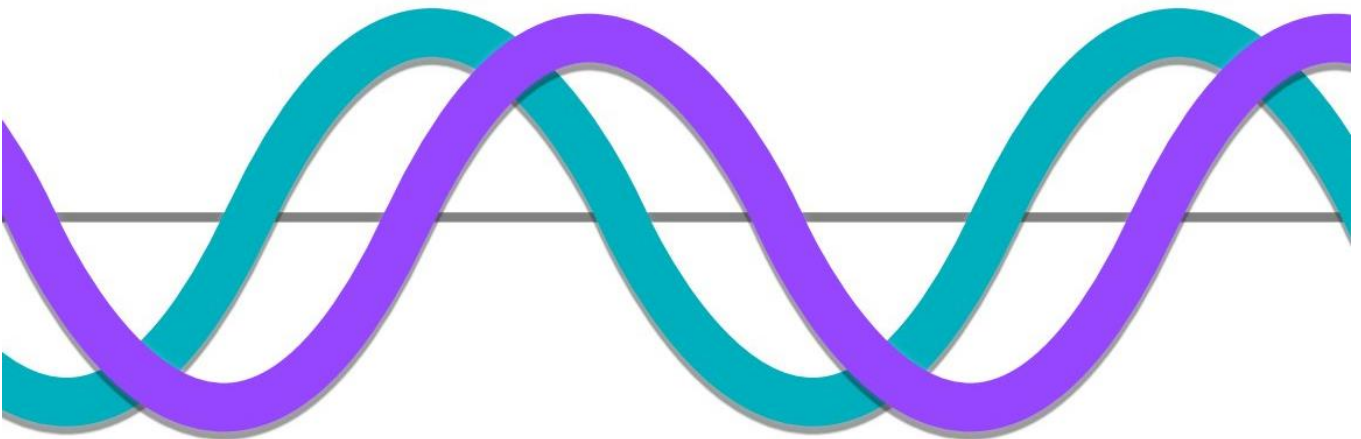


AC Response of an RLC Network



By

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Introduction

Power factor correction (PFC) is an essential aspect of electrical power systems designed to improve the efficiency and reduce energy consumption in various applications.

In systems consisting of resistive, inductive, and capacitive (RLC) elements, PFC is used to improve efficiency by minimising losses and improving power transfer between source and load.

This report aims to discuss the basic concepts of PFC in RLC circuits, including the importance of power factor, methods of PFC, and the impact of PFC on energy efficiency. The report will also examine the effects PFC has on circuit resonance and maximum power transfer.

Overall, the report highlights the significance of PFC in optimising the performance of RLC circuits, reducing energy wastage, and promoting sustainable energy practices.

***This report will outline results and conclusions drawn from pre-set tasks outline in assignment “AC response of an RLC Network” for module ACDA5001.*

Task 1)

This initial task requires an RLC circuit to be created using Proteus software. Components within this circuit are to be determined by a unique student ID, in this case, 2218705. Using this ID, the components values for this task are as follows:

- RSRC – 2.2Ω
- Load Inductor – 187mH
- Load Resistor - 05Ω

This circuit was to be supplied by a $400\text{VRMS} / 50\text{Hz}$ supply. Initially, no power correction capacitor was connected to this circuit.

Using this information, the RLC network shown in figure 1 was created.

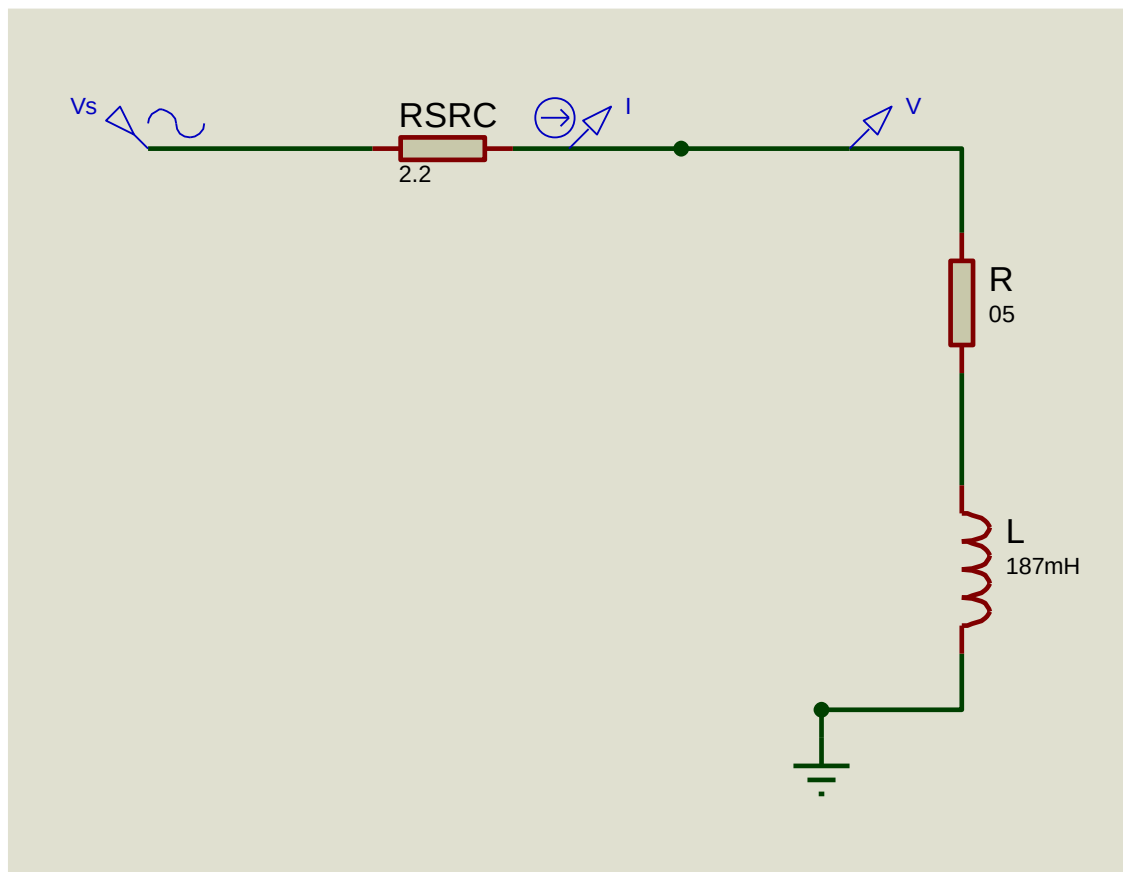


Figure 1 - Initial RLC Network

Section 1)

To determine the initial phase difference within this circuit, proteus was used to plot the load voltage against current. For ease of analysis, the graph was scaled to show a single cycle as shown within figure 2.

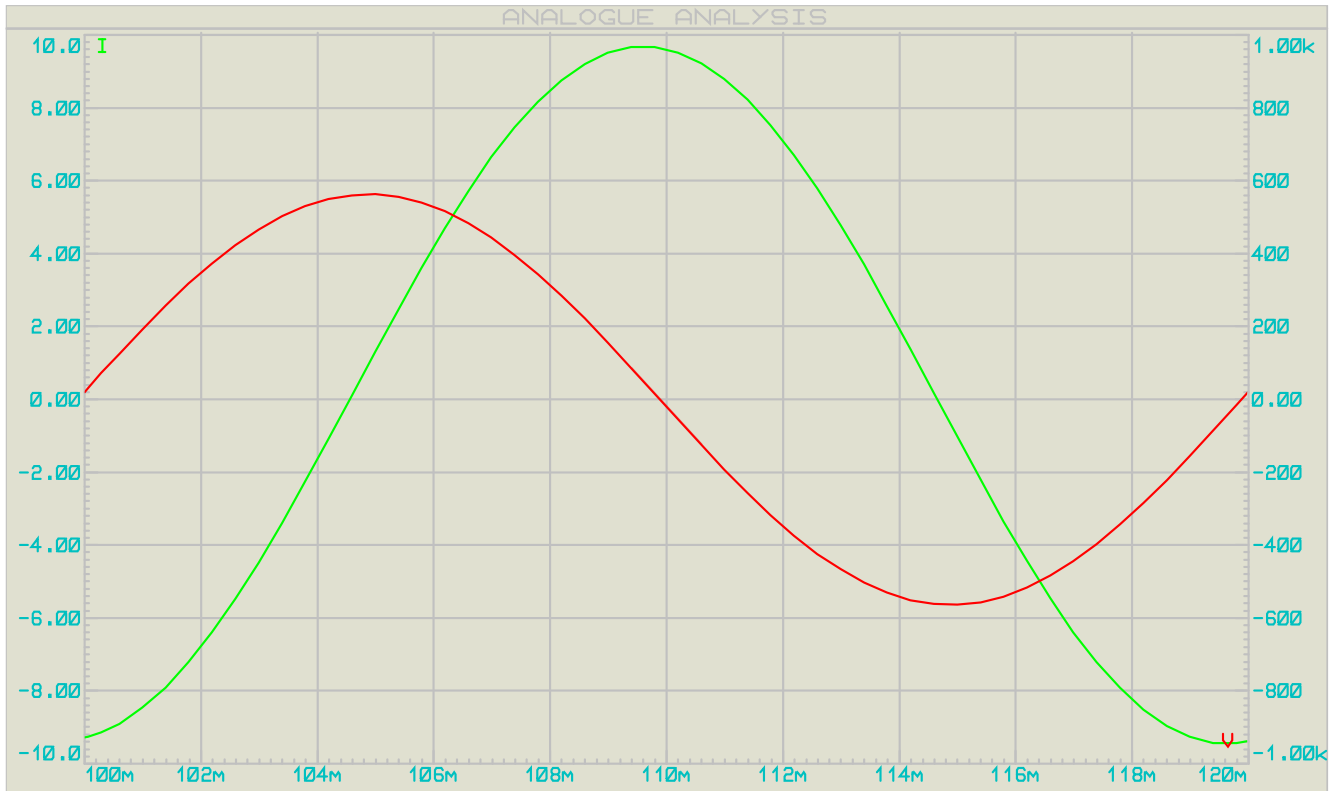


Figure 2 - Initial Phase Difference of the RLC Network

To determine phase difference, the following equation was used:

$$\text{Phase Angle } (\theta) = \frac{\text{Delay Between Waveforms } (t)}{\text{One Period } (T)} \times 2\pi$$

The delay between waveforms (t), can be determined by inspection of corresponding points on each waveform.

In this instance, 0 volts (i.e., the beginning of the voltage cycle) was measured at 100mS whilst the corresponding 0 amps occurred at 104.6mS. This gave a total delay of 4.60mS.

From a known frequency value of 50Hz, the period of each waveform can also be determined:

$$Period (T) = \frac{1}{Frequency (f)} = \frac{1}{50} = 20mS$$

Using these values, the phase difference for this circuit could be calculated.

$$Phase Angle (\theta) = \frac{4.60mS}{20mS} \times 2\pi = 1.445 rads$$

To convert this value to degrees, the answer can simply be multiplied by 180 / Pi.

$$1.445 rads \times \frac{180}{\pi} = Phase Angle of 82.79^\circ$$

As the current shown in figure 2 occurs after the voltage, the current is said to be lagging the voltage at a phase angle of 82.79°.

Section 2)

The second part of this task requires the active, reactive, and apparent powers to be calculated for this circuit.

Using the graph created in figure 2, the proteus software measures a peak value of 9.67A (I_m) within this network. For the power to be calculated, this result will need to be converted to its RMS value. This can be achieved by simply dividing the peak value by root two.

$$RMS\ Current\ (I) = \frac{Peak\ Current\ (I_m)}{\sqrt{2}} = \frac{9.67}{\sqrt{2}} = 6.84\ A$$

Per the initial task instruction, RMS voltage is 400V. Using the following formulas, the power for this network could be calculated.

Active Power (P):

$$P = VI \cos \theta = 400 \times 6.84 \times \cos(82.79) = 343.385\ W$$

Reactive Power (Q):

$$Q = VI \sin \theta = 400 \times 6.84 \times \sin(82.79) = 2714.366\ VAR$$

Apparent Power (S):

$$S = VI = 400 \times 6.84 = 2736\ VA$$

Section 3)

Power factor is the ratio between active power and apparent power in an electrical system and defines system efficiency. It is a measure of the system's ability to utilise total available power.

An inefficient system will lose power through reactive components and therefore, have a low power factor. If reactive components are present within a system, reactance will cause the current to lag or lead the voltage as shown in figure 2. This causes several drawbacks to system operation which will be discussed in greater detail later in the report.

The initial power factor for this RLC circuit can be found using the phase angle calculated in section 1.

$$\text{Power Factor} = \cos(\text{Phase Angle}) = \cos(82.79) = 0.126$$

From this result, it can be seen that the initial RLC circuit is highly inefficient. A power factor this low indicates that the circuit is only utilising a very small portion of available energy in a useful manner. The rest of this power is being lost via reactive components. This can be seen when inspecting the calculated active and reactive powers found in section 2.

Power factor is measured on a scale between 0 and 1. A power factor of 1 would indicate a system is operating at perfect efficiency hence, systems are designed to operate as close to a power factor of 1 as possible. In real world applications, a power factor of 1 cannot be achieved as some losses are unavoidable. As a result, systems are normally designed to operate at a targeted power factor of at least >0.90 [1].

In contrast to this, the RLC circuits calculated power factor of 0.126 highlights very poor power efficiency that required rectification via power factor correction.

Section 4)

To increase power factor within this RLC network, the initial phase difference calculated in section 1 needs to be examined. The phase angle of the current was calculated to be 82.79° lagging the voltage. This lagging effect is created by the inductive reactance of the 187mH inductor.

To counteract the lagging effects of inductive devices, a capacitor needs to be introduced across the load. The capacitor will introduce capacitive reactance into this circuit that will lead the voltage by 90° , thus counteracting the inductive reactance. By varying the value of this capacitor, the resultant current can be pulled closer to phase with the voltage.

For this task, it has been requested that the power factor be improved to 0.95.

To obtain a power factor of 0.95, phase angle of $\cos(0.95)$ will be required.

$$\cos(0.95) = 18.2^\circ$$

Using the trigonometric relationship between real, reactive, and apparent power, the following calculations can be used to find the required reactive power to correct the power factor to 0.95 [2]. The power values used were calculated in section 2.

$$\tan(\theta) = \frac{\text{Reactive Power (Q)}}{\text{Active Power (P)}}$$

$$\tan(18.2) = \frac{\text{VAR}(\text{corrective})}{343.385W}$$

$$\text{VAR}(\text{corrective}) = 343.385 \times 0.329 = 112.974 \text{ VAR}$$

To find the capacitive reactance required to obtain a power factor of 0.95, the reactive power calculated needs to be subtracted from the original reactive power of the circuit [2].

$$\text{VAR}_{\text{Capacitor}} (Q_C) = \text{VAR}(\text{original}) - \text{VAR}(\text{corrective})$$

$$Q_C = 2714.366 - 112.974 = 2601.392 \text{ VAR}$$

Using the following series of equations, the capacitive reactance and hence the required capacitor value can be found [2].

As the capacitor will be connected in parallel with the load, the branch voltage is required. Due to a voltage drop across component RSRC, this value will differ slightly from the original RMS voltage of 400V. Using the voltmeter connected in proteus, as shown in figure 1, the peak voltage (V_m) is found to be 563V.

$$\text{RMS Capacitor Voltage (V}_C) = \frac{\text{Peak Voltage (V}_m)}{\sqrt{2}} = \frac{563}{\sqrt{2}} = 398.10 \text{ V}$$

$$\text{Reactive Power } (Q_c) = \frac{\text{RMS Capacitor Voltage}^2 (V_c)}{\text{Capacitive Reactance } (X_c)}$$

$$X_c = \frac{398.10^2 (V_c)}{2601.392 (Q_c)} = 60.92\Omega$$

$$X_c = \frac{1}{2\pi f c}$$

$$\text{Capacitor Value} = \frac{1}{2\pi f X_c} = \frac{1}{2\pi \times 50 \times 60.92} = 52.25\mu F$$

Therefore, to improve circuit power factor to 0.95, a 52.25μF capacitor can be connected across the load.

Using the proteus simulation created from task 1, this power factor correction capacitor was added across the load as shown in figure 3.

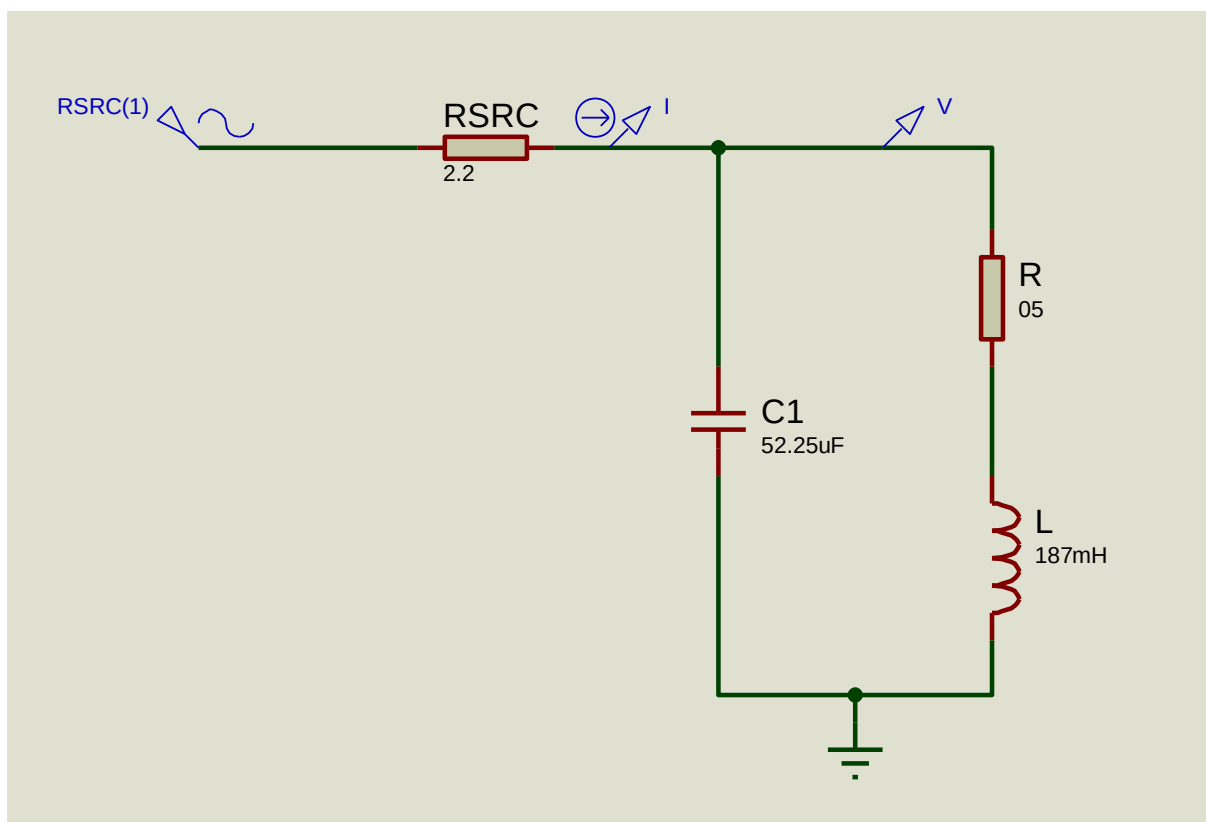


Figure 3 - Power Factor Correction of RLC Network

With the power factor correction capacitor installed, the voltage and current phase difference was again able to be plotted using proteus. These results are shown in figure 4.

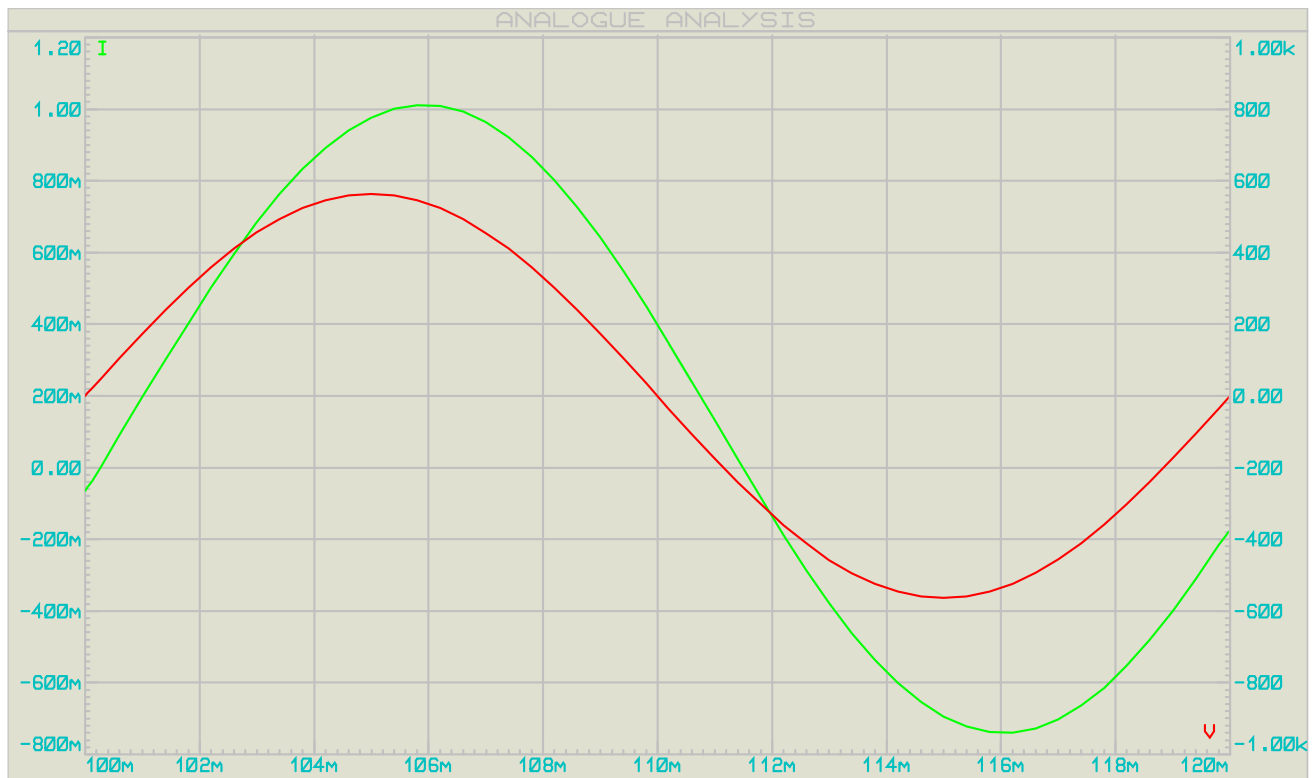


Figure 4 - Phase Difference Post Power Factor Correction

By analysing this data, it can be seen that there has been a significant shift in current phase. There is now only a 0.7mS delay between corresponding zero points on these waveforms opposed to the 4.6mS observed prior to power factor correction.

There has also been a substantial reduction in current measured from 9.67A (Im) down to 1.01A (Im). This occurs due to the reduction of circuit impedance as the capacitive reactance introduced, counteracts the inductive reactance. It is this reaction that allows the system to operate more efficiently as power factor increases.

Sections 5 and 6)

The next section of this task, it is asked for capacitor values to be plotted against power factor and phase angle on separate graphs.

To achieve this, the capacitor value within the proteus simulation was varied across the desired range. The simulation was then run, and the phase angle calculated in an identical fashion shown in section 1.

$$\text{Phase Angle } (\theta) = \frac{\text{Delay Between Waveforms } (t)}{\text{One Period } (T)} \times 2\pi$$

Once the phase angle was known the power factor was calculated.

$$\text{Power Factor} = \cos(\theta)$$

These calculations were performed for capacitor values ranging between 10-100 μ F. The results table created from these calculations is shown below in figure 5,

Capacitor Value (μ F)	Phase Angle ($^{\circ}$)	Power Factor
10	82.79	0.136
20	82.79	0.136
30	82.79	0.136
40	70.19	0.339
50	35.98	0.809
60	53.97	0.588
70	73.8	0.279
80	79.18	0.188
90	81.02	0.156
100	82.79	0.136

Figure 5 – Capacitor vs Phase Angle vs Power Factor Result Table

Using these results, the following two graphs were created.

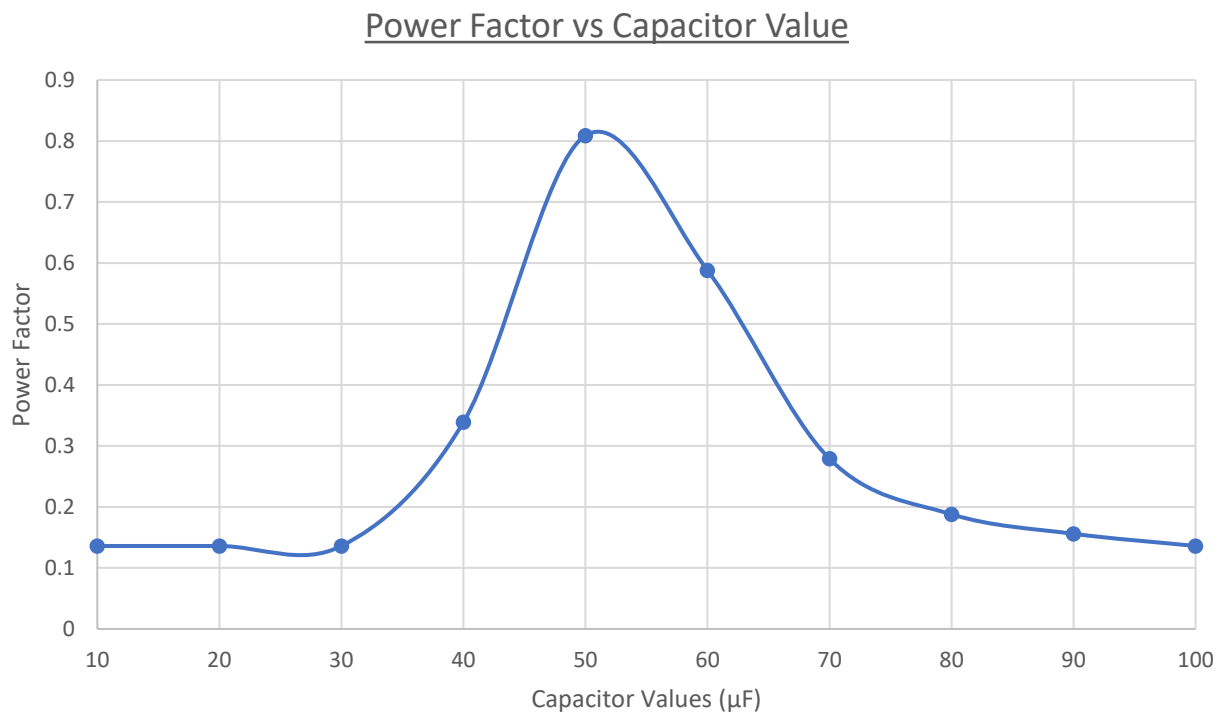


Figure 7 - Graph Showing Capacitor Value vs Power Factor

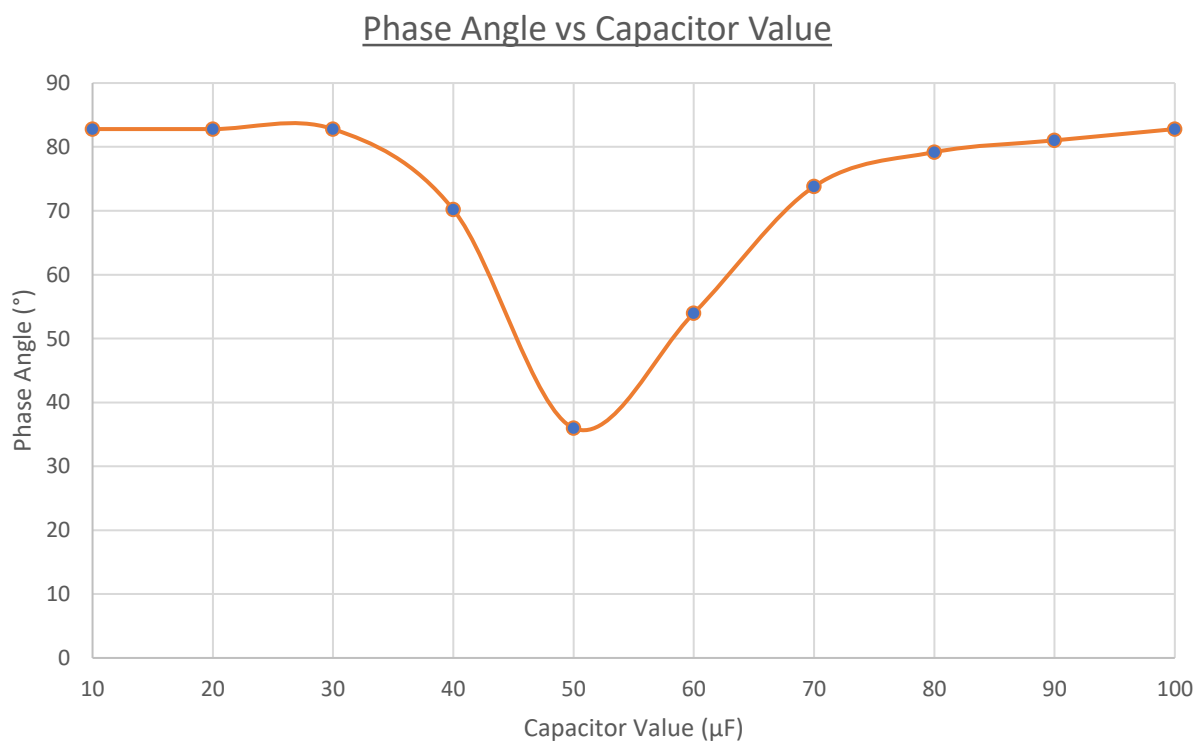


Figure 6 - Graph Showing Capacitor Value vs Phase Angle

When analysing these graphs, it can be seen that a maximum power factor and minimum phase angle is achieved at a capacitor value of approximately $50\mu\text{F}$. This shows very clear evidence that a singular capacitor value will produce optimised efficiency within this specific RLC network.

If this capacitor value were calculated, the circuit could reach a point that all reactive components would cancel each other out creating a purely resistive system. There would therefore be no phase difference from the supply and current flow would be minimised.

At this point, the circuit will be in a state known as resonance.

Section 7)

As the power factor approaches 1, the energy stored within the capacitor and inductor will decrease. This is because a power factor of 1 indicates that capacitive and inductive reactance's are equal (i.e., $X_L = X_C$) and therefore, circuit impedance is equal to total resistance. As no reactive energy is being utilised within the circuit, it can be seen that minimal energy is being store within the capacitor or inductor.

Using this information, it can be deduced that as power factor approaches 1, reactive power within a circuit will tend to zero hence, energy stored within reactive components will reduce.

Task 2)

For task two, phasor diagrams are required to be drawn for the RLC network created in task 1.

Four phasor diagrams are required:

- 1) At 25Hz below resonant frequency
- 2) At Resonance frequency
- 3) At 25Hz above resonant frequency
- 4) At 50Hz

Resonant frequency refers to the frequency at which capacitive and inductive reactance within a circuit will be equal. This will create a resonant circuit without altering reactive component values.

The relationship between frequency and reactance can be seen in the below equations for capacitive and inductive reactance.

$$\text{Inductive Reactance } (X_L) = 2\pi fL$$

$$\text{Capacitive Reactance } (X_C) = \frac{1}{2\pi fC}$$

To begin this task, the resonant frequency of this circuit needs to be calculated. This was done using the following formula [3].

$$\text{Resonant Frequency } (f_r) = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$\text{Resonant Frequency } (f_r) = \frac{1}{2\pi} \sqrt{\frac{1}{187\text{mH} \times 52.25\mu\text{F}} - \frac{7.2^2}{187\text{mH}^2}}$$

$$\text{Resonant Frequency } (f_r) = 50.67\text{Hz}$$

As within task 1, the following calculation will require the branch voltage across the capacitor and load. Per the initial equation on page 8, this equates to 398.10V.

Phasor Diagram at 25.67Hz:

To complete each of these phasor diagrams the following method was used,

$$\text{Inductive Reactance } (X_L) = 2\pi fL = 2\pi \times 25.67 \times 187\text{mH} = 30.16\Omega$$

$$\text{Capacitive Reactance } (X_C) = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 25.67 \times 52.25\mu\text{F}} = 118.05\Omega$$

$$\text{Load Impedance } (Z_{LR}) = \sqrt{R^2 + X_L^2} = \sqrt{5^2 + 30.16^2} = 30.57\Omega$$

$$\text{Load Current } (I_{LR}) = \frac{V_{LR}}{Z_{LR}} = \frac{398.10}{30.57} = 13.02\text{A}$$

$$\text{Capacitor Current } (I_C) = \frac{V_C}{X_C} = \frac{398.10}{118.05} = 3.37\text{A}$$

At this frequency, $I_{LR} > I_C$ therefore the resultant current will lag the voltage.

$$\text{Load Phase Angle } (\theta_1) = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{30.16}{5}\right) = 80.59^\circ \text{ Lagging}$$

Using proteus, it can be seen that the resultant current has a magnitude of 13.5A (I_m). This equates to an RMS current of 9.55A (I). Using this value, the resultant current phase angle can be found.

$$\text{Resultant Current Phase Angle } (\theta) = \tan^{-1}\left(\frac{I_{LR} - I_C \sin \theta_1}{I_{LR} \cos \theta_1}\right)$$

$$\theta = \tan^{-1}\left(\frac{13.02 - 3.37 \sin 80.59}{13.02 \cos 80.59}\right) = 77.62^\circ \text{ Lagging}$$

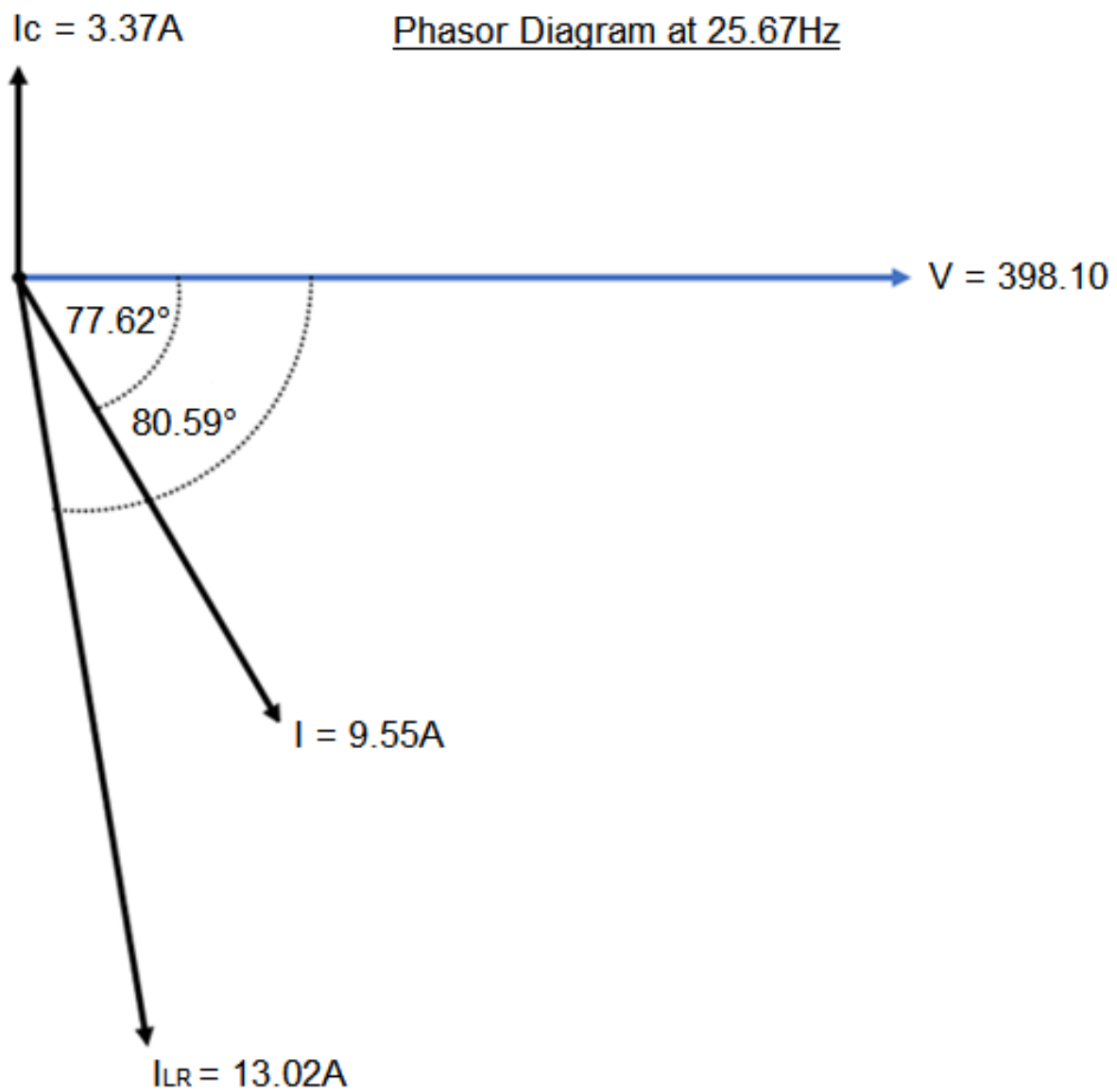


Figure 8 - Phasor Diagram at 25.67Hz

Phasor Diagram at Resonant Frequency (50.67Hz):

$$\text{Inductive Reactance } (XL) = 2\pi fL = 2\pi \times 50.67 \times 187\text{mH} = 59.54\Omega$$

$$\text{Load Impedance } (Z_{LR}) = \sqrt{R^2 + XL^2} = \sqrt{5^2 + 59.43^2} = 59.64\Omega$$

$$\text{Load Current } (I_{LR}) = \frac{V_{LR}}{Z_{LR}} = \frac{398.10}{59.64} = 6.68\text{A}$$

At resonance, load current is equal to capacitor current therefore, I_C is also 6.68A

$$\text{Load Phase Angle } (\theta_1) = \tan^{-1}\left(\frac{XL}{R}\right) = \tan^{-1}\left(\frac{59.54}{5}\right) = 85.20^\circ \text{ Lagging}$$

Using proteus, it can be seen that the resultant current has a magnitude of 788mA (I_m). This equates to an RMS current of 557mA (I).

Finally, at resonance, resultant phase angle will equal 0° .

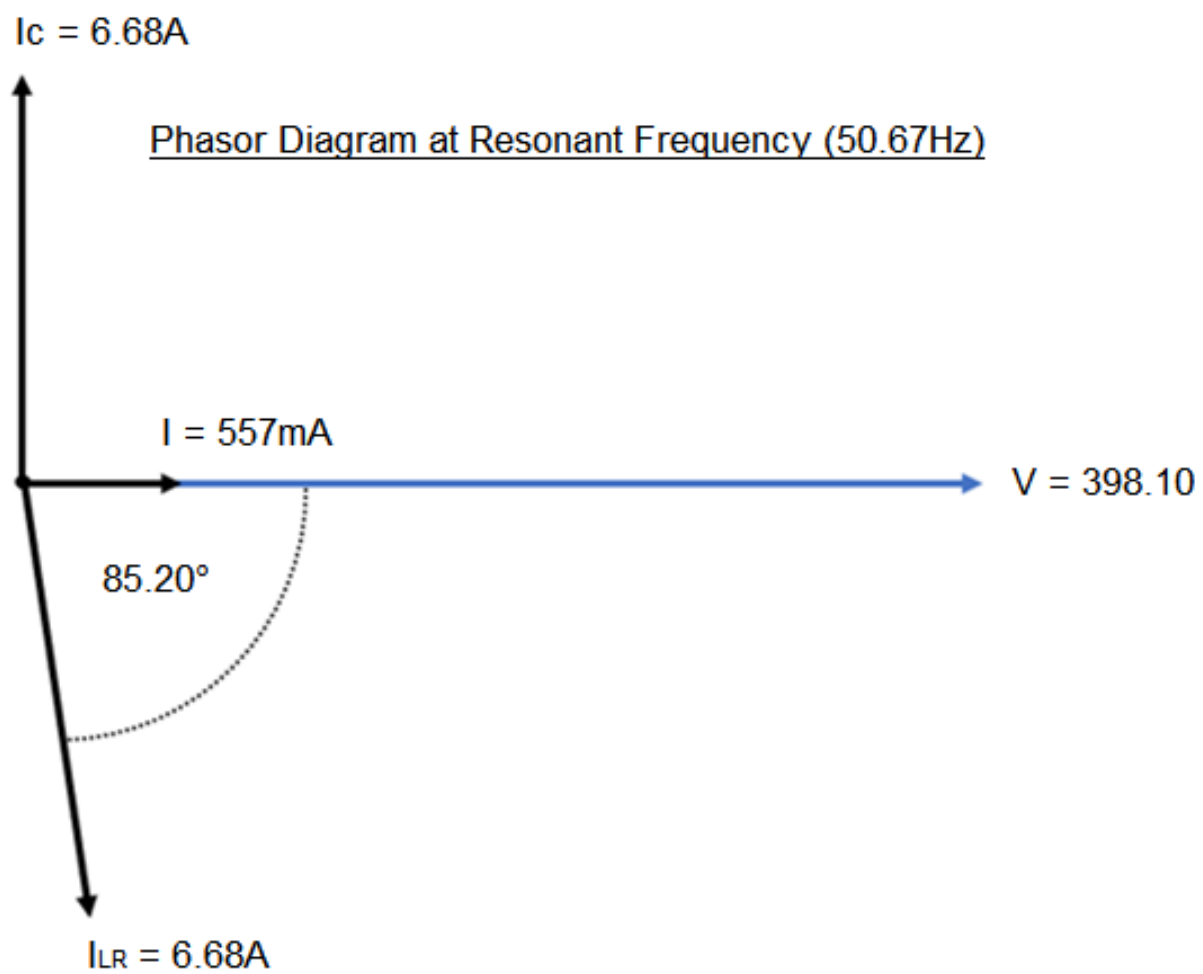


Figure 9 - Phasor Diagram at Resonant Frequency (50.67Hz)

Phasor Diagram at 75.67Hz:

$$\text{Inductive Reactance } (X_L) = 2\pi fL = 2\pi \times 75.67 \times 187\text{mH} = 88.91\Omega$$

$$\text{Capacitive Reactance } (X_C) = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 75.67 \times 52.25\mu\text{F}} = 40.25\Omega$$

$$\text{Load Impedance } (Z_{LR}) = \sqrt{R^2 + X_L^2} = \sqrt{5^2 + 88.91^2} = 89.05\Omega$$

$$\text{Load Current } (I_{LR}) = \frac{V_{LR}}{Z_{LR}} = \frac{398.10}{89.05} = 4.47\text{A}$$

$$\text{Capacitor Current } (I_C) = \frac{V_C}{X_C} = \frac{398.10}{40.25} = 9.89\text{A}$$

At this frequency, $I_C > I_{LR}$ therefore the resultant current will lead the voltage.

$$\text{Load Phase Angle } (\theta_1) = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{89.05}{5}\right) = 86.79^\circ \text{ Lagging}$$

Using proteus, it can be seen that the resultant current has a magnitude of 7.71A (I_m). This equates to an RMS current of 5.61A (I). Using this value, the resultant current phase angle can be found.

$$\text{Resultant Current Phase Angle } (\theta) = \tan^{-1}\left(\frac{I_C - I_{LR} \sin \theta_1}{I_{LR} \cos \theta_1}\right)$$

$$\theta = \tan^{-1}\left(\frac{9.89 - 4.47 \sin 86.79}{4.47 \cos 86.79}\right) = 87.36^\circ \text{ Leading}$$

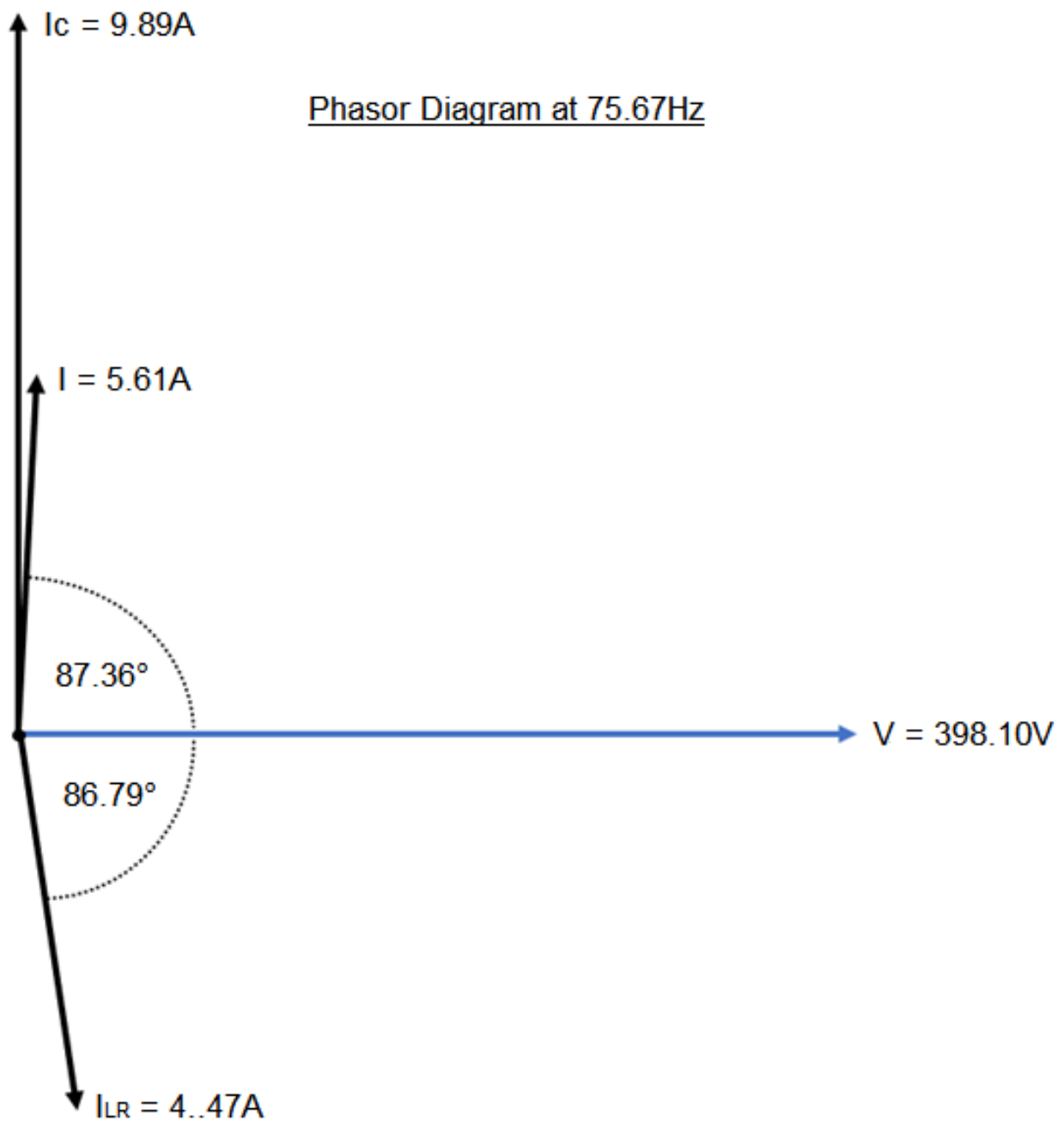


Figure 10 - Phasor Diagram at 75.67Hz

Phasor Diagram at 50Hz:

$$\text{Inductive Reactance } (X_L) = 2\pi fL = 2\pi \times 50 \times 187\text{mH} = 58.75\Omega$$

$$\text{Capacitive Reactance } (X_C) = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 52.25\mu\text{F}} = 60.92\Omega$$

$$\text{Load Impedance } (Z_{LR}) = \sqrt{R^2 + X_L^2} = \sqrt{5^2 + 58.75^2} = 58.96\Omega$$

$$\text{Load Current } (I_{LR}) = \frac{V_{LR}}{Z_{LR}} = \frac{398.10}{58.75} = 6.78\text{A}$$

$$\text{Capacitor Current } (I_C) = \frac{V_C}{X_C} = \frac{398.10}{61.00} = 6.53\text{A}$$

At this frequency, $I_{LR} > I_C$ therefore the resultant current will lag the voltage.

$$\text{Load Phase Angle } (\theta_1) = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{58.75}{5}\right) = 85.14^\circ \text{ Lagging}$$

Using proteus, it can be seen that the resultant current has a magnitude of 835mA (I_m). This equates to an RMS current of 590mA (I). Using this value, the resultant current phase angle can be found.

$$\text{Resultant Current Phase Angle } (\theta) = \tan^{-1}\left(\frac{I_{LR} - I_C \sin \theta_1}{I_{LR} \cos \theta_1}\right)$$

$$\theta = \tan^{-1}\left(\frac{6.78 - 6.53 \sin 85.14}{6.78 \cos 85.14}\right) = 25.46^\circ \text{ Lagging}$$

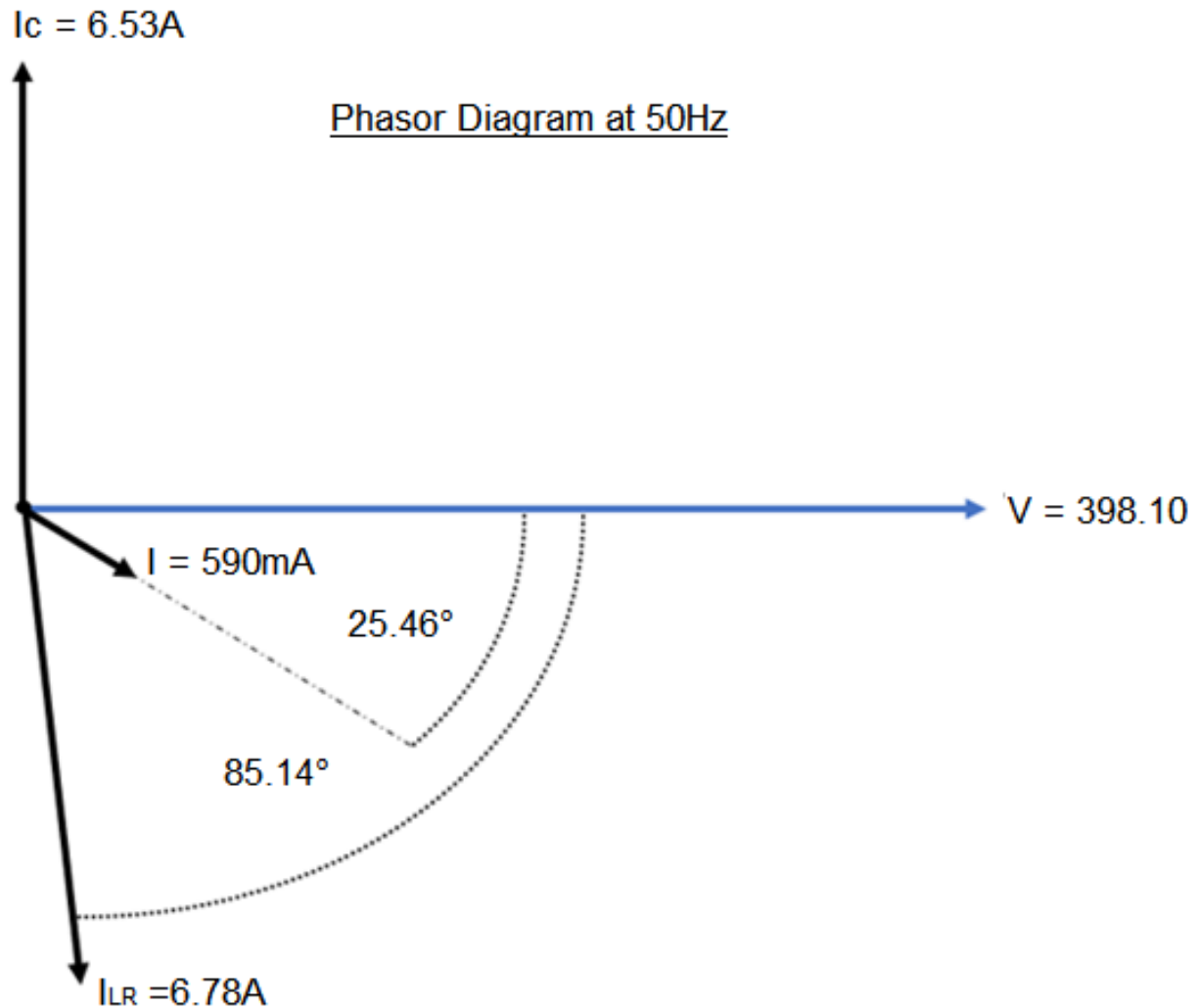


Figure 11 - Phasor Diagram at 50Hz

Phasor Diagram Analysis:

From these phasor diagrams, the following conclusions can be made:

- At resonance, it was already known that the phase angle would be 0° and that current flow would be minimised. This was highlighted in figure 9.
- When analysing figure 8, it can be seen that as frequency reduces below resonance, the resultant current will tend to lag further from the voltage.
- The opposite effect can be seen in figure 10, whereby the resultant current begins to lead the voltage as frequency increases past resonance.

Task 3)

Resonance occurs when all reactive components within a circuit cancel each other out creating a purely resistive circuit. This removes phase difference between circuit voltage and current eliminating losses created from reactive components. The systems therefore runs more efficiently and with a power factor very close to 1.

Within task one, a capacitor was added across the load as the initial current was lagging the supply voltage. The addition of a capacitor counteracted the inductive reactance within the circuit and reduced the phase difference from the supply. By calculating the required value of this capacitor, a specific power factor could be achieved by bringing the circuit closer to resonance. These results were observed in figure 4.

As resonance eliminates reactive losses within a circuit, maximum power transfer can occur between the load and supply. At this point, source impedance is equal to that of the load resulting in minimum circuit impedance.

For task one, power factor was increased to bring the circuit closer to resonance. By doing so, the circuit impedance was reduced and losses from reactive components were lessened. This allowed more power to be effectively used by the load.

As resonance is required for maximum power transfer to occur, it can be deduced that as power factor approaches 1, the circuit approaches resonance and thus, power transfer to the load becomes more efficient.

Conclusion

In conclusion, understanding power factor, resonance, and maximum power transfer is essential in the operation of electrical systems. Power factor correction can enhance the power quality in a system, reduce energy consumption, and minimise operating costs.

System resonance will occur when impedance is minimised through the elimination of reactive elements. Utilising power factor correction, systems can be brought closer to their resonant state and therefore, perform far more efficiently. This was demonstrated within task one, whereby a capacitor was connected across the load to do just this.

Finally, maximum power transfer occurs at resonance when the resistance of the load is equal to the source. Circuit impedance is therefore minimised resulting in increased current flow and optimal conditions for maximum power transfer to occur.

In summary, having a comprehensive understanding of power factor, resonance, and maximum power transfer, along with correction techniques, guarantees efficient and reliable operation of electrical systems.

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