

12 Pages

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1	Student Number	
2	Title of Examination Paper	CLASS TEST 2
3	Module Code	PSDE5008
4	Date and Time of Examination	
5	Examination Venue	
6	Seat Number	
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List the numbers of the questions you have attempted	Office Use
1	35
2	20
3	0
Total	55

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Surname INCE (Block Capitals) Forename(s) DANIEL

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PATENT PENDING
 INFOSAFE®

1. Fourier Transform

$$V(t) = 7e^{4t} \text{ for } t < 0$$

$$V(t) = 7e^{-4t} \text{ for } t > 0$$

$$\hat{V}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^0 7e^{4t} \times e^{-j\omega t} dt + \frac{1}{\sqrt{2\pi}} \int_0^{\infty} 7e^{-4t} \times e^{-j\omega t} dt$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^0 7e^{(4-j\omega)t} dt + \frac{1}{\sqrt{2\pi}} \int_0^{\infty} 7e^{-(4+j\omega)t} dt$$

$$= \frac{1}{\sqrt{2\pi}} \left[\frac{7e^{(4-j\omega)t}}{(4-j\omega)} \right]_{-\infty}^0 + \frac{1}{\sqrt{2\pi}} \left[\frac{7e^{-(4+j\omega)t}}{-(4+j\omega)} \right]_0^{\infty}$$

$$= \frac{1}{\sqrt{2\pi}} \left[\frac{7}{(4-j\omega)} e^0 - \frac{7}{(4-j\omega)} e^{-\infty} \right] + \frac{1}{\sqrt{2\pi}} \left[\frac{7}{-(4+j\omega)} e^{-\infty} - \frac{7}{-(4+j\omega)} e^0 \right]$$

Since $e^0 = 1$ and $e^{-\infty} = 0 \dots$

$$\rightarrow \frac{1}{\sqrt{2\pi}} \left[\frac{7}{(4-j\omega)} + \frac{7}{(4+j\omega)} \right]$$

$$= \frac{1}{\sqrt{2\pi}} \left[\frac{7}{(4-j\omega)} + \frac{7}{(4+j\omega)} \right] = \frac{1}{\sqrt{2\pi}} \left[\frac{7(4+j\omega) + 7(4-j\omega)}{(4-j\omega)(4+j\omega)} \right]$$

$$= \frac{1}{\sqrt{2\pi}} \left[\frac{28 + 7j\omega + 28 - 7j\omega}{16 + 4j\omega - 4j\omega - j^2\omega^2} \right] \quad \begin{array}{l} \text{since } j^2 = -1 \\ -j^2 = 1 \end{array}$$

$$\rightarrow \frac{1}{\sqrt{2\pi}} \left[\frac{56}{16 + \omega^2} \right]$$

35
35

3. Fourier Series:

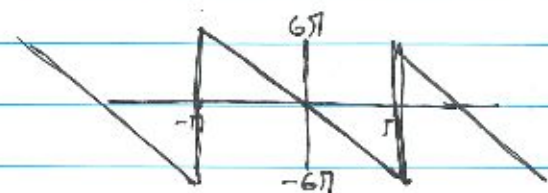
$$f(x) = -6x \quad (-\pi < x < 0)$$

$$f(x) = 6x \quad (0 < x < \pi) \quad \text{and} \quad f(x+2\pi) = f(x)$$

$$-\pi \rightarrow -6(-\pi) = 6\pi$$

$$\pi \rightarrow -6(\pi) = -6\pi$$

ODD. $\times$



$\times$ So $a_0 = 0$
and $a_n = 0$ $\times$

$$b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin nx \, dx$$

$$= \frac{2}{\pi} \int_0^{\pi} (6x) \sin nx \, dx \quad \text{Integrate by parts:}$$

$$u = 6x \quad \frac{dv}{dx} = \sin nx$$

$$\frac{du}{dx} = 6 \quad v = -\frac{1}{n} \cos nx$$

$$\text{so...} \int_0^{\pi} 6x \sin nx \, dx = (6x) \left(-\frac{1}{n} \cos nx\right) - \int \left(-\frac{1}{n} \cos nx\right) (6) \, dx$$

$$= -\frac{6x}{n} \cos nx - \int -\frac{6}{n} \cos nx \, dx$$

$$= -\frac{6x}{n} \cos nx + \frac{6}{n^2} \sin nx$$

$$= \frac{2}{\pi} \left[-\frac{6x}{n} \cos nx + \frac{6}{n^2} \sin nx \right]_0^{\pi}$$

$$= \frac{2}{\pi} \left[-\frac{6\pi}{n} \cos n\pi + \frac{6}{n^2} \sin n\pi \right] + \frac{2}{\pi} \left[-\frac{6(0)}{n} \cos 0 + \frac{6}{n^2} \sin 0 \right]$$

Since $\sin \pi$ and $\sin 0 = 0 \dots$

$$\hookrightarrow \frac{2}{\pi} \left[-\frac{6\pi}{n} \cos n\pi \right] \rightarrow \frac{-12}{n} \cos n\pi = b_n$$

$$\frac{0}{40}$$

$$\text{so...} \quad f(x) = \sum_{n=1}^{\infty} \left(\frac{-12}{n} \cos n\pi \right)$$

$$\text{so...} \quad y_{n=1} \quad b_1 = \frac{-12}{1} \cos \pi = 12$$

$$y_{n=2} \quad b_2 = \frac{-12}{2} \cos 2\pi = -6$$

$$y_{n=3} \quad b_3 = \frac{-12}{3} \cos(3\pi) = -4$$

$$y_{n=4} \quad b_4 = \frac{-12}{4} \cos(4\pi) = 3$$

$$\text{So, } f(x) = \sum_{n=1}^{\infty} \left(\frac{-12}{1} \cos(\pi) + \frac{-12}{2} \cos(2\pi) + \frac{-12}{3} \cos(3\pi) + \frac{-12}{4} \cos(4\pi) \right) \dots$$

2. Odd, Even or Neither.

i, $6\sin x + 2$ $x = -2$ $6\sin(-2) + 2 = -3.456$
 $x = 2$ $6\sin(2) + 2 = 7.456$ NEITHER (no symmetry)

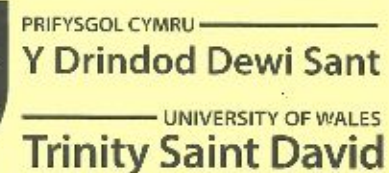
ii, $x^2 - 3x$ $x = -2$ $-2^2 - 3(-2) = -122$ ODD ✓
 $x = 2$ $2^2 - 3(2) = 122$

iii, $5 - 3\cos x$ $x = -2$ $5 - 3\cos(-2) = 6.248$ EVEN ✓
 $x = 2$ $5 - 3\cos(2) = 6.248$

iv, x^3 $x = -2$ $(-2)^3 = -8$ ODD ✓
 $x = 2$ $(2)^3 = 8$

v, $\sin^2(x)$ $x = -2$ $\sin^2(-2) = 0.757$ ODD $\times$
 $x = 2$ $\sin^2(2) = -0.757$

$$\frac{20}{25}$$



8 Pages

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[illegible]

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Surname Jones (Block Capitals) Forename(s) Sean Jones

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PATENT PENDING

$$\textcircled{1} \hat{f}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-j\omega x} dx$$

$$f = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} 7e^{4t} e^{-j\omega t} dt + \int_0^{\infty} 7e^{-4t} e^{-j\omega t} dt$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^0 7e^{(4-j\omega)t} dt + \int_0^{\infty} 7e^{(-4-j\omega)t} dt$$

$$= \frac{1}{\sqrt{2\pi}} \left[\frac{7}{4-j\omega} e^{(4-j\omega)t} \right]_{-\infty}^0 + \left[\frac{7}{-4-j\omega} e^{(-4-j\omega)t} \right]_0^{\infty}$$

$$= \frac{1}{\sqrt{2\pi}} \left[\frac{7}{4-j\omega} e^{(4-j\omega)(0)} - \frac{7}{4-j\omega} e^{(4-j\omega)(-\infty)} + \frac{7}{-4-j\omega} e^{(-4-j\omega)(\infty)} - \frac{7}{-4-j\omega} e^{(-4-j\omega)(0)} \right]$$

↑ ignore the line

$$= \frac{7}{-4-j\omega} e^{(-4-j\omega)(0)}$$

$$= \frac{1}{\sqrt{2\pi}} \left[\frac{7}{4-j\omega} - 0 + 0 - \frac{7}{-4-j\omega} \right]$$

$$= \frac{1}{\sqrt{2\pi}} \left[\frac{7}{4-j\omega} - \frac{7}{-4-j\omega} \right]$$

$$= \frac{7(-4-j\omega) - 7(4-j\omega)}{(4-j\omega)(-4-j\omega)}$$

$$= \frac{-28 - 7j\omega - 28 + 7j\omega}{-16 - 4j\omega + 4j\omega + j\omega^2} = \frac{0}{j\omega^2 - 16}$$

24
35

②

i) $6\sin(2)+2 = 7.456$ $6\sin(-2)+2 = -3.456$ Neither ✓

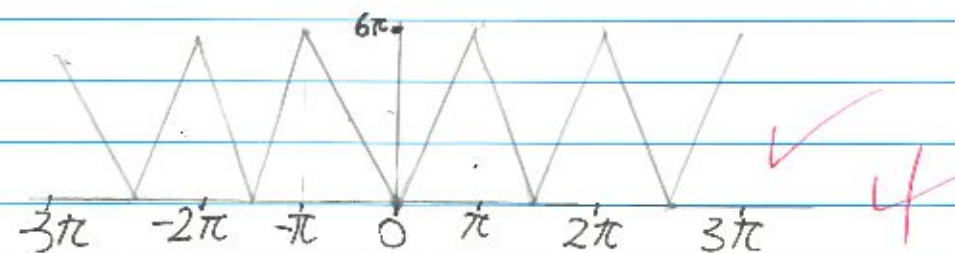
ii) $(2)^7-3(2) = 122$ $(-2)^7-3(-2) = -122$ odd ✓

iii) $5-3\cos(2) = 6.25$ $5-3\cos(-2) = 6.25$ even ✓

iv) $(2)^3 = 8$ $(-2)^3 = -8$ odd ✓

v) $\sin^2(2) = 0.789$ $\sin^2(-2) = 0.789$ even ✗

$\frac{20}{25}$ ✓



$-6(-\pi) = 6\pi$ $6(\pi) = 6\pi$ this is even ✓

$B_n = 0$ ✓

$a_0 = \frac{2}{\pi} \int_0^\pi f(x) dx$

$= \frac{2}{\pi} \int_0^\pi (x+2\pi) dx$

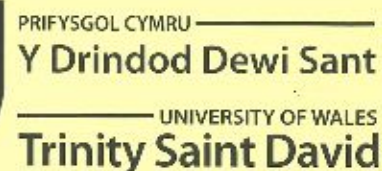
$= \frac{2}{\pi} \left[\frac{x^2}{2} + 2\pi x \right]_0^\pi$

$a_n = \frac{2}{\pi} \int_0^\pi f(x) \cos nx dx$

$a_n = \frac{2}{\pi} \int_0^\pi f(x) dx$

0.

$\frac{5}{40}$ ✓



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Surname	(Block Capitals) Forename(s)
Davies	Madoc Leo

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PATENT PENDING

1. Find the Fourier transform of the function

$$v(t) = 7e^{4t} \quad \text{for } t < 0$$

$$v(t) = 7e^{-4t} \quad \text{for } t > 0$$

$$\begin{aligned} \hat{v}(\omega) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} 7e^{4t} x e^{j\omega x} + \frac{1}{\sqrt{2\pi}} \int_0^{\infty} 7e^{-4t} x e^{-j\omega x} \\ &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} 7e^{(4+j\omega)x} + \frac{1}{\sqrt{2\pi}} \int_0^{\infty} 7e^{(-4-j\omega)x} \\ &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} 7e^{(4+j\omega)x} + \frac{1}{\sqrt{2\pi}} \int_0^{\infty} 7e^{(-4-j\omega)x} \end{aligned} \quad \begin{matrix} 4 \\ 3 \end{matrix}$$

$$= \frac{1}{\sqrt{2\pi}} \left[\frac{1}{4+j\omega} 7e^{(4+j\omega)x} \right]_{-\infty}^{\infty} + \frac{1}{\sqrt{2\pi}} \left[\frac{1}{-4-j\omega} 7e^{(-4-j\omega)x} \right]_{-\infty}^{\infty} \quad 4$$

$$= \frac{1}{\sqrt{2\pi}} \left[\frac{1}{4+j\omega} 7e^0 \frac{1}{4+j\omega} e^{-\infty} \right] + \frac{1}{\sqrt{2\pi}} \left[\frac{1}{-4-j\omega} 7e^{-\infty} \frac{1}{-4-j\omega} e^0 \right]$$

$$= \frac{1}{\sqrt{2\pi}} \left[\frac{7(4+j\omega + 4+j\omega)}{4+j\omega + 4+j\omega} \right] \quad 4$$

$$= \frac{1}{\sqrt{2\pi}} \left[\frac{7(4\omega + j\omega^2 + 4\omega + j\omega^2)}{12\omega + 4j\omega^2 + 4j\omega^2 + j^2\omega^2} \right] \quad X$$

$$= \frac{1}{\sqrt{2\pi}} \left[\frac{7(4\omega + 4\omega)}{12\omega + \omega^2} \right]$$

$$= \frac{1}{\sqrt{2\pi}} \left[\frac{56\omega}{12\omega + \omega^2} \right] \quad X$$

Ans

$$\frac{15}{35}$$

2. State whether the following functions have odd, even or no symmetry. Showing how you arrived at the

i) $6 \sin x + 2$

$$6 \sin(2) + 2 = 7.46$$

$$6 \sin(-2) + 2 = -3.46$$

Neither

ii) $x^7 - 3x$

$$2^7 - 3(2) = 122$$

$$-2^7 - 3(-2) = -122$$

Odd

iii) $5 - 3 \cos x$

$$5 - 3 \cos(2) = 6.24$$

$$5 - 3 \cos(-2) = 6.24$$

Even

iv) x^3

$$2^3 = 8$$

$$-2^3 = -8$$

odd

v) $\sin^2(x)$

$$\sin(2)^2 = 0.83$$

$$\sin(-2)^2 = 0.83$$

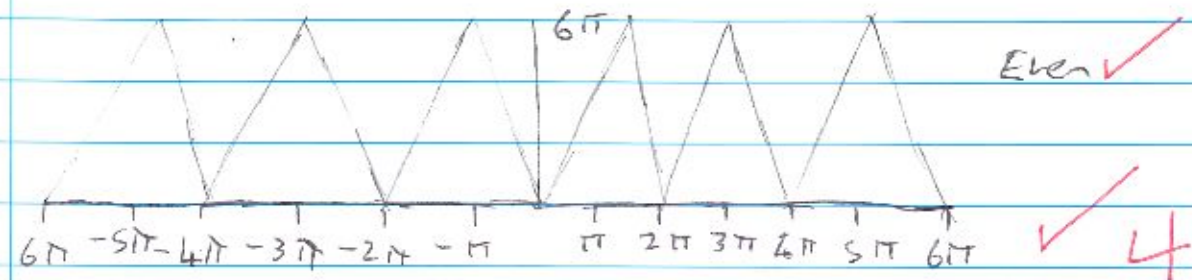
Even

25.
25

3. Sketch the graph and determine the Fourier series expansion function and find the first 4 non zero terms of the series.

$$f(x) = -6x \quad (-\pi < x < 0)$$

$$f(x) = 6x \quad (0 < x < \pi) \quad \text{and} \quad f(x+2\pi) = f(x)$$



$$a_0 = \frac{2}{\pi} \int_0^{\pi} 6x$$

$$= \frac{2}{\pi} \left[6x \right]_0^{\pi}$$

$$\text{Let } u =$$

$$= \frac{2}{\pi} [6(\pi)] - \frac{2}{\pi} [6(0)]$$

$$= 6\pi$$

3

$$a_n = \frac{2}{\pi} \int_0^{\pi} 6x \cos nx \, dx$$

✓ 2

integrate by parts

$$\text{Let } u = 6x$$

$$\frac{du}{dx} = \cos nx$$

$$\frac{du}{dx} = 6$$

$$v = \frac{1}{n} \sin nx$$

✓ 4

$$\int 6x \cos nx = \frac{2}{\pi} \left[(6x) \left(\frac{1}{n} \sin nx \right) - \int (6) \left(\frac{1}{n} \sin nx \right) \right]$$

$$= \frac{2}{\pi} \left(\frac{6x}{n} \sin nx + \left(\frac{6}{n^2} \cos nx \right) \right)$$

✓ 4

3.

$$= \frac{2}{\pi} \left[\frac{6(\pi)}{n} \sin nx + \frac{6}{n^2} \cos nx \right] - \frac{2}{\pi} \left[\frac{6(0)}{n} \sin nx + \frac{6}{n^2} \cos nx \right]$$

or

$$= 2 \left[\frac{6}{n^2} \cos n\pi \right] - 2 \frac{2}{\pi} \left[\frac{6}{n^2} \right]$$

✓

$$= \frac{12}{n^2} \cos n\pi - \frac{12}{\pi n^2}$$

X

$$a_n = \frac{12}{n^2} \cos n\pi$$

X

$$\text{If } n=1 \quad a_1 = \frac{12}{(1)^2} \cos(1)\pi = \frac{12}{1^2} (-1) = -\frac{12}{1^2}$$

$$\text{If } n=2 \quad a_2 = \frac{12}{(2)^2} \cos(2)\pi = \frac{12}{2^2} (1) = \frac{12}{2^2}$$

$$\text{If } n=3 \quad a_3 = \frac{12}{(3)^2} \cos(3)\pi = \frac{12}{(3)^2} (-1) = -\frac{12}{3^2}$$

$$\text{If } n=4 \quad a_4 = \frac{12}{(4)^2} \cos(4)\pi = \frac{12}{(4)^2} (1) = \frac{12}{4^2}$$

$$\text{If } n=5 \quad a_5 = -\frac{12}{5^2}$$

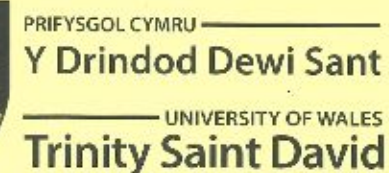
$$f(x) = \frac{1}{2} a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$f(x) = \frac{1}{2} 6\pi + 12 \cos x - 12 \cos 3x - 12 \cos 5x$$

X

20
40

✓



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2	25
3	39
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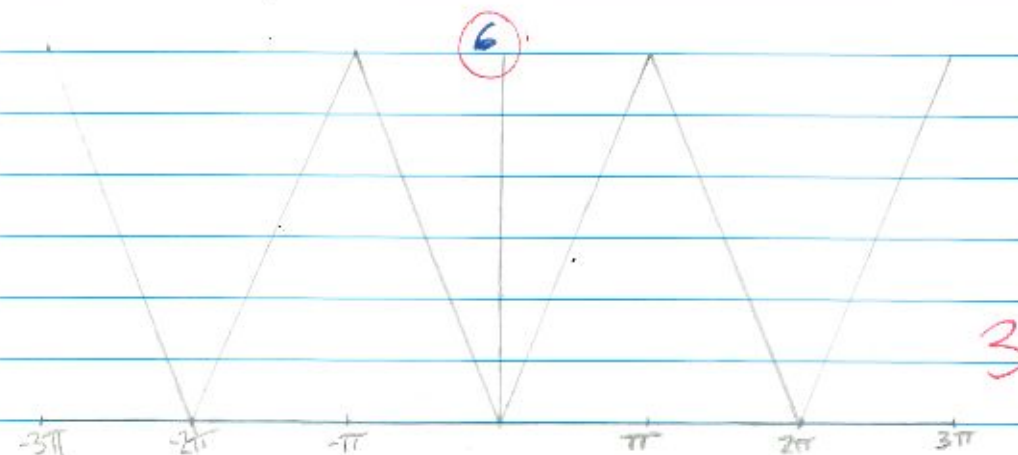
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Surname **DAVIELS** Forename(s) **TRYSTAN**

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PATENT PENDING

3.

 $\therefore$ even function ✓ $\therefore b_n = 0$ ✓

$$a_0 = \frac{2}{\pi} \int_0^{\pi} f(x) dx$$

$$= \frac{2}{\pi} \int_0^{\pi} 6x dx$$

$$= \frac{2}{\pi} \left[\frac{6x^2}{2} \right]_0^{\pi}$$

$$= \frac{2}{\pi} \left[\frac{6\pi^2}{2} \right] - \frac{2}{\pi} \left[\frac{6 \cdot 0^2}{2} \right]$$

$$= \frac{2}{\pi} \left[\frac{6\pi^2}{2} \right] - [0]$$

$$= 2 \left[\frac{6\pi}{2} \right]$$

$$= \frac{12\pi}{2}$$

$$a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx dx$$

$$= \frac{2}{\pi} \int_0^{\pi} 6x \cos nx dx$$

3.

$$= \frac{2}{\pi} \int_0^{\pi} 6x \cos nx \, dx \quad \checkmark$$

int by parts :-

$$\text{Let } u = 6x \quad \frac{dv}{dx} = \cos nx \, dx$$

$$\frac{du}{dx} = 6$$

$$v = \frac{1}{n} \sin nx$$

$$6x \cos nx \, dx = (6x) \left(\frac{1}{n} \sin nx \right) - \int \left(\frac{1}{n} \sin nx \right) (6 \, dx)$$

$$= \frac{6x}{n} \sin nx - \int \frac{6}{n} \sin nx$$

$$= \frac{6x}{n} \sin nx + \frac{6}{n^2} \cos nx$$

$$\frac{2}{\pi} \left[\frac{6x}{n} \sin nx + \frac{6}{n^2} \cos nx \right]_0^{\pi}$$

$$= \frac{2}{\pi} \left[\frac{6(\pi)}{n} \sin(n\pi) + \frac{6}{n^2} \cos(n\pi) \right] \quad \checkmark$$

$$- \frac{2}{\pi} \left[\frac{6(0)}{n} \sin(n \cdot 0) + \frac{6}{n^2} \cos(n \cdot 0) \right]$$

$$\text{as } \cos(n \cdot 0) = 1$$

$$= \frac{2}{\pi} \left[\frac{6}{n^2} \cos(n\pi) \right] - \frac{2}{\pi} \left[\frac{6}{n^2} \right]$$

$$= \frac{12}{\pi n^2} [\cos(n\pi) - 1] \quad \checkmark$$

$$3. \frac{12}{\pi n^2} [\cos(n\pi) - 1]$$

If $n=1, b=1$

$$= \frac{12}{\pi(1)^2} [\cos(\pi) - 1] = \frac{12}{\pi(1)} (-2) = -\frac{24}{\pi(1)} \checkmark$$

If $n=2, b=2$

$$= \frac{12}{\pi(2)^2} [\cos(2\pi) - 1] = \frac{12}{\pi(4)} (0) = 0 \checkmark$$

If $n=3, b=3$

$$= \frac{12}{\pi(3)^2} [\cos(3\pi) - 1] = \frac{12}{\pi(9)} (-2) = -\frac{24}{\pi(9)} \checkmark$$

If $n=4, b=4$

$$= \frac{12}{\pi(4)^2} [\cos(4\pi) - 1] = \frac{12}{\pi(16)} (0) = 0 \checkmark$$

If $n=5, b=5$

$$= \frac{12}{\pi(5)^2} [\cos(5\pi) - 1] = \frac{12}{\pi(25)} (-2) = -\frac{24}{\pi(25)}$$

If $n=6, b=5$

$$= \frac{12}{\pi(6)^2} [\cos(6\pi) - 1] = \frac{12}{\pi(36)} (0) = 0$$

If $n=7, b=7$

$$= \frac{12}{\pi(7)^2} [\cos(7\pi) - 1] = \frac{12}{\pi(49)} (-2) = -\frac{24}{\pi(49)}$$

$$3. F(x) = \frac{1}{2} a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$f(x) = 0.5 \times \frac{12\pi}{2}$$

$$= \frac{6\pi}{2} - \frac{24}{(1)\pi} \cos x - \frac{24}{(9)\pi} \cos 3x - \frac{24}{(25)\pi} \cos 5x$$

$$+ \frac{24}{(49)\pi} \cos 7x \dots$$

4 terms.

$$\frac{39}{40} \checkmark$$

1. $V(t) = 7e^{4t}$ for $t < 0$
 $V(t) = 7e^{-4t}$ for $t > 0$

$$\hat{F}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-j\omega x} dx$$

$$= \int_{-\infty}^0 7e^{4t} \times e^{-j\omega t} dt +$$

$$\int_0^{\infty} 7e^{-4t} \times e^{-j\omega t} dt. \quad \checkmark 5$$

$$= \int_{-\infty}^0 7e^{(4-j\omega)t} dt + \int_0^{\infty} 7e^{-(4+j\omega)t} dt \quad \checkmark 3$$

$$= \left[\frac{7}{(4-j\omega)} \right] + \left[\frac{7}{-(4+j\omega)} \right]_0 \quad 1$$

$$= \left[\frac{14 - 28 + j\omega + 28 - j\omega}{-8 + j\omega - 4j\omega - \omega^2} \right]$$

$$= \left[\frac{14}{-8 - \omega^2} \right] \quad X$$

$$\hat{F}(\omega) = \left[\frac{14}{-8 - \omega^2} \right] \quad X$$

2.

i) $6 \sin x + 2$

$$= 6 \sin(2) + 2 = 7.456 \quad \text{No Symmetry as answers and signs are different}$$

$$= 6 \sin(-2) + 2 = -3.456$$

ii) $x^2 - 3x$

$$= 2^2 - 3(2) = 122 \quad \text{odd as answer is same but signs are different}$$

$$= -2^2 - 3(-2) = -122$$

iii) $5 - 3 \cos x$

$$= 5 - 3 \cos(2) = 6.248 \quad \text{even, as answer and signs are the same.}$$

$$= 5 - 3 \cos(-2) = 6.248$$

iv) x^3

$$= 2^3 = 8 \quad \text{odd, as answer is the same but the signs are different.}$$

$$= (-2)^3 = -8$$

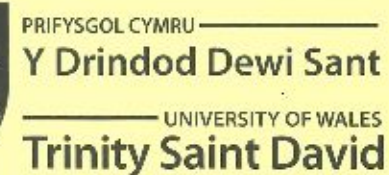
v) $\sin^2(x)$

$$= \sin(2)^2 = 0.8268 \quad \text{even, as answer and signs are the same.}$$

$$= \sin(-2)^2 = 0.8268$$

9/35

25/25



8 Pages

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1	Student Number	
2	Title of Examination Paper	CLASS TEST 2
3	Module Code	PSDES008
4	Date and Time of Examination	
5	Examination Venue	
6	Seat Number	
7	Number of Answer Books handed in	

List the numbers of the questions you have attempted	Office Use
1	27
2	20
3	0
Total	47

Please complete this section and SEAL before the examination starts.

Surname MULDER (Block Capitals) Forename(s) CHRISTOPHER

(Gweler yr ochr arall am y fersiwn Cymraeg)

PATENT PENDING

$$\textcircled{1} \quad \begin{aligned} V(t) &= 7e^{4t} & t < 0 \\ V(t) &= 7e^{-4t} & t > 0 \end{aligned}$$

$$\hat{f}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-j\omega x} dx$$

$$\hat{f}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \cancel{7e^{4t}} \times e^{-j\omega x} dx + \frac{1}{\sqrt{2\pi}} \int_0^{\infty} 7e^{-4t} \times e^{-j\omega x} dx \quad \checkmark \quad 5$$

$$\cancel{\frac{1}{\sqrt{2\pi}} \int_{-\infty}^0 7e^{4t} e^{-j\omega x} dx}$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^0 7e^{(4-j\omega)t} dx + \frac{1}{\sqrt{2\pi}} \int_0^{\infty} 7e^{-(4+j\omega)t} dx \quad \checkmark \quad 3$$

$$= \frac{1}{\sqrt{2\pi}} \left[\frac{7}{4-j\omega} e^{(4-j\omega)t} \right]_{-\infty}^0 + \frac{1}{\sqrt{2\pi}} \left[\frac{7}{-(4+j\omega)} e^{-(4+j\omega)t} \right]_{0}^{\infty} \quad \checkmark \quad 5$$

$$= \frac{1}{\sqrt{2\pi}} \left[\frac{7}{4-j\omega} e^0 - \frac{7}{4-j\omega} e^0 \right] + \frac{1}{\sqrt{2\pi}} \left[\frac{7}{-(4+j\omega)} e^0 - \frac{7}{-(4+j\omega)} e^0 \right] \quad 5$$

$$= \frac{1}{\sqrt{2\pi}} \left[\frac{7}{(4-j\omega)} + \frac{7}{(4+j\omega)} \right] \quad 3$$

$$= \frac{1}{\sqrt{2\pi}} \left[\frac{27+j\omega + 27-j\omega}{16+4j\omega-4j\omega-j^2\omega^2} \right] \quad 2$$

$$= \frac{1}{\sqrt{2\pi}} \left[\frac{27}{16+\omega^2} \right] \quad 4$$

$$\frac{27}{35} \quad \checkmark$$

②

i) $6 \sin x + 2$

$$f(30) = 6 \sin(30) + 2 = -3.928$$

$$f(-30) = 6 \sin(-30) + 2 = 7.928$$

Different number & sign = neither

ii) $x^2 - 3x$

$$f(2) = (2)^2 - 3(2) = 122$$

Same number different

sign = Odd

$$f(-2) = (-2)^2 - 3(-2) = -122$$

iii) $5 - 3 \cos x$

$$f(60) = 5 - 3 \cos(60) = 7.857$$

$$f(-60) = 5 - 3 \cos(-60) = 7.857$$

Same number & sign = even

iv) x^3

$$f(2) = (2)^3 = 8$$

Same number different sign

$$f(-2) = (-2)^3 = -8$$

= odd

v) $\sin^2(x)$

$$f(30) = \sin^2(30) = 21.24$$

Same number

different sign = odd

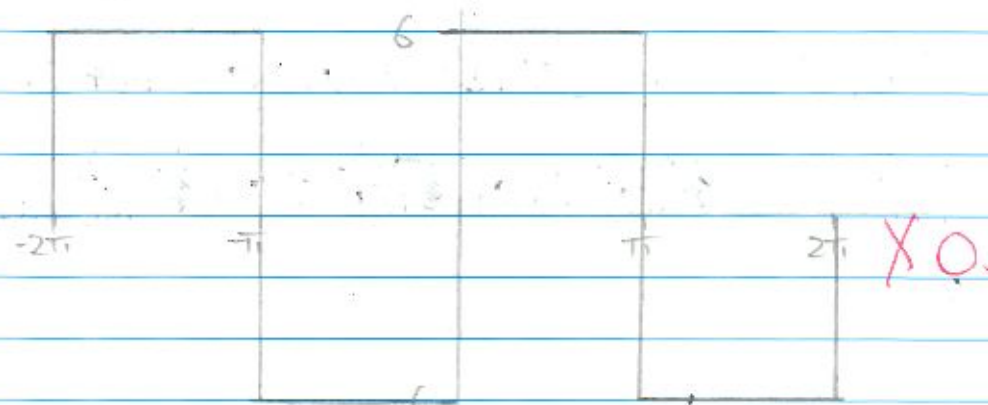
$$f(-30) = \sin^2(-30) = -21.24$$

③

$$f(x) = -6x \quad (-\pi < x < 0)$$

$$f(x) = 6x \quad (0 < x < \pi)$$

$$f(x+2\pi) = f(x)$$

Not symmetrical $\therefore$ function is odd.

$$a_0 = 0; a_n = 0$$

$$b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin nx \, dx$$

$$b_n = \frac{2}{\pi} \int_0^{\pi} 6 \sin nx \, dx$$

$$= \frac{2}{\pi} \left[\frac{6}{n} \cos nx \right]_0^{\pi}$$

$$= \frac{2}{\pi} \left[\frac{6}{n} \cos n\pi \right] - \frac{2}{\pi} \left[\frac{6}{n} \cos 0 \right]$$

$$= -\frac{12}{\pi n} \cos n\pi + \frac{12}{\pi n}$$

$$b_n = \frac{12}{\pi n} [\cos n\pi - 1]$$

$$n=1 \quad b_1 = \frac{12}{\pi(1)} [\cos(1)\pi - 1] = \frac{12}{\pi} (-2) = -\frac{24}{\pi}$$

$$n=2 \quad b_2 = \frac{12}{\pi(2)} [\cos 2\pi - 1] = \frac{12}{\pi(2)} (0) = 0$$

20/25

$$n=3 \quad b=3 \quad \frac{12}{\pi(3)} [\cos 3\pi - 1] = \frac{12}{\pi(3)} (-2) = \frac{-24}{\pi(3)}$$

$$n=4 = 0$$

$$n=5 = \frac{-24}{\pi(5)}$$

$$f(x) = \frac{1}{2} a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$f(x) = b_1 \sin 1x + b_2 \sin 2x + b_3 \sin 3x + b_4 \sin 4x \dots$$

$$f(x) = \frac{-24}{\pi(1)} \sin x + \frac{-24}{\pi(3)} \sin 3x + \frac{-24}{\pi(5)} \sin 5x + \frac{24}{\pi(7)} \sin 7x \dots$$

X

0/40



8 Pages

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4	Date and Time of Examination	
5	Examination Venue	
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7	Number of Answer Books handed in	

List the numbers of the questions you have attempted	Office Use
1	0
2	25
3	5
Total	30

Please complete this section and SEAL before the examination starts.

Surname ~~Walter~~ EDGELL (Block Capitals) Forename(s) ALED CHARLES

Complete
section,
remove
strip, fold as
indicated
and SEAL.

PATENT PENDING

10. Fourier transform.

$$V(t) = 7e^{4t} \quad \text{for } t < 0$$

$$V(t) = 7e^{-4t} \quad \text{for } t > 0$$



$$F(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} 7e^{-j\omega x} dx$$

$$[-j\omega x 7e^{-j\omega x}]_0^{\infty} - [-j\omega x 7e^{j\omega x}]_{-\infty}^0$$

$$[-0 + 7e^{-j\omega \cdot \infty}] + j\omega \cdot 0 - [-j\omega \cdot \infty 7e^{-j\omega \cdot \infty} + j\omega \cdot \infty 7e^{-j\omega \cdot \infty}]$$

Since $e = 1$

$$\frac{1}{\sqrt{2\pi}} \int 7e^{-j\omega x} dx = \frac{7 \cdot 2\pi \cdot 2}{\sqrt{2\pi} \cdot 2\pi \cdot 2}$$

$$= \frac{14}{\sqrt{2\pi}} \cdot \frac{4\pi}{2\pi}$$

$$= \frac{4.9}{\sqrt{2}} \cdot 2$$

X

O.



2.

i) $6 \sin x + 2$ if $x=2$ then $= 7.46$
if $x=-2$ then $= -3.46$

Therefore this is neither odd nor even ✓

ii) $x^7 - 3x$ if $x=2$ $= 122$
if $x=-2$ $= -122$

As they are the opposite to each other they are odd ✓

iii) $5 - 3 \cos x$ if $x=2$ $= 6.25$
if $x=-2$ $= 6.25$

As both numbers are the same the function is even ✓

iv) x^3 if $x=2$ $= 8$
if $x=-2$ $= -8$

As the numbers are same but one is positive and one is negative it is an odd function ✓

v) $\sin^2(x)$ if $x=2$ $= 0.83$
if $x=-2$ $= 0.83$

As both numbers are the same and are both positive the function is even. ✓

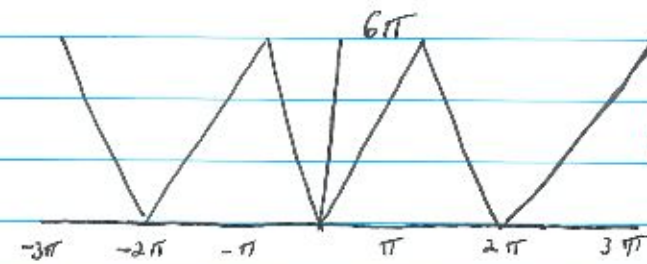
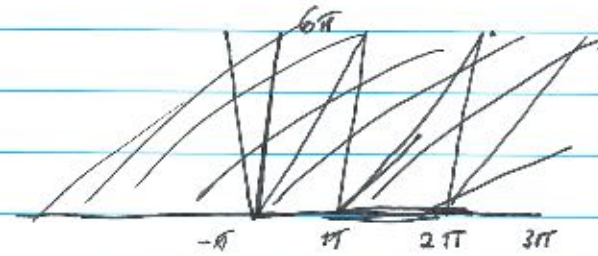
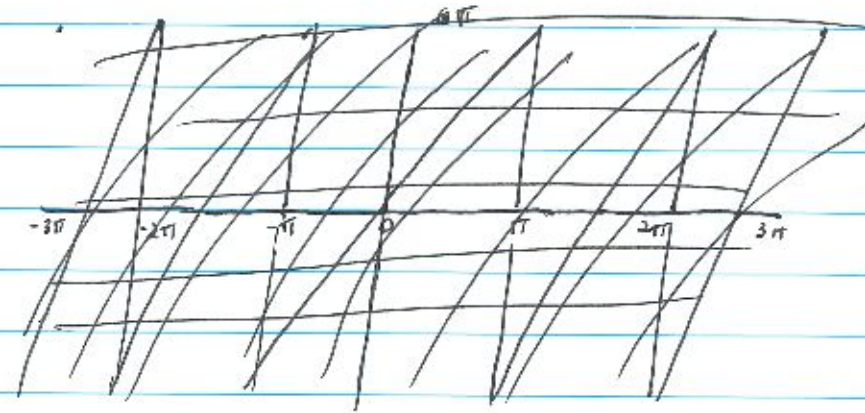
25
25

3.

$$f(x) = -6x \quad (-\pi < x < 0)$$

$$f(x) = 6x \quad (0 < x < \pi)$$

$$f(x+2\pi) = f(x)$$



Even function therefore $b_n = 0$

$$a_0 = \frac{2}{\pi} \int_0^{\pi} x + 2\pi \, dx$$

$$= \frac{2}{\pi} \left[\frac{x^2}{2} + \frac{2\pi x}{2} \right]_0^{\pi}$$

$$= \frac{2}{\pi} \left[\frac{\pi^2}{2} + \frac{2\pi^2}{2} \right] - \frac{2}{\pi} \left[\frac{0^2}{2} + \frac{2\pi \cdot 0}{2} \right]$$

$$= [3\pi] - [2\pi] = \pi$$

$$a_n = \frac{2}{\pi} \int_0^{\pi} (x+2\pi) \cos nx \, dx$$

Integrating by parts

$$u = x+2\pi \quad \frac{du}{dx} = 1$$

$$\frac{dv}{dx} = \cos nx \quad v = \frac{1}{n} \sin nx$$

$$\int (x+2\pi) \cos nx \, dx = (x+2\pi) \left(\frac{1}{n} \sin nx \right) - \int \left(\frac{1}{n} \sin nx \right) (1) \, dx$$

$$= \frac{x+2\pi}{n} \sin nx + \int \frac{1}{n} \sin nx \, dx$$

$$(2\pi) \left(\frac{1}{n} \sin nx \right) dx$$

Integrating by parts again

$$u = 2\pi \quad \frac{du}{dx} = 0$$

$$\frac{dv}{dx} = \frac{1}{n} \sin nx \quad v = -\frac{1}{n^2} \cos nx$$

$$= \left(\frac{1}{n^2} \cos nx \right) (2\pi) = \frac{2}{n^2} \cos nx$$

Sub into original

$$a_n = \frac{2}{\pi} \left[\frac{x+2\pi}{n} \sin nx + \left(\frac{2}{n^2} \cos nx \right) \right]_0^{\pi}$$

$$= \frac{2}{\pi} \left[\frac{\pi+2\pi}{n} \sin n\pi + \frac{2}{n^2} \cos n\pi \right] - \frac{2}{\pi} \left[\frac{0+2\pi}{n} \sin n(0) + \frac{2}{n^2} \cos n(0) \right]$$

$$a_n = \frac{2}{n^2} \cos n\pi$$

$n=1$	$a=1$	$\frac{2}{(1)^2}$	$\cos(1)\pi = (-1)$	$= -\frac{2}{(1)^2}$
$n=2$	$a=2$	$\frac{2}{(2)^2}$	$\cos(2)\pi = (1)$	$= \frac{2}{(2)^2}$
$n=3$	$a=3$	$\frac{2}{(3)^2}$	$\cos(3)\pi = (-1)$	$= -\frac{2}{(3)^2}$
$n=4$	$a=4$	$\frac{2}{(4)^2}$	$\cos(4)\pi = (1)$	$= \frac{2}{(4)^2}$
$n=5$	$a=5$	$\frac{2}{(5)^2}$	$\cos(5)\pi = (-1)$	$= -\frac{2}{(5)^2}$
$n=6$	$a=6$	$\frac{2}{(6)^2}$	$\cos(6)\pi = (1)$	$= \frac{2}{(6)^2}$
$n=7$	$a=7$	$\frac{2}{(7)^2}$	$\cos(7)\pi = (-1)$	$= -\frac{2}{(7)^2}$
$n=8$	$a=8$	$\frac{2}{(8)^2}$	$\cos(8)\pi = (1)$	$= \frac{2}{(8)^2}$

$$f(x) = \frac{1}{2} a_0 + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx \right)$$

$$f(x) = \frac{1}{2} \pi + \frac{2}{2^2} \cos 2x + \frac{2}{4^2} \cos 4x + \frac{2}{6^2} \cos 6x + \frac{2}{8^2} \cos 8x \dots$$

0.

5.
✓