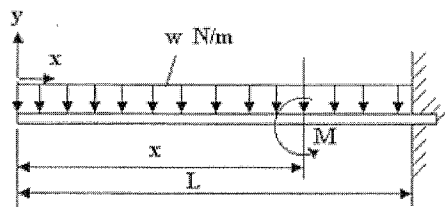


Max Slope

$$\frac{FL^2}{2EI}$$

Max Deflection

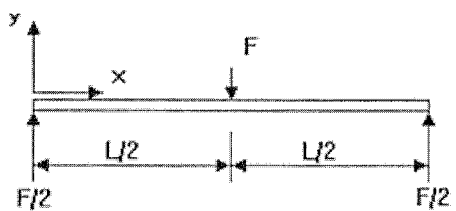
$$\frac{FL^3}{3EI}$$



$$\frac{wL^3}{6EI}$$

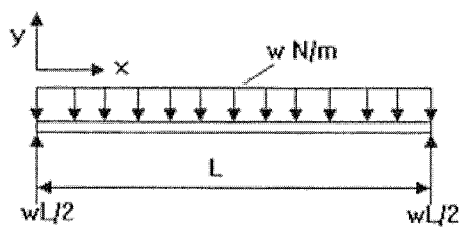
$$\frac{wL^4}{8EI}$$

Simply Supported Beams:



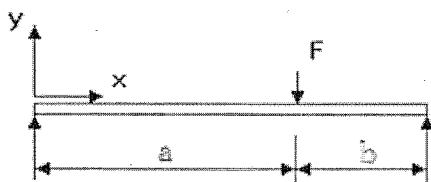
$$\frac{FL^2}{16EI}$$

$$\frac{FL^3}{48EI}$$



$$\frac{wL^3}{24EI}$$

$$\frac{5wL^4}{384EI}$$



$$\frac{Fb}{6EIL}(2L^2 - 3Lb + b^2)$$

$$\frac{Fa^2b^2}{3EIL}$$

Q1) $\sigma_x = 40 \times 10^6 \text{ N/m}^2$ } ✓

⑥ $\sigma_y = -30 \times 10^6 \text{ N/m}^2$ }

$\nu = 0.25$

$E = 200 \times 10^9 \text{ N/m}^2$ ✓

$$E_x = \frac{1}{E} (\sigma_x - \nu \sigma_y) = \frac{1}{200 \times 10^9} (40 \times 10^6 - (0.25 \times -30 \times 10^6)) = 237.5 \mu\epsilon$$

(tensile extension)

$$E_y = \frac{1}{E} (\sigma_y - \nu \sigma_x) = \frac{1}{200 \times 10^9} (-30 \times 10^6 - (0.25 \times 40 \times 10^6)) = -200 \mu\epsilon$$

(compression).

-2×10^{-4}

Q2) $E_x = 0.56 \mu\epsilon = 0.56 \times 10^{-6}$ } ✓

⑥ $E_y = -0.80 \mu\epsilon = -0.8 \times 10^{-6}$ }

$\nu = 0.25$

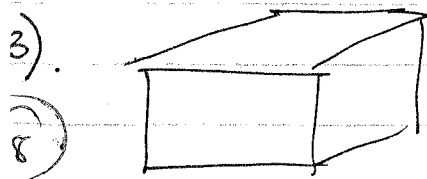
$E = 205 \times 10^9 \text{ N/m}^2$ ✓

$$\sigma_x = \frac{E}{1-\nu^2} (E_x + \nu E_y) = \frac{205 \times 10^9}{1-0.25^2} (0.56 \times 10^{-6} + (0.25 \times -0.80 \times 10^{-6})) = 78.72 \text{ kPa}$$

tensile

$$\sigma_y = \frac{E}{1-\nu^2} (E_y + \nu E_x) = \frac{205 \times 10^9}{1-0.25^2} (-0.8 \times 10^{-6} + (0.25 \times 0.56 \times 10^{-6})) = -144.32 \text{ kPa}$$

(compression)



$l = 200 \times 10^{-3} \text{ m}$

$E = 163.2 \text{ GPa}$

$\nu = 0.3$

$\Delta V = ?$

$P = 50 \times 10^6 \text{ N/m}^2$ ✓

$$K = \frac{E}{3(1-2\nu)} = \frac{163.2 \times 10^9}{3(1-0.6)}$$

$K = 136 \times 10^9 \text{ Pa}$

$$K = \frac{\sigma}{\epsilon} = \frac{-P}{\epsilon_v}$$

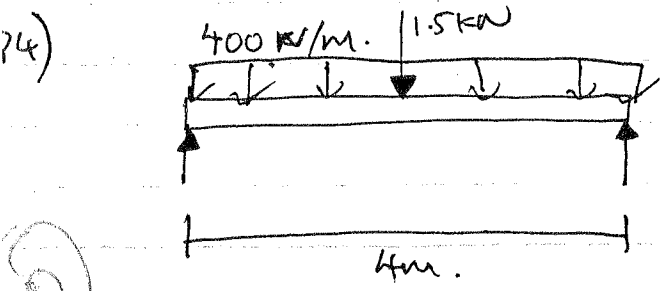
$$\epsilon_v = \frac{-P}{K} = \frac{-50 \times 10^6}{136 \times 10^9}$$
 ✓

$\epsilon_v = -367.65 \mu\epsilon$ ✓

$$\epsilon_v = \frac{\Delta V}{V}$$

$\Delta V = \epsilon_v V = -367.65 \times 10^{-6} \times (200 \times 10^{-3})^3$ ✓

$\Delta V = -2.941 \times 10^{-6} \text{ m}^3 = -2941 \text{ mm}^3$ ✓



$$F = -1.5 \text{ kN.}$$

$$w = -400 \text{ N/m.}$$

$$L = 4 \text{ m}$$

$$EI = 8 \text{ MN/m}^2 = 8 \times 10^6 \text{ N/m}^2.$$

Point Load

$$y = \frac{FL^3}{48EI} = \frac{-1500 \times 4^3}{48 \times 8 \times 10^6} = -0.25 \times 10^{-3} \text{ m.}$$

$$\checkmark \quad 48 \times 8 \times 10^6 = -0.25 \text{ mm.} \quad \checkmark$$

u.d.l.

$$y = \frac{5wL^4}{384EI} = \frac{5 \times -400 \times 4^4}{384 \times 8 \times 10^6} = -0.167 \times 10^{-3} \text{ m}$$

$$\checkmark \quad 384 \times 8 \times 10^6 = -0.167 \text{ mm.} \quad \checkmark$$

i) Total $y = -0.25 - 0.167 = -0.417 \text{ mm.} \quad \checkmark$

ANS.

ii) Total $y = -1 \times 10^{-3} = y_{PL} + y_{udl}. \quad \checkmark$

$$y_{PL} = -1 \times 10^{-3} + 0.167 \times 10^{-3} = -0.833 \times 10^{-3} \text{ m.} \quad \checkmark$$

$$y_{PL} = \frac{FL^3}{48EI} \quad F_{\max} = \frac{y_{PL} \times 48EI}{L^3} = \frac{-0.833 \times 10^{-3} \times 48 \times 8 \times 10^6}{4^3} \quad \checkmark$$

$$F_{\max} = -4.998 \text{ kN.} \quad \checkmark$$

$$\text{Increase} = -4998 - (-1500) = -3.498 \times 10^3 \text{ N}$$

$$= -3.498 \text{ kN.} \quad \checkmark$$

Q5)

$$D = 0.65 \text{ m}$$

$$L = 1.2 \text{ m}$$

$$t = 0.008 \text{ m}$$

$$\eta_L = 0.75$$

$$\eta_c = 0.4$$

$$\sigma_{\text{max}} = 75 \times 10^6 \text{ N/m}^2$$

$$\sigma_c = \frac{PD}{2t\eta_L}$$

$$\sigma_L = \frac{PD}{4t\eta_c}$$

$$P_{\text{max}} = \frac{\sigma_c \times 2t\eta_L}{D}$$

$$P_{\text{max}} = \frac{\sigma_L \times 4t\eta_c}{D}$$

$$P_{\text{max}} = \frac{75 \times 10^6 \times 2 \times 0.008 \times 0.75}{0.65}$$

$$P_{\text{max}} = \frac{75 \times 10^6 \times 4 \times 0.008 \times 0.4}{0.65}$$

$$= 13.85 \text{ Bar.}$$

$$= 14.77 \text{ Bar.}$$

$$1.385 \text{ MPa}$$

$$14.77 \text{ MPa.}$$

$$\text{Hence } P_{\text{max}} = \underline{13.85 \text{ Bar}}$$

$$13.85 \text{ Bar} = 1.3 \text{ MPa}$$

