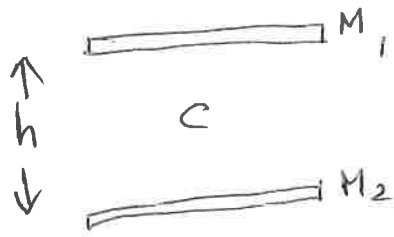


Gurney equation for asymmetric sandwich:

(1)



M_1 mass/unit area
 M_2 mass/unit area
 c mass/unit area of H.E.
 $\therefore \rho = \text{mass/unit vol} = \frac{c}{h}$

Explosive products impart
 and $-v_2$ to M_2 .
 explosive combustion products
 and $-v_2$

velocity v_1 to M_1
 Assume velocity of
 varies linearly between v_1

$$v(y) = \frac{(v_1 + v_2)y}{h} - v_2$$

Apply Conservation of Momentum:

$$0 = M_1 v_1 - M_2 v_2 + \int_0^h \rho v(y) dy$$

$$0 = M_1 v_1 - M_2 v_2 + \frac{c}{h} \int_0^h \left[\frac{(v_1 + v_2)y}{h} - v_2 \right] dy$$

$$0 = M_1 v_1 - M_2 v_2 + \frac{c}{h} \left[\frac{(v_1 + v_2)y^2}{2h} - v_2 y \right]_0^h$$

$$0 = M_1 v_1 - M_2 v_2 + \frac{c}{h} \left[\frac{(v_1 + v_2)h^2}{2h} - v_2 h \right]$$

$$0 = M_1 v_1 - M_2 v_2 + c \left[\frac{(v_1 + v_2)}{2} - v_2 \right]$$

$$0 = M_1 v_1 - M_2 v_2 + c \left[\frac{v_1 + v_2 - 2v_2}{2} \right]$$

$$0 = 2M_1 v_1 - 2M_2 v_2 + c [v_1 - v_2] \quad (2)$$

collect v_2 terms on LHS

$$v_2 (2M_2 + c) = ~~2M_1 v_1~~ v_1 (2M_1 + c)$$

$$v_2 = v_1 \frac{(2M_1 + c)}{(2M_2 + c)}$$

Divide top & bottom of RHS by c

$$v_2 = v_1 \frac{\left(\frac{2M_1}{c} + 1\right)}{\left(\frac{2M_2}{c} + 1\right)}$$

$$v_2 = A v_1 \quad \text{--- (1)}$$

$$\text{where } A = \frac{\left(\frac{2M_1}{c} + 1\right)}{\left(\frac{2M_2}{c} + 1\right)}$$

Now Conservation of Energy

(3)

Let $E = \text{energy/unit mass}$

$\therefore EC = \text{energy/unit area}$

$$EC = \frac{1}{2} M_1 v_1^2 + \frac{1}{2} M_2 v_2^2 + \frac{1}{2} \int_0^h \rho v(y)^2 dy$$

Let's consider the integral in detail then come back to this equation for energy balance

$$\begin{aligned} & \frac{1}{2} \frac{\rho}{h} \int_0^h \left[\frac{(v_1 + v_2)y}{h} - v_2 \right]^2 dy \\ &= \frac{1}{2} \frac{\rho}{h} \int_0^h \left[\frac{(v_1 + v_2)^2 y^2}{h^2} + v_2^2 - \frac{2v_2(v_1 + v_2)y}{h} \right] dy \\ &= \frac{1}{2} \frac{\rho}{h} \left[\frac{(v_1 + v_2)^2 h^3}{3h^2} + v_2^2 h - \frac{2v_2(v_1 + v_2)h^2}{2h} \right] \\ &= \frac{1}{2} \rho \left[\frac{(v_1 + v_2)^2}{3} + 3v_2^2 - 3v_2(v_1 + v_2) \right] \\ &= \frac{\rho}{2} \left[\frac{v_1^2 + v_2^2 + 2v_1v_2}{3} + 3v_2^2 - 3v_1v_2 - 3v_2^2 \right] \\ &= \frac{\rho}{2} \left[\frac{v_1^2 + v_2^2 - v_1v_2}{3} \right] \end{aligned}$$

Now put this back into energy equation

$$EC = \frac{1}{2} M_1 v_1^2 + \frac{1}{2} M_2 v_2^2 + \frac{c}{2} \left[\frac{v_1^2 + v_2^2 - v_1 v_2}{3} \right] \quad (4)$$

But remember that $v_2 = A v_1$ from (1)

$$\therefore EC = \frac{1}{2} M_1 v_1^2 + \frac{1}{2} M_2 A^2 v_1^2 + \frac{c}{2} \left[\frac{v_1^2 + A^2 v_1^2 - A v_1^2}{3} \right]$$

Multiply both sides by 2:

$$2EC = v_1^2 \left[M_1 + A^2 M_2 + \frac{c}{3} (1 - A + A^2) \right]$$

i.e.,

$$v_1^2 = \frac{2EC}{M_1 + A^2 M_2 + \frac{c}{3} (1 - A + A^2)}$$

Divide top & bottom of RHS by c :

$$v_1^2 = \frac{2E}{\frac{M_1}{c} + \frac{A^2 M_2}{c} + \frac{(1 - A + A^2)}{3}}$$

(5)

$$\text{But } 1 - A + A^2 = \frac{A^3 + 1}{A + 1}$$

Proof:

$$\begin{aligned} 1 - A + A^2 &= \frac{(A+1)(1 - A + A^2)}{(A+1)} \\ &= \frac{A^3 + 1}{A + 1} \end{aligned}$$

∴

$$V_1^2 = \frac{2E}{\frac{M_1}{c} + \frac{A^2 M_2}{c} + \frac{1}{3} \frac{(A^3 + 1)}{(A + 1)}}$$

$$\text{Let } \alpha = \frac{1}{\frac{M_1}{c} + \frac{A^2 M_2}{c} + \frac{1}{3} \frac{(A^3 + 1)}{(A + 1)}}$$

$$\therefore V_1^2 = 2E \alpha$$

$$\therefore V_1 = \sqrt{2E} \cdot \sqrt{\alpha}$$