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12 Pages

1. Please give the information requested in the boxes below.
2. Enter your name in the space provided to the right, fold over and seal down (using the label provided) to hide your name.
3. The answers must be legibly written. The Examiners can either ignore illegible script or have it typed and the expense of typing charged to the candidate.
4. Write your answers in ink.
5. Begin each answer on a fresh page. Do all rough work in the answer book and cross it through.
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1	Student Number	1707513/1
2	Title of Examination Paper	Principles thesis
3	Module Code	GOAC 5001
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5	Examination Venue	hall
6	Seat Number	
7	Number of Answer Books handed in	1

[illegible]

(Gweler yr ochr arall am y fersiwn Cymraeg)

② Question ②

$$f(x) = \begin{cases} -3 & -\pi < x < 0 \\ 3 & 0 \leq x < \pi \end{cases} \quad \text{and} \quad f(x+2\pi) = f(x)$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) dx \quad \checkmark$$

$$a_0 = \frac{1}{2\pi} \left(\int_{-\pi}^0 -3 dx + \int_0^{\pi} 3 dx \right) \quad \checkmark$$

$$a_0 = \frac{1}{2\pi} \left(-3(x)_{-\pi}^0 + 3(x)_0^{\pi} \right) \quad \checkmark$$

$$= -\frac{3}{2\pi} (x)_{-\pi}^0 + \frac{3}{2\pi} (x)_0^{\pi} \quad \checkmark$$

$$= -\frac{3}{2\pi} (\pi) + \frac{3}{2\pi} (\pi)$$

$$= -\frac{3}{2} + \frac{3}{2}$$

$$= 0 \quad \checkmark$$

80

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \quad \checkmark$$

$$= \frac{1}{\pi} \int_{-\pi}^0 -3 \cos nx dx + \frac{1}{\pi} \int_0^{\pi} 3 \cos nx dx \quad \checkmark$$

$$= \frac{1}{\pi} \left(\frac{-3 \sin nx}{n} \right)_{-\pi}^0 + \frac{1}{\pi} \left(\frac{3 \sin nx}{n} \right)_0^{\pi} \quad \checkmark$$

$$= \frac{1}{n\pi} \left(-3 \sin nx \right)_{-\pi}^0 + \frac{1}{n\pi} \left(3 \sin nx \right)_0^{\pi} \quad \checkmark$$

$$= -\frac{3}{n\pi} (\sin 0 - \sin(-\pi)) + \frac{3}{n\pi} (\sin(\pi) - \sin 0) \quad \checkmark$$

$$= 0 \quad \checkmark$$

Q2

20

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \, dx \quad \checkmark$$

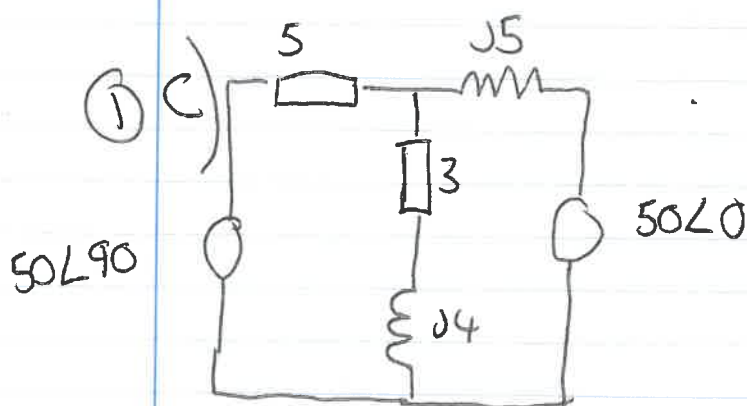
$$b_n = \frac{1}{\pi} \int_{-\pi}^0 -3 \sin nx \, dx + \frac{1}{\pi} \int_0^{\pi} 3 \sin nx \, dx \quad \checkmark$$

$$b_n = \frac{1}{\pi} \left(-3 \left(\frac{-\cos nx}{n} \right) \right)_{-\pi}^0 + \frac{1}{\pi} \left(3 \left(\frac{-\cos nx}{n} \right) \right)_{0}^{\pi} \quad \checkmark$$

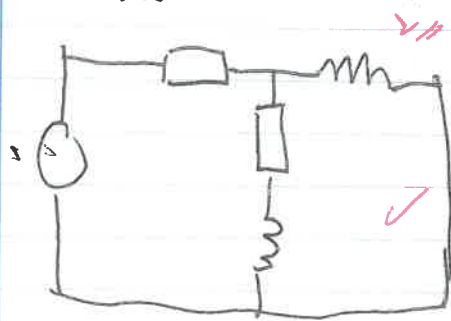
$$b_n = -\frac{3}{\pi} \left(\frac{-\cos 0}{n} - \frac{-\cos(-\pi)}{n} \right) + \frac{3}{\pi} \left(\frac{-\cos \pi}{n} - \frac{-\cos 0}{n} \right) \quad \checkmark$$

~~$b_n = 0$ when even~~

$b_n = 0$ when even



Re draw



$$Z_T = \frac{5(3+j4)}{5+3+j4} + 5j \quad \times$$

$$Z_T = \frac{15+j20}{8+j4} + 5j \quad \times$$

convert to polar

$$= \frac{25 \angle 53.13}{8.44 \angle 26.57} = 2.97 \angle 26.56$$

conv to rec

$$= (5.81 + j2.9) + j5$$

$$Z_T = 5.81 + j7.9$$

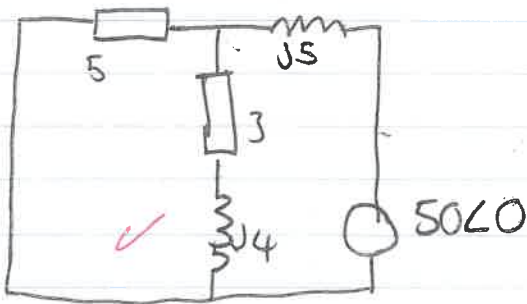
$$I = \frac{V}{Z_T} = \frac{50 \angle 90^\circ}{5.81 + j7.9} = \frac{50 \angle 90^\circ}{11.83 \angle 41.88^\circ}$$

error carried forward

$$I = 4.23 \angle 48.12^\circ$$

$$I_1 = \frac{5}{5 + 3 + j4} \times 4.23 \angle 48.12^\circ$$

$$I_1 = \frac{5 \angle 0^\circ}{8.944 \angle 26.57^\circ} = (0.56 \angle -26.57^\circ)(4.23 \angle 48.12^\circ) = 2.37 \angle 21.55^\circ \quad \text{Rec} = 2.2 + j0.87$$



$$Z_T = \frac{j5(3 + j4)}{j5 + j4 + 3} + 5 = \frac{j15 - 20}{3 + j4} + 5$$

conv to pol

$$\frac{25 \angle 143.13^\circ}{4.45 \angle 71.56^\circ} = 2.65 \angle 71.57^\circ + 5$$

convert to Rec

$$= 0.84 + j2.51 + 5$$

$$Z_T = 5.84 + j2.51$$

$$I = \frac{V}{Z} = \frac{50 \angle 0^\circ}{6.36 \angle 23.25^\circ} = 7.86 \angle -23.25^\circ$$

error carried forward

$$I_2 = \frac{j5}{j5 + 3 + j4} \times 7.86 \angle -23.25^\circ$$

$$= \frac{5 \angle 90^\circ}{4.45 \angle 71.56^\circ} = 0.53 \angle 18.44^\circ \times 7.86 \angle -23.25^\circ = 4.17 \angle -4.81^\circ$$

Rec

$$= 4.16 - j0.35$$

$$\begin{aligned} I_1 + I_2 &= (2.204 + j6.87) + (4.16 - j0.35) \\ &= 6.364 + j6.52 \end{aligned}$$

Q3 ③ a) $1 + 2t - \frac{1}{3}t^4$

i) $1 + 2t - \frac{1}{3}t^4$

$$\frac{1}{s} + \frac{2}{s^2} - \frac{1}{3} \frac{4!}{s^5} \quad \checkmark \quad 2$$

~~$$\frac{3!}{s^4} - \frac{1}{3} \frac{4!}{s^5}$$~~

ii) $5e^{2t} - 3e^{-t}$

$$= 5 \left(\frac{1}{s+2} \right) - 3 \left(\frac{1}{s} \right) = 2(2) = 4 \quad \times$$

iii) $6\sin(3t) - 4\cos(5t)$

$$= 6 \left(\frac{3}{s^2+3^2} \right) - 4 \left(\frac{s}{s^2+5^2} \right) \quad 6 \left(\frac{1}{3} \right) - 4 \left(\frac{s}{s^2} \right)$$

$$= 2 - 4 \left(\frac{s}{s^2} \right) \quad \times$$

iv) $e^{-2t} \sin(4t)$

$$\left(\frac{1}{s+2} \right) \times \left(\frac{4}{s^2+4^2} \right) = s^2 + 4^2 + 4s + 8 \quad \times$$

v) $\cos(2t) + 3e^{-4t}$

$$\frac{s}{s^2+2^2} + 3 \left(\frac{1}{s+4} \right) \quad \checkmark \quad 2$$

b) $\frac{dy}{dt} + 2y = 12 \quad y:0 \quad t:0$

~~$L\left[\frac{df(t)}{dt}\right] = sf(s) - f(0)$~~

$sY(s) - y(0) + 2Y(s) = \frac{12}{s} \quad \checkmark$

$s^2Y(s) + 2sY(s) = 12$

$Y(s)(s^2 + 2s) = 12 \quad \times$

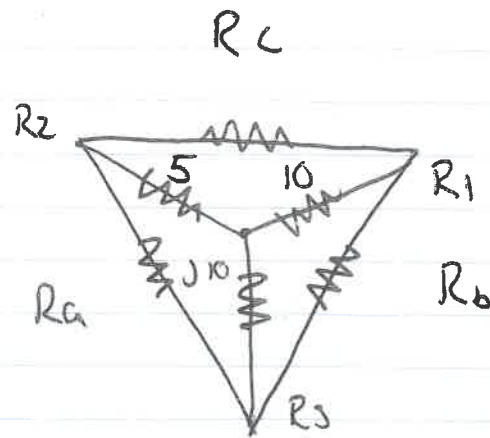
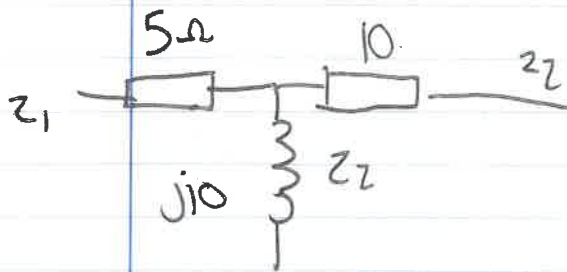
$Y(s) = \frac{12}{s(s+2)} \quad \times$

Q3

b

L

① a) Star to delta



~~Ra = R1 + R2 + R3~~

$$R_a = \frac{(5 \times 10) + (10 \times j10) + (j10 \times 5)}{10} = \frac{15 + 100j + j50}{10}$$

$$= \frac{15 + j150}{10} = 1.5 + j15$$

~~$\frac{150 - j1500}{-100} = -1.5 - j15$~~

~~$R_b = 15 + j100 + j50$~~

$$R_b = \frac{15 + j150}{5} = 3 + j30$$

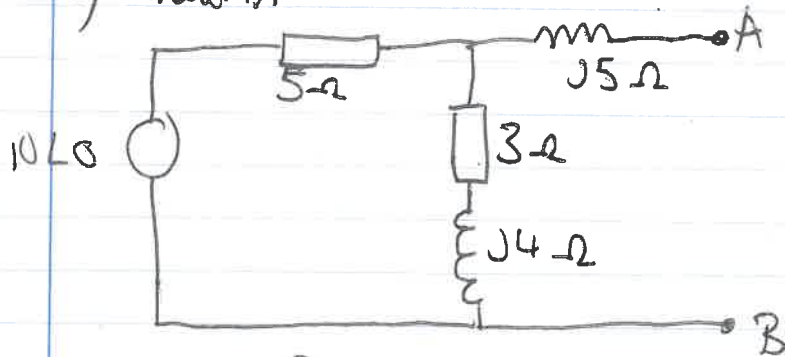
error carried forward

$$R_c = \frac{15 + j100 + j50}{j10} = \frac{15 + j150}{j10} \times \frac{-j10}{-j10} = \frac{-150j - 1500j^2}{-100j^2}$$

$$= \frac{150j + 1500}{100} = 1.5j + 15$$

$$= 15 + j1.5$$

b) Thevenin



$$V_{TH} = \frac{R_1}{R_1 + R_2} \times V_s$$

$$V_{TH} = \frac{5}{5 + (3 + j4)} \times 10\angle 0$$

$$= \frac{5\angle 0}{8.94\angle 26.57} \times 10\angle 0 = 0.56\angle -26.57 \times 10\angle 0 = 5.6\angle -26.57$$

$$R_{TH} = \frac{5(3 + j4)}{5 + 3 + j4} + j5 = \frac{15 + j20}{8 + j4} + j5$$

convert to pol

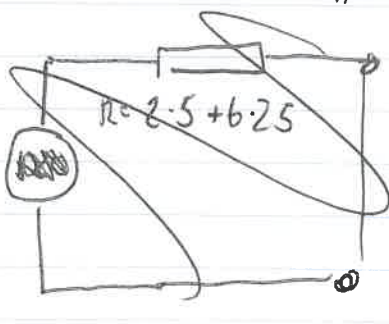
$$\frac{25\angle 53.13}{8.94\angle 26.57}$$

$$= 2.8\angle 26.56$$

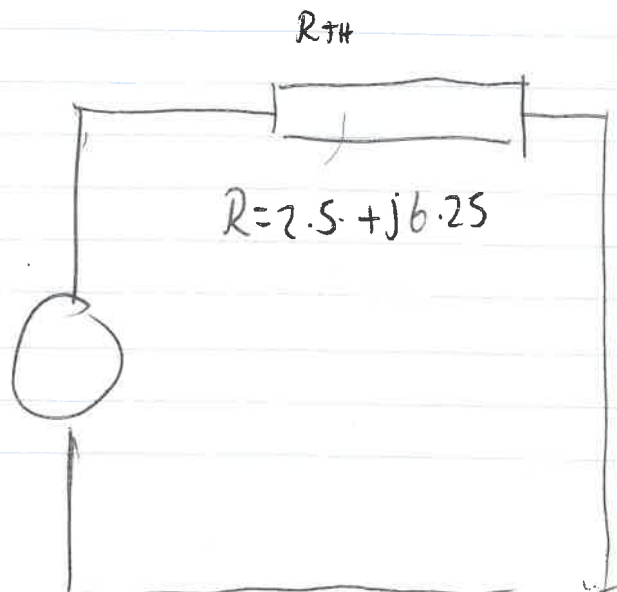
conv to REC

$$= 2.5 + j1.25 + j5$$

$$R_{TH} = 2.5 + j6.25\Omega$$



$$V = 5\angle -26.57$$



Q1
D
5

(4) a) $v(t) = 30 + 20 \cos(180\pi t + 45) + 10 \cos(120\pi t - 45)$
 $i(t) = 6 + 4 \cos(180\pi t + 10) - 2 \cos(120\pi t - 60)$

$$P_{AV} = \frac{(30)(6)}{2} \cos(10 - 45) + \frac{(10)(2)}{2} \cos(60 - 45)$$

$$= 73.72 + 9.67$$

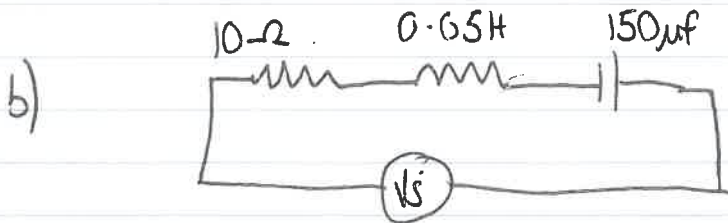
$$P_{AV} = 83.37W$$

$$V_{rms} = \sqrt{\frac{30^2}{2} + \frac{10^2}{2}} = 15.8111 \dots$$

$$= 15.81V$$

$$I_{rms} = \sqrt{\frac{6^2}{2} + \frac{2^2}{2}} = 3.162 \dots$$

$$= 3.16A$$



$$X_C = \frac{1}{j2\pi fC} = \frac{1}{j2\pi \times 15 \times 10^{-6}} = -j9.425$$

(put j in at end)

$$= 0.1061 \dots$$

$$= j0.11$$

$$X_L = j2\pi fL = j2\pi \times 50 \times 0.05$$

$$= 15.707 \dots$$

$$= j15.71$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{10^2 + (15.71 - 0.11)^2}$$

$$= 10.7619 \dots$$

$$Z = jR + X_L + X_C$$

$$= j10.76$$

Supply current $= \frac{V}{Z}$

$$= \frac{100}{10.76} = 9.2919 \dots$$

$$= 9.29A$$

Q4
a1
011

Q4
b1
311

b) $\frac{100 \angle 0}{10 \angle 57.57} = 10 \angle -57.57$ ~~$= 9.2436 \angle -57$~~ $= 4.20 \angle -8.3$
 Rec
 $10.87 \angle 8.3^\circ$ ~~$10.87 \angle 57.57$~~ Supply $= 9.10 - j1.33 \text{ A}$

$Z = R + X_L + X_C = 28.82$
 $\tan^{-1}(10.76)$
 $= 84.69^\circ \text{ phase angle}$

$P = I^2 R$

$P = (9.10 - j1.33)^2 \times j10.76$

~~$= 82.81 - 1.7689j$~~

~~$= 82.81 + 1.7689j$~~

~~$= 82.81 - 1.7689j$~~

~~$= 82.81 + 1.7689j$~~

~~$= 84.578...$~~

$= 84.58 \text{ W}$