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Question 1



$\Rightarrow$ cylinder axis

Piston mass = 0.24 kg

rod mass = 0.33 kg

shells = 0.02 kg

Stroke = 90 mm throw = 45 mm

crank = 5 kg

rod length = 140 mm

a)

The horizontal first order force along the cylinder axis can be represented as 2 counter-rotating constant amplitude vectors, (next page).

$$F = m_{\text{rod}} \cdot r \omega^2 \cos \theta$$

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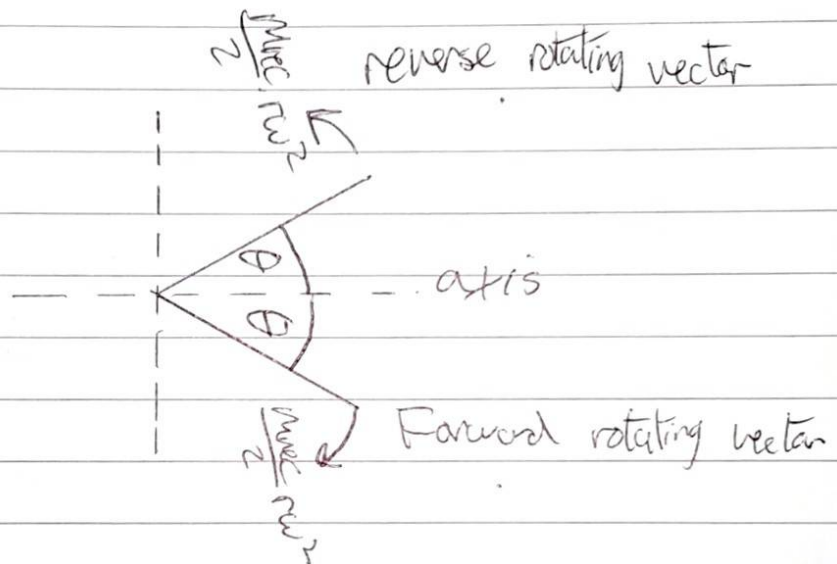
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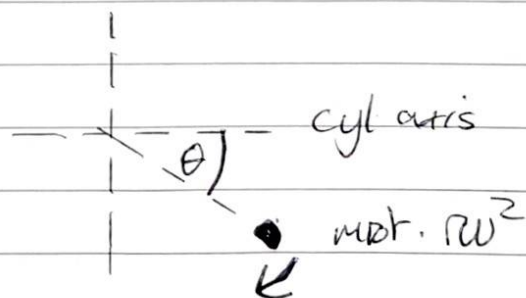
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Question 2

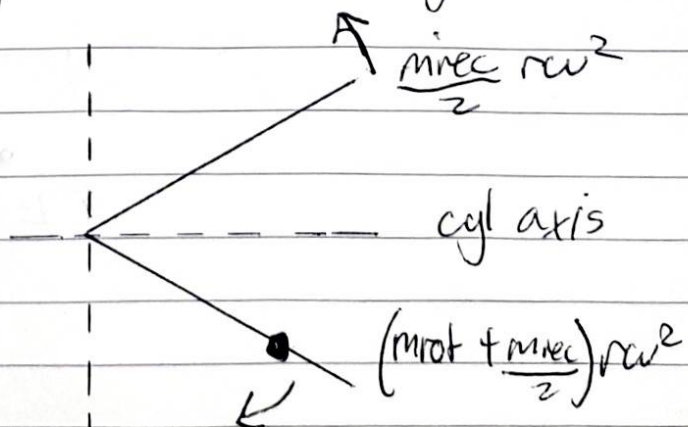
a)



Because of the rotating con-rod with the crankpin, a centrifugal force forward is created



Total first order forces are therefore



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Question 2

rod = 0.35 piston = 0.24

a) Forward - first order - kg·mm

$$\text{Forward} = (m_{\text{rod}} + \frac{m_{\text{rec}}}{2}) r \omega^2$$

$$m_{\text{rod}} = \frac{2}{3} \text{ rod mass} = 0.23 \text{ kg}$$

$$m_{\text{rec}} = \text{Piston} + \frac{1}{3} \text{ rod} = 0.35 \text{ kg}$$

$$\text{FMM} = \cancel{18.45} \text{ kg·mm} \quad \text{single} = 18.675 \text{ kg·mm}$$

$$\text{MM total} = \cancel{36.9} \text{ kg·mm} \quad \text{both} = \underline{37.35} \text{ kg·mm}$$

b)

Balance 100% forward moments on crankshaft

$$37.35 = m_{\text{crank}} \cdot x$$

$$\frac{37.35}{5} = x$$

$$x = 7.47 \text{ mm}$$



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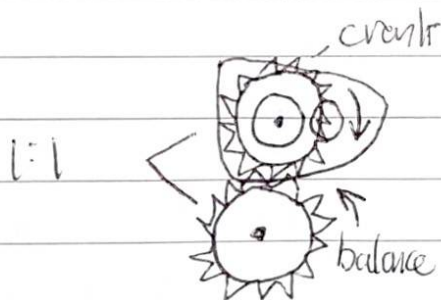
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Question 1

c) Reverse mass moment = $\frac{m_{rec}}{2} r \omega^2$

$$T_H = \frac{m_{rec}}{2} r + \frac{m_{rec}}{2} r = m_{rec} \cdot r = 15.75 \text{ kg mm}$$

d)



drive the balance with a 1:1 ratio gear to maintain ~~sp~~ same speed in opposite direction (counter-rotating)

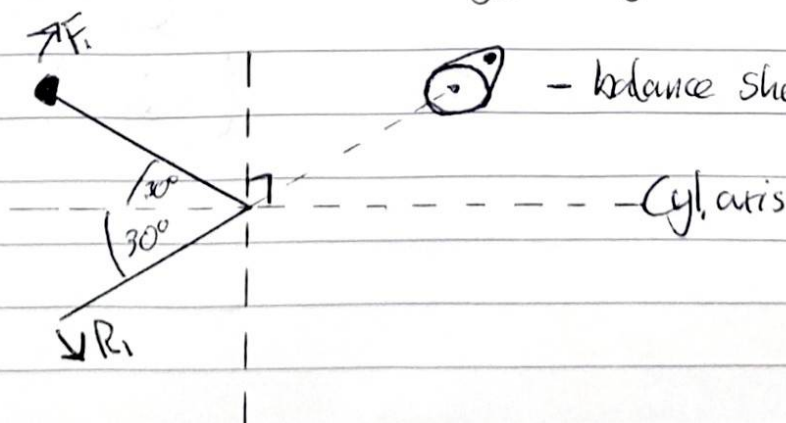
e)

$$15.75 = 2.5 \cdot y$$

$$\frac{15.75}{2.5} = y$$

$$y = 6.3 \text{ mm offset of C.O.G}$$

f)



- balance shift opposite reverse vector

- Cyl. axis

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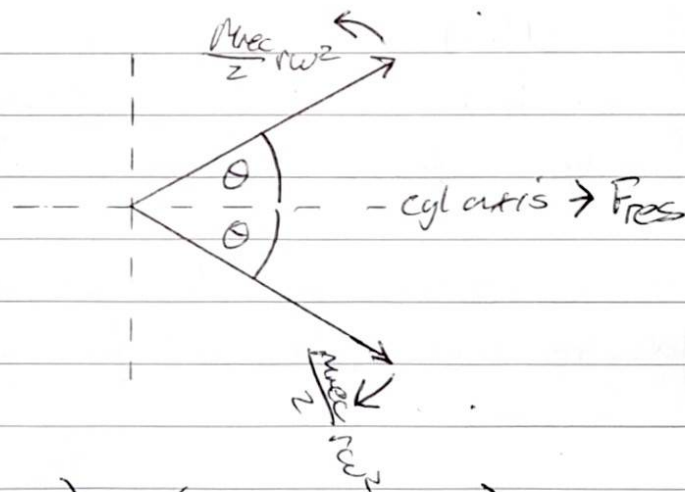
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Question 1

- g) If a balance shaft cannot be fitted, then one option is to under balance the crankshaft to retain a horizontal acting (cylinder axis) vibrations.

$$M_{\text{crank}} \cdot x \cdot \omega^2 = m_{\text{rod}} \cdot r \cdot \omega^2$$



$$F_{\text{res}} = \left(\frac{m_{\text{rod}}}{2} r \omega^2 \cos \theta \right) + \left(\frac{m_{\text{rod}}}{2} \cdot r \omega^2 \cos \theta \right)$$
$$= m_{\text{rod}} \cdot r \omega^2 \cos \theta$$

$$M_{\text{crank}} \cdot x = m_{\text{rod}} \cdot r$$

$$M_{\text{crank}} = 0.24 \cdot 45 = \underline{10.8 \text{ Kg} \cdot \text{mm}}$$

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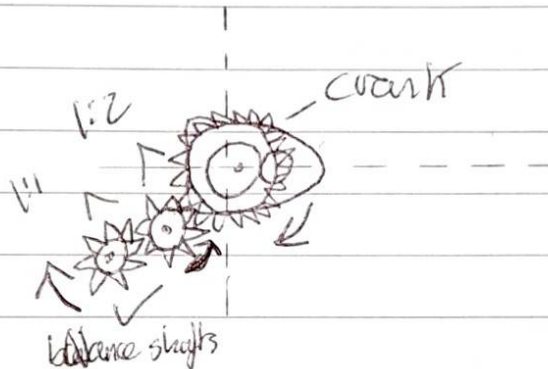
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Question 1

h)



gear driven with a ratio of 1:2 and then 1:1
so that the balance shafts rotate at twice
engine speed (2ω).

counter rotating for reverse second order and
forward rotating (clockwise) for forward second order.

$$\text{Second order forces} = \frac{1}{2} \frac{1}{L} m_{rec} \cdot r \omega^2 \cos(2\theta)$$

$$\omega^2 \cdot 2.53 = m_{bal} \cdot y \cdot (2\omega)^2$$

$$\omega^2 \cdot 2.53 = 4 \cdot m_{bal} \cdot y \cdot \omega^2$$

$$2.53 = 4 \cdot m_{bal} \cdot y$$

$$\frac{2.53}{4} = M_{balance} | \text{single} | \quad \text{MM}_{balance}$$

$$\text{For both pins} = \frac{1}{L} m_{rec} \cdot r \omega^2$$

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Question 1

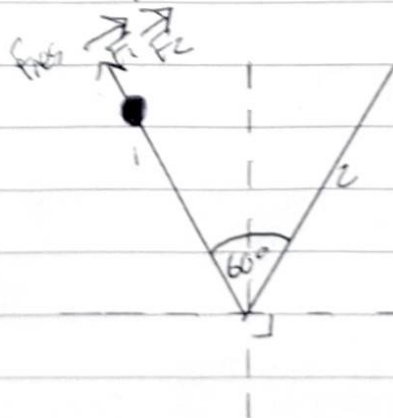
h) $5.0625 = 4 \cdot m_{bal} \cdot y$

$$\frac{5.0625}{4} = M_{balance\ shaft}$$

balance shaft mass moment = 1.27 kg mm

Question 2

- a) Piston mass = 0.24 kg rod mass = 0.36 kg
stroke = 80 mm throw = 40 mm (r)
rod length = 140 mm



$$\text{First order force} = 2 \left(m_{rot} + \frac{m_{rec}}{2} \right) r \omega^2$$

$$m_{rot} = 0.24 \text{ kg}$$

$$m_{rec} = 0.36 \text{ kg}$$

$$F_{res} = 33.6 \text{ kg mm}$$

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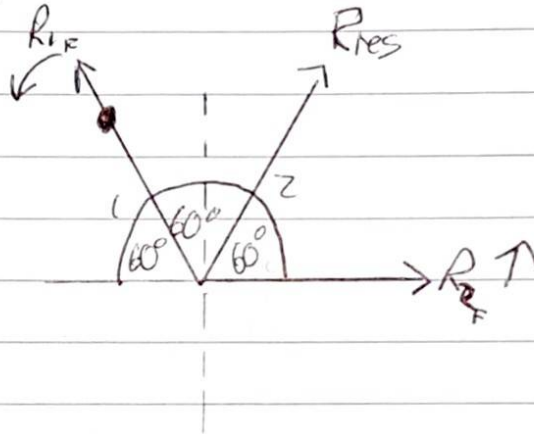
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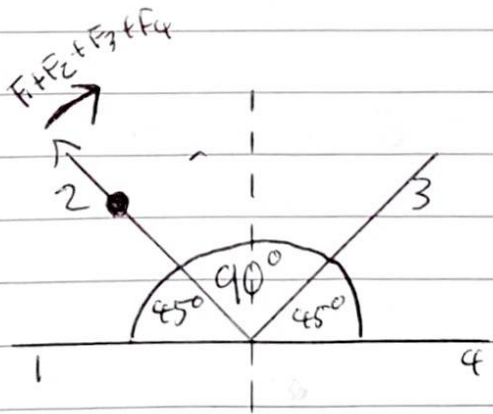
Question (2)

b)



$$\begin{aligned}\text{Reverse result} &= \frac{m_{rec}}{2} r \cos 60 + \frac{m_{rec}}{2} r \cos 60 \\ &= 7.2 \text{ kg mm}\end{aligned}$$

c)



$$\begin{aligned}F_{res} &= 4 \left(m_{rod} + \frac{m_{rec}}{2} \right) r \\ &= \underline{\underline{67.2 \text{ kg mm}}}\end{aligned}$$

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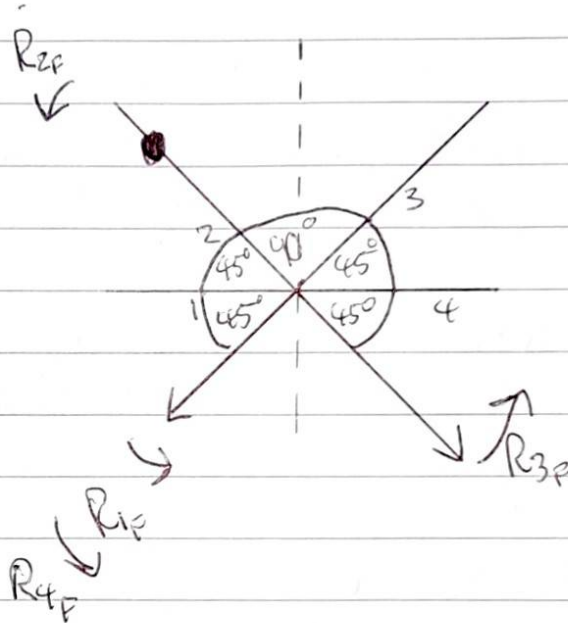
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Question 2

d)



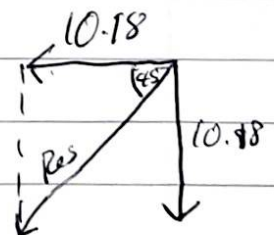
$$\text{Verticle } \uparrow = \frac{m_{rec}}{2} r \cos 45 = R_2$$

$$\text{Verticle } \downarrow = 3\left(\frac{m_{rec}}{2} r \cos(45)\right) = R_1 + R_4 + R_3$$

$$\begin{aligned} \text{Verticle res} &= R_1 + R_4 + R_3 - R_2 = 2R \\ &= 2\left(\frac{m_{rec}}{2} r \cos 45\right) = 10.18 \text{ kgmm} \end{aligned}$$

$$\begin{aligned} \text{Horizontal} &= 3\left(\frac{m_{rec}}{2} r \cos 45\right) = 3R - \text{left} \\ \text{Right} &= \left(\frac{m_{rec}}{2} r \cos 45\right) = 1R \end{aligned}$$

$$\begin{aligned} \text{Horizontal res} &= 3R - 1R = 2R \\ &= 10.18 \text{ kg} \cdot \text{mm} \end{aligned}$$



Resultant force = 14.4 kgmm at 45° below horizontal

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Question 2

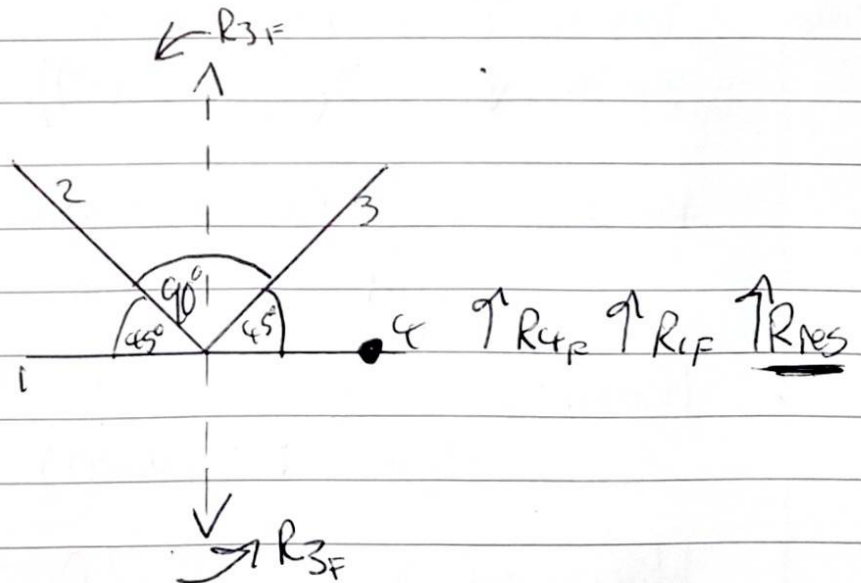
- e) First order forward would be balanced on the crankshaft by offsetting the centre of gravity creating the same mass moment

Reverse first order will be balanced with a counter rotating shaft with ~~an~~ an offset Centre of gravity same magnitude of mass moment

$$MM_{\text{crank}} = 67.2 \text{ kg} \cdot \text{mm}$$

$$MM_{\text{balance}} = 14.4 \text{ kg} \cdot \text{mm}$$

2)



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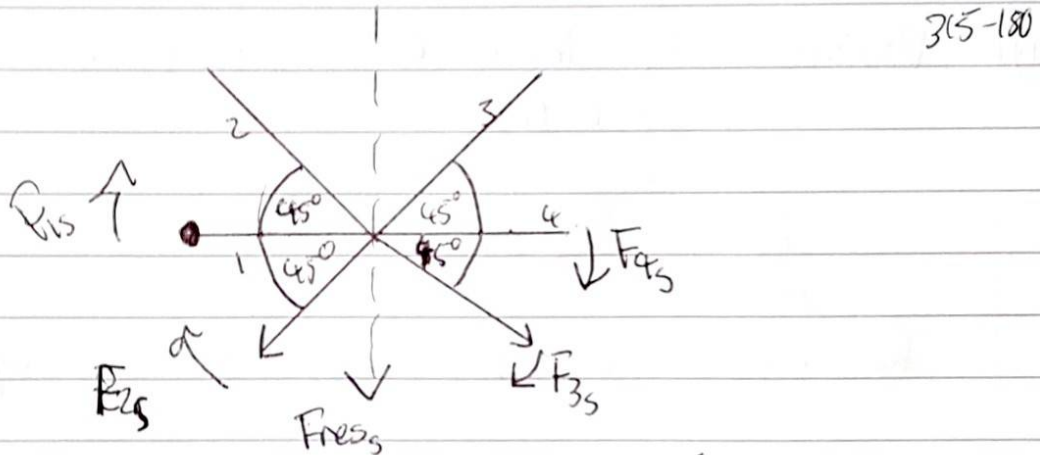
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Question 2

g)



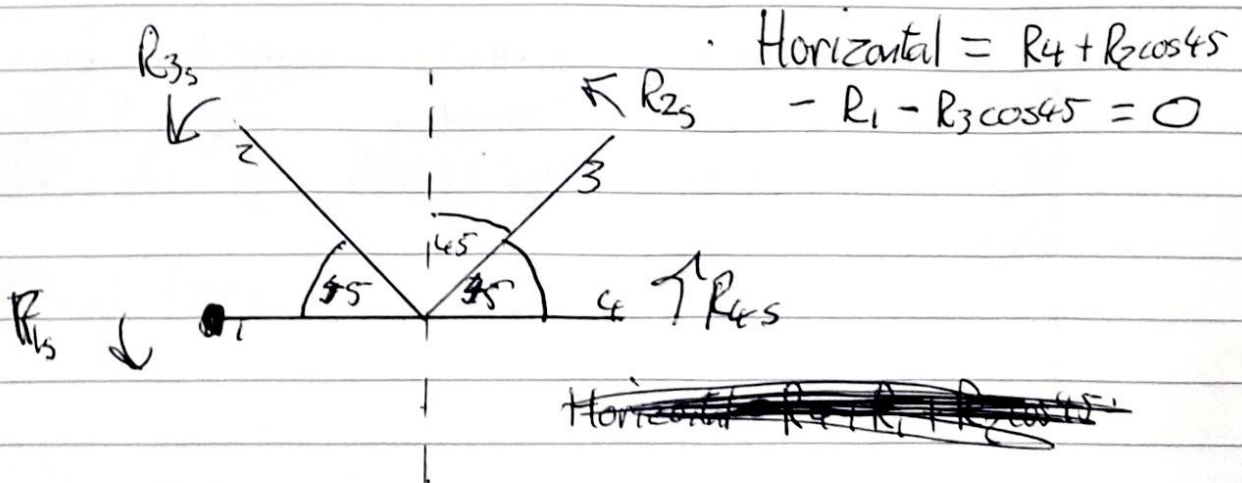
$$\text{Horizontal} = F_1 + F_2 \cos 45^\circ - F_4 - F_3 \cos 45^\circ = 0$$

$$\text{Vertical} = F_2 \cos 45^\circ + F_3 \cos 45^\circ = 1.41F$$

$$\text{Second order} = \frac{1}{2} I \omega^2 = F$$

$$= 2.9 \text{ kg} \cdot \text{mm} \text{ vertically down}$$

h)



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Question 2

$$\begin{aligned} \text{h) } V_{\text{vertical}} &= R_2 \cos 45 + R_3 \cos 45 \quad \text{q} \\ &= 1.41R \end{aligned}$$

$$R = \frac{1}{2} m r \omega^2$$

$$\text{Resultant} = 2.9 \text{ kg} \cdot \text{mm} \quad \text{vertically up}$$

Second order forces balance out with $2.9 \text{ kg} \cdot \text{mm}$ up and $2.9 \text{ kg} \cdot \text{mm}$ down

i)

The resultant cancels out because they occur at the same amplitude of 2ω and the magnitude is the same at $2.9 \text{ kg} \cdot \text{mm}$.

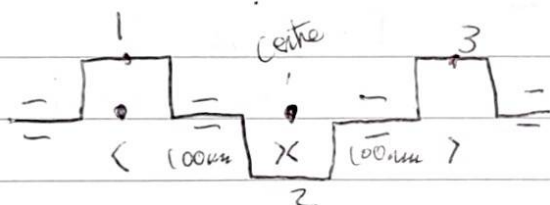
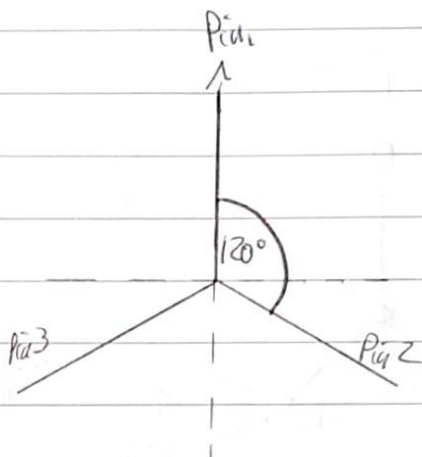
They shouldn't change direction because horizontal will consistently cancel themselves as will vertical because of the 45° cylinder axis.

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Question 4



piston mass = 0.24 kg

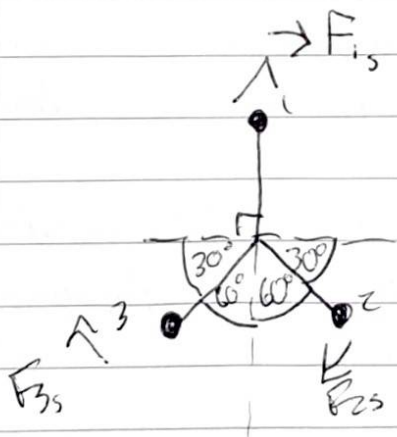
rod mass = 0.3 kg

length = 150 mm

stroke = 90 mm, throw = 45 mm

base = 100 mm

a)



$$Resultant = 100F + 100F \cos 60$$

$$Resultant = 150F$$

$$= 150 \times (2.3)$$

$$= 345 \text{ kg} \cdot \text{mm} \cdot \text{mm}$$

$$115 \text{ kg} \cdot \text{mm} \cdot \text{mm}$$

$$F = \frac{1}{2} \sqrt{I \cdot \omega^2}$$

$$M_{rec} = 0.24 + \frac{1}{3} 0.3 = 0.34 \text{ kg}$$

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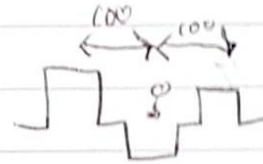
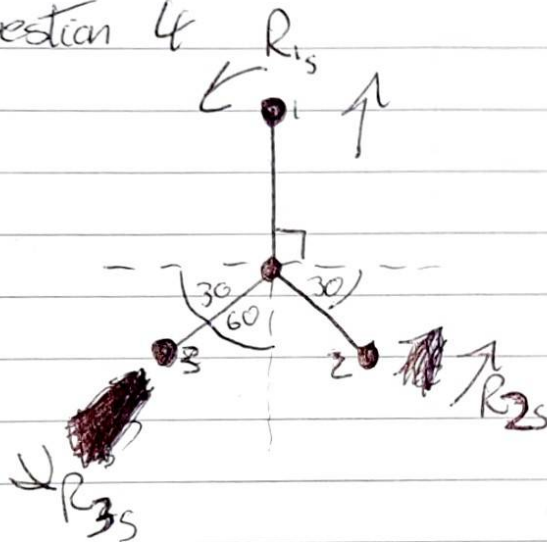
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Question 4

b)



$$R_{res} = 100R + 100R \cos 60$$

$$R_{res} = 50R$$

$$R_{resultant} = \underline{115 \text{ kg mm s}^{-2}}$$

$$R = \frac{1}{2} \frac{1}{\omega} \frac{d\omega}{dt} r \omega^2$$
$$= 2.3 \text{ kg mm s}^{-2}$$

Forward would rotate the crankshaft clockwise, as would the Reverse mass moment meaning the balance would be required to rotate counter clockwise.

This counter rotating mass moment equivalent can be achieved using the pulley and flywheel with an offset centre of gravity.

This is how Ford balance their fox engine.