



12 Pages

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3. The answers must be legibly written. The Examiners can either ignore illegible script or have it typed and the expense of typing charged to the candidate.
4. Write your answers in ink.
5. Begin each answer on a fresh page. Do all rough work in the answer book and cross it through.
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Complete, fold and seal
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SURNAME:

Pesarillo

FORENAME(S):

Ronan

1	Student Number	PES15826963
2	Title of Examination Paper	Electrical Principles Theory
3	Module Code	GOEE5001
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6	Seat Number	
7	Number of Answer Books handed in	1

List the numbers of the questions you have attempted	Office Use
1)	9
2)	9
3)	20
4)	24
Total	62/100

62% ✓

C.M.

(Gweler yr ochr arall am y fersiwn Cymraeg)

1) a) $v = 60e^{-t} = L \frac{di}{dt} + Ri$
if $L = 6, R = 12$ Initial Time = 0s

$\mathcal{L}\left[\frac{di(t)}{dt}\right] = sF(s) - f(0)$ ✓

Q1
9/30
Q1a
2/20

$6 \frac{di}{dt} + 12i = ?$

$s \times 6/s - 60e^{-t}$
or $\frac{-60}{s+1}$

$\frac{60}{s+1} \xrightarrow{L} = s6 + \frac{12}{s}$ Point of Mark Reduction.

$60 \mathcal{L}(e^{-t}) = 6 \mathcal{L}\left(\frac{di}{dt}\right) + 12 \mathcal{L}(i)$

$\left(\frac{12}{s}\right)^2 = 12^2 + 2(12 \times s^{-1}) + s^{-2}$

$\frac{60}{s+1} = \frac{6}{s^2} + \frac{12}{s}$

Confirmed: $\mathcal{L}(12) = \frac{12}{s}$ for v $\mathcal{L}(60e^{-t}) = \frac{60}{s+1}$ for R

$\frac{60}{s+1} = \frac{6}{s^2} + \frac{12}{s}$

$60 = (s+1)\left(\frac{6}{s^2} + \frac{12}{s}\right)$
 $60 = \frac{6s+6}{s^2} + \frac{12s+12}{s}$

$60s^2 = 6s + 6 + 12s^2 + 12s$
 $60s^2 = 12s^2 + 18s + 6$
 $0 = -48s^2 + 18s + 6$
 $0 = 6(-8s^2 + 3s + 1)$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = -8, b = 3, c = 1$
 $F(s) = \mathcal{L}[F(t)]$
 $60e^{-0} = 60 \times 1 = 60$

$\frac{-3 \pm \sqrt{9 - (4 \times -8 \times 1)}}{-16} = \frac{-3 \pm \sqrt{9 + 32}}{-16}$

$= \frac{-3 \pm \sqrt{41}}{-16} = \frac{3 \pm \sqrt{41}}{16}$
 $+ = 0.587695264...$
 $- = -0.212895264...$

$\frac{60}{s+1} = -54 + \frac{12}{s}$

$\frac{60}{s+1} = 6 + \frac{12}{s} - \frac{60}{s+1}$
 $\frac{120}{s+1} - \frac{12}{s} = 6$

$\mathcal{L}\left(\frac{di}{dt}\right) = s \times \frac{6}{s} - 60$

$54 = \frac{12A}{s} - \frac{60B}{s+1}$

$\rightarrow 6 = \frac{120A}{s+1} - \frac{12B}{s}$
 $\rightarrow 6 = \frac{120A(s)}{s(s+1)} - \frac{12B(s+1)}{s(s+1)}$

$\rightarrow 6 - 60 = -54$

$54(s)(s+1) = 12(s+1)A - 60(s)B$

if $s = 0, 54(0)(1) = 12(1)A$
 $-12 = A$

if $s = 0, 6 = -12B(1)$
 $\frac{6}{-12} = B = -0.5$

$\frac{120 \times 0.05}{s+1} - \frac{12 \times -0.5}{s}$
 $= \frac{6}{s+1} + \frac{6}{s}$

if $s = -1, 54(-1)(0) = -60(-1)B$
 $0 = 60B$

if $s = 1, 6 = 120A$
 $\frac{6}{120} = A = 0.05$

$\mathcal{L}^{-1}\left(\frac{6}{s+1}\right) = 6e^{-t}$
 $\mathcal{L}^{-1}\left(\frac{6}{s}\right) = 6$

Alternatively...

Therefore: $I = 6 + 6e^{-t}$

$54 = 12(1)A$

$54 = -60(-1)B$

$\frac{54}{12} = 4.5 = A$

$\frac{54}{60} = 0.9 = B$

See next page for 1) b)

1) b) Laplace Transforms of the following functions:

i) $e^{3t} + 12 \xrightarrow{\mathcal{L}} \frac{1}{s-3} + \frac{12}{s}$; $e^{-at} \xrightarrow{\mathcal{L}} \frac{1}{s+a}$ & $1 \text{ or } v(t) \xrightarrow{\mathcal{L}} \frac{1}{s}$

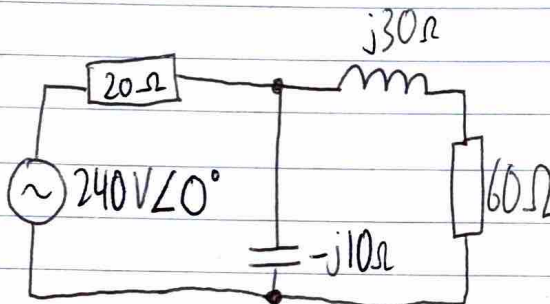
ii) $\cos(5t) + 2\sin(2t) \xrightarrow{\mathcal{L}} \frac{s}{s^2+5^2} + \frac{2}{s^2+2^2} = \frac{s}{s^2+25} + \frac{2}{s^2+4}$; $\cos(\omega t) = \frac{s}{s^2+\omega^2}$ & $\sin(\omega t) = \frac{\omega}{s^2+\omega^2}$

iii) $(t^3+4t+2)e^{3t} \text{ or } e^{3t}(t^3+4t+2) \xrightarrow{\mathcal{L}} \frac{6}{(s-3)^4} + \frac{4t}{(s-3)^3} + \frac{2}{(s-3)^2} = \frac{6}{(s-3)^4} + \frac{6}{(s-3)^3} + \frac{2}{(s-3)^2}$

iv) $5e^{5t}\cos(2t) - e^{3t} \xrightarrow{\mathcal{L}} \frac{5(s-5)}{(s-5)^2+2^2} = \frac{5s-25}{(s-5)^2+4} = \frac{5s-25}{s^2-10s+29}$

v) $2e^{-2t}\sin(4t) \xrightarrow{\mathcal{L}} \frac{4}{(s+2)^2+4^2} = \frac{4}{s^2+4s+20}$

2) Fig. 2



$$I^2 R = P$$

$$V^2/R = P$$

$$VI = P$$

$$V/R = I$$

$$V/I = R$$

a) $\frac{(j30+60)(-j10)}{j30+60-j10} = \frac{300-j600}{60+j20} \times \frac{60-j20}{60-j20} = \frac{18000+j^2 12000-j6000-j36000}{3600-j400}$

$= \frac{6000-j42000}{3600-j400} = \frac{6000-j42000}{4000} = \frac{6-j42}{4} = \frac{3-j21}{2} = Z_{Tot}$

b) $240\angle 0^\circ$ in Cartesian form: $\frac{\sqrt{13}}{2} \angle 33.6900675^\circ$, $\frac{\sqrt{13}}{2} = 1.80277563$
 $1.8027756 \angle 33.6900675^\circ = 133.128047 \angle -33.6900675^\circ = I_{Tot}$

c) $240 \div (L-C_{Tot-is}) = 12A?$
 $V/R = I \quad (j30-j10-j20)$

error carried forward

d) $240^2 \div (20+60=80) = 720W?$
 $V^2 \div R_{Tot-is} \quad V^2/R = P$

Q320/25

3) $f(x) = \begin{cases} -3 & \text{if } -\pi \leq x < 0 \\ 3 & \text{if } 0 \leq x < \pi \end{cases}$ and $f(x+2\pi) = f(x)$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx = \frac{1}{\pi} \int_0^{\pi} f(x) dx + \frac{1}{\pi} \int_{-\pi}^0 f(x) dx$$

$$= \frac{1}{\pi} \int_0^{\pi} 3 dx + \frac{1}{\pi} \int_{-\pi}^0 -3 dx$$

$$= \frac{1}{\pi} [3x]_0^{\pi} + \frac{1}{\pi} [-3x]_{-\pi}^0$$

$$\frac{3\pi}{\pi} + \frac{-3\pi}{\pi} = 0 = 3 - 3$$

$$a_0 = 0$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx) dx = \frac{1}{\pi} \int_0^{\pi} f(x) \cos(nx) dx + \int_{-\pi}^0 f(x) \cos(nx) dx$$

$$\sin\left(\frac{3\pi}{2}\right) = -1$$

$$\sin\left(\frac{\pi}{2}\right) = 1$$

$$\sin\left(\frac{\pi}{2}\right) \neq \sin\left(\frac{3\pi}{2}\right)$$

$$\sin(0) = 0$$

$$\sin(n\pi) = 0$$

$$\text{Considering } n \text{ is at least integer} = \frac{3}{\pi} \left(\frac{\sin(n\pi)}{n} - \frac{\sin(0)}{n} \right) + \frac{-3}{\pi} \left(\frac{\sin(0)}{n} + \frac{\sin(-n\pi)}{n} \right)$$

$$a_n = 0$$

See next page for more.

3)

$$f(x) = \begin{cases} -3 & \text{if } -\pi \leq x < 0 \\ 3 & \text{if } 0 \leq x < \pi \end{cases} \text{ and } f(x+2\pi) = f(x)$$

$$a_0 = 0 \quad \checkmark \quad \text{See previous page for written submissions' evidence.}$$

$$a_n = 0 \quad \checkmark$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(nx) dx = \frac{1}{\pi} \int_0^{\pi} f(x) \sin(nx) dx + \int_{-\pi}^0 f(x) \sin(nx) dx$$

$$= \frac{1}{\pi} \int_0^{\pi} 3 \sin(nx) dx + \frac{1}{\pi} \int_{-\pi}^0 -3 \sin(nx) dx$$

$$\cos(0) = 1$$

$$\cos(\pi) = -1$$

$$\cos(2\pi) = 1$$

$$\vdots$$

$$= \frac{3}{\pi} \left[\frac{\cos(nx)}{n} \right]_0^{\pi} + \frac{-3}{\pi} \left[\frac{\cos(nx)}{n} \right]_{-\pi}^0$$

$$= \frac{3}{\pi} \left(\frac{\cos(\pi n)}{n} - \frac{\cos(0)}{n} \right) + \frac{-3}{\pi} \left(\frac{\cos(0)}{n} + \frac{\cos(-\pi n)}{n} \right)$$

$$= \frac{3}{\pi} \cdot \frac{\cos(\pi n)}{n} - \frac{3}{\pi} \cdot \frac{\cos(-\pi n)}{n}$$

$$\text{If } n=1 (\text{odd}), \frac{3 \cos(\pi)}{\pi} - \frac{3 \cos(-\pi)}{\pi} \quad \square - (\square) = 2\square \quad \text{Therefore} = \frac{6 \cos(\pi n)}{\pi n} = \frac{6}{\pi n} \cos(\pi n)$$

re-interpret as if n is exponential? Multiply by 2 in case. $\checkmark$

$$\text{If } n=2 (\text{even}), \frac{3 \cos(2\pi)}{2\pi} - \frac{3 \cos(2\pi)}{2\pi} \quad \square^2 - (\square)^2 = 0, \text{cancels out.} \quad \frac{12 \cos(\pi n)}{\pi n}$$

Fourier Series of Given Functions:

$$a_0 + a_1 + a_2 + a_3 \dots$$

$$+ b_1 + b_2 + b_3 \dots$$

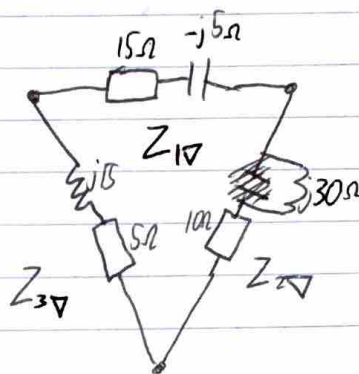
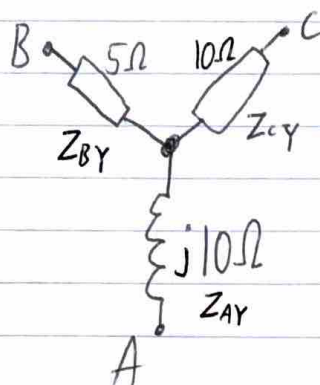
$$0 + 0 + 0 + 0 \dots$$

$$+ \frac{6}{\pi} \cos(\pi) + \frac{2}{\pi} \cos(3\pi) + \frac{1.2}{\pi} \cos(5\pi) \dots$$

$$0 + 0 + 0 + 0 \dots$$

$$+ \frac{12}{\pi} \cos(\pi) + \frac{4}{\pi} \cos(3\pi) + \frac{2.4}{\pi} \cos(5\pi) \dots$$

4) a) Convert the following star network into a delta network:



$$Z_{1\Delta} = \frac{Z_{1Y}Z_{2Y} + Z_{2Y}Z_{3Y} + Z_{3Y}Z_{1Y}}{Z_{1Y}} \rightarrow \frac{j10 \times 5 + 5 \times 10 + 10 \times j10}{j10} = \frac{50 + j150}{j10} \times \frac{j10}{-j10} = \frac{-j500 - j^2 1500}{-j^2 100} = \frac{1500 - j500}{100} = 15 - j5 = Z_{1\Delta}$$

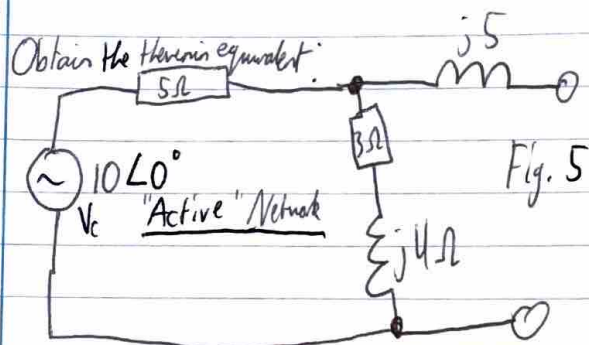
$$Z_{2\Delta} = \frac{50 + j150}{5} \times \frac{Z_{1Y}Z_{2Y} + Z_{2Y}Z_{3Y} + Z_{3Y}Z_{1Y}}{Z_{2Y}} \rightarrow = 10 + j30$$

$$Z_{3\Delta} = \boxed{} \text{ except denominator is } Z_{3Y} \rightarrow \frac{50 + j150}{10} = 5 + j15$$

See next page for 4 b).

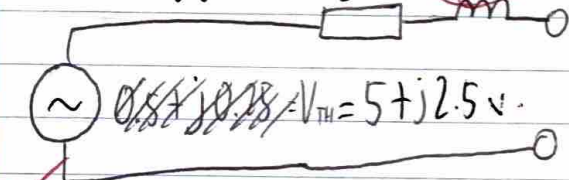
Q4
24/25
Q4a
10/10

4) b)



$$V_{Th} = \frac{3+j4}{5+3+j4} \times 10 \angle 0^\circ = \frac{3+j4}{8+j4} \times \frac{8-j4}{8-j4} \times 10 = \frac{24-j^2 16 + j32-j12}{64-j^2 16} \times 10 = \frac{40+j20}{80} \times 10 = \frac{2+j}{4} \times 10 = 5+j2.5$$

$$Z_{Th} = \frac{(3+j4) \times 5}{5+3+j4} = \frac{15+j20}{8+j4} \times \frac{8-j4}{8-j4} = \frac{120-j^2 80 + j160-j60}{64-j^2 16} = \frac{200+j100}{80} = \frac{10+j5}{4} = 2.5+j1.25$$

14/15

$$\frac{2}{4} = 0.50$$

$$\frac{1}{4} = 0.25$$

$$\frac{5}{4} = 1.25$$

$$\frac{10}{4} = 2.50$$

$$\frac{20}{4} = 5.00$$

Q4b

14/15