

Stress Analysis

2003580
ABC DEFG

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Q1 $\epsilon_x = 20 \times 10^{-6}$ $\epsilon_y = -18 \times 10^{-6}$
 $\nu = 0.28$ $E = 170 \times 10^9$

i) $\sigma_x = \frac{E}{1-\nu^2} \times (\epsilon_x + \nu \epsilon_y)$

$$\sigma_x = \frac{170 \times 10^9}{1-0.28^2} \times (20 \times 10^{-6} + (0.28 \times -18 \times 10^{-6}))$$

$$= 1.84 \times 10^{11} \times 1.5 \times 10^{-5}$$

$$\sigma_x = 2.76 \times 10^6 \text{ Pa (tensile)}$$

$\sigma_y = \frac{E}{1-\nu^2} \times (\epsilon_y + \nu \epsilon_x)$

$$\sigma_y = \frac{170 \times 10^9}{1-0.28^2} \times (-18 \times 10^{-6} + (0.28 \times 20 \times 10^{-6}))$$

$$= 1.84 \times 10^{11} \times -12.4 \times 10^{-6}$$

$$\sigma_y = -2.28 \times 10^6 \text{ Pa (compressive)}$$

ii) $\sigma_x = 88 \times 10^6$ $\sigma_y = -40 \times 10^6$

$\epsilon_x = \frac{1}{E} \times (\sigma_x - \nu \sigma_y)$

$$\epsilon_x = \frac{1}{170 \times 10^9} \times (88 \times 10^6 - (0.28 \times -40 \times 10^6))$$

$$= 5.88 \times 10^{-12} \times 99.2 \times 10^6$$

$$\epsilon_x = 583.3 \times 10^{-6}$$

$\epsilon_y = \frac{1}{E} \times (\sigma_y - \nu \sigma_x)$

$$\epsilon_y = \frac{1}{170 \times 10^9} \times (-40 \times 10^6 - (0.28 \times 88 \times 10^6))$$

$$= 5.88 \times 10^{-12} \times -64.64 \times 10^6$$

$$\epsilon_y = -380.1 \times 10^{-6}$$

Q2. $\sigma_x = 105 \times 10^6 \text{ Pa}$ $\sigma_y = 208 \times 10^6 \text{ Pa}$
 $\tau_{xy} = 125 \times 10^6 \text{ Pa}$

i $\sigma_1 = \frac{(\sigma_x + \sigma_y)}{2} + \sqrt{\frac{(\sigma_x - \sigma_y)^2}{4} + \tau_{xy}^2}$

$\sigma_1 = \frac{(105 \times 10^6 + 208 \times 10^6)}{2} + \sqrt{\frac{(105 \times 10^6 - 208 \times 10^6)^2}{4} + 125 \times 10^6^2}$

$\sigma_1 = 156.5 \times 10^6 + 182 \times 10^6 = 191.19 \times 10^6$

$\sigma_1 = 347.69 \times 10^6 \text{ Pa}$

$\sigma_2 = \frac{(\sigma_x + \sigma_y)}{2} - \sqrt{\frac{(\sigma_x - \sigma_y)^2}{4} + \tau_{xy}^2}$

$\sigma_2 = 156.5 \times 10^6 - 191.19 \times 10^6$

$\sigma_2 = -34.69 \times 10^6 \text{ Pa}$

ii $\tau_{\max} = \sqrt{\frac{(\sigma_x - \sigma_y)^2}{4} + \tau_{xy}^2}$

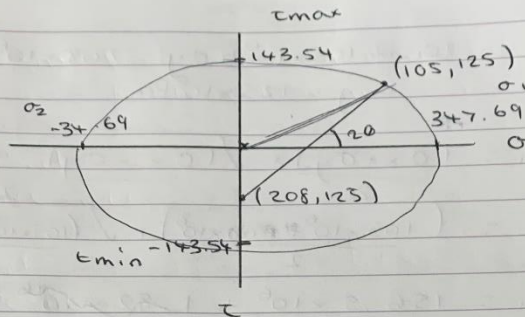
$\tau_{\max} = \sqrt{\frac{(105 \times 10^6 - 208 \times 10^6)^2}{4} + 125 \times 10^6^2}$

$\tau_{\max} = 143.54 \times 10^6 \text{ Pa}$

$\tau_{\min} = -\sqrt{\frac{(\sigma_x - \sigma_y)^2}{4} + \tau_{xy}^2}$

$\tau_{\min} = -\sqrt{\frac{(105 \times 10^6 - 208 \times 10^6)^2}{4} + 125 \times 10^6^2}$

$\tau_{\min} = -143.54 \times 10^6 \text{ Pa}$



Q3. $R_A = 1969.4 \text{ N}$ $R_B = 2580.6 \text{ N}$

i. calculating moments

$\sum A_3 + R_2 =$

$$m = m_1(x) + P_1(x - a_1) + P_2(x - a_2) + P_3(x - a_3)$$

$\sum A_3 + R_2 =$

ii $EI \frac{d^2y}{dx^2} = 1969.4[x]^2 - 450[x - 0.5]^2 + 450[x - 1.5]^2 - 2300[x - 1.5]^2 - 800[x - 1.5]^2 + 800[x - 2.5]^2 - 1000[x - 2.5]^2 + A$

$EI \frac{dy}{dx} = \frac{1969.4 \cdot 2}{6} x^3 - \frac{450 \cdot 2}{6} [x - 0.5]^3 + \frac{450 \cdot 2}{6} [x - 1.5]^3$

$$- \frac{2300 \cdot 2}{6} [x - 1.5]^3 - \frac{800 \cdot 2}{6} [x - 1.5]^3 + \frac{800 \cdot 2}{6} [x - 2.5]^3 - \frac{1000 \cdot 2}{6} [x - 2.5]^3 + Ax$$

$$\text{iii} \quad EIy = \frac{1969.4x^3}{6} - \frac{450[x-0.5]^4}{24} + \frac{450[x-1.5]^4}{24} - \frac{2300[x-1.5]^4}{24} - \frac{800[x-1.5]^4}{24} + \frac{800[x-2.5]^4}{24} - \frac{1000[x-2.5]^4}{24} + Ax + B$$

$$\text{iv} \quad @ x = 0 \quad y = 0 \\ B = 0 - 0 + 0 - 0 - 0 + 0 - 0 \\ B = 0$$

$$@ x = 3.1 \quad y = 0 \quad \text{sub in } EIy$$

$$EIy = \frac{1969.4[3.1]^3}{6} - \frac{450[3.1-0.5]^4}{24} + \frac{450[3.1-1.5]^4}{24} - \frac{2300[3.1-1.5]^4}{24} - \frac{800[3.1-1.5]^4}{24} + \frac{800[3.1-2.5]^4}{24} - \frac{1000[3.1-2.5]^4}{24} + A(3.1) + B$$

$$= 9778.4 - 856.83 + 122.88 - 628.1 - 218.5 + 12.42 - 5.4 + 3.1A \\ = 8204.87 + 3.1A \\ - 3.1A = 8204.87$$

$$A = \frac{8204.87}{-3.1} = -2646.73$$

$$\text{EI}y = 8204.87 - 2646.73 \\ = 5558.14$$

$$y = \frac{EIy}{EI} = \frac{5558.14}{EI}$$

✓ @ centre $x = 1.58 \times 10^3 \text{ mm}$

$$EIy = \frac{1969.4 [1.58]^3}{6} - \frac{450 [1.58 - 0.5]^4}{24} + \frac{450 [1.58 - 1.5]^4}{24} \\ - \frac{2300 [1.58 - 0.5]^4}{24} - \frac{800 [1.58 - 1.5]^4}{24} \\ + \frac{800 [1.58 - 2.5]^4}{24} - \frac{1000 [1.58 - 2.5]^4}{24} + A(1.58) + B$$

$$EIy = 1294.65 - 25.51 + 7.68 \times 10^{-4} - 3.93 \times 10^3 \\ - 1.37 \times 10^3 + 23.9 - 29.8 + 1.5A$$

$$EIy = 1263.23 + 1.5A \\ 1263.23 + 1.5 = -2646.73 \\ = 3911.46$$

$$y = \frac{EIy}{EI} = \frac{3911.46}{1.58 \times 10^6} = 2.48 \times 10^{-3}$$

Q4 $d = 0.28\text{m}$ $L = 0.6\text{m}$ $nL = 0.58$
 $n_c = 0.60$ $P = 14.5 \times 10^5 \text{N/m}^2$ $\sigma_{\text{max}} = 150 \times 10^6 \text{N/m}^2$

i $\sigma_c = \frac{P \times d}{2 \times t \times nL}$

$t = \frac{P \times d}{2 \times \sigma_c \times nL}$

$= \frac{14.5 \times 10^5 \times 0.28}{2 \times 150 \times 10^6 \times 0.58}$

$t = 2.33 \times 10^{-3} \text{m}$

ii $\sigma_c = \frac{P \times d}{2 \times t \times n_c}$

$t = \frac{P \times d}{2 \times \sigma_c \times n_c}$

$= \frac{14.5 \times 10^5 \times 0.28}{2 \times 150 \times 10^6 \times 0.6}$

$t = 2.26 \times 10^{-3} \text{m}$ 1.13×10^{-3}

iii $t_{\text{min}} = 2.33 \times 10^{-3} \text{m}$

Q5 $L = 1.38 \text{ m}$ $d_o = 0.01905$
 $E = 200 \times 10^9 \text{ Pa}$ $d_i = 0.01905 - 1.2 \times 10^{-3} - 1.2 \times 10^{-3}$
 $= 0.01665$

i $A = \frac{\pi d^2}{4} = \frac{\pi \times 0.01905^2}{4}$
 $A = 2.85 \times 10^{-4} \text{ m}^2$

ii $A = \frac{\pi d^2}{4} = 6.46 \times 10^{-9}$

iii $A = \frac{\pi}{4} (d_o^2 - d_i^2) = 6.73 \times 10^{-5} \text{ m}^2$

$A = \frac{\pi}{4} \times (d_o^2 - d_i^2)$
 $A = \frac{\pi}{4} \times (0.01905^2 - 0.01665^2) = 6.73 \times 10^{-5} \text{ m}^2$

ii $n = 0.25$

iii $F_{cr} = \frac{n \times \pi^2 \times E \times I}{L^2}$

$I = \frac{\pi}{64} (d_o^4 - d_i^4)$

$= \frac{\pi}{64} \times (0.01905^4 - 0.01665^4) = 2.69 \times 10^{-9} \text{ m}^4$

$F_{cr} = \frac{0.25 \times \pi^2 \times 200 \times 10^9 \times 2.69 \times 10^{-9}}{1.38^2}$

$F_{cr} = 2088.3 \text{ N}$ 697.05 N

$$iv \quad k = \frac{1}{A} = \frac{2.69 \times 10^{-9}}{6.73 \times 10^{-5}} = 3.4 \times 10^{-5}$$

$$SR = \frac{L}{K} = \frac{1.38}{3.4 \times 10^{-5}} = 34500$$