

Q2 a)

i) prove $\gamma = \frac{Q}{AG}$

from $U = \frac{1}{2} Q \delta$ knowing that $\delta = \int \gamma ds$

$$U = \frac{1}{2} Q \int \gamma ds$$

$$G = \frac{\tau}{\gamma} \rightarrow \text{we know that } \tau = \frac{Q}{A}$$

we can rewrite it as $G = \frac{Q}{A\gamma}$

therefore $\boxed{\gamma = \frac{Q}{AG}}$

ii) prove that $U = \frac{Q^2 L}{2AG}$

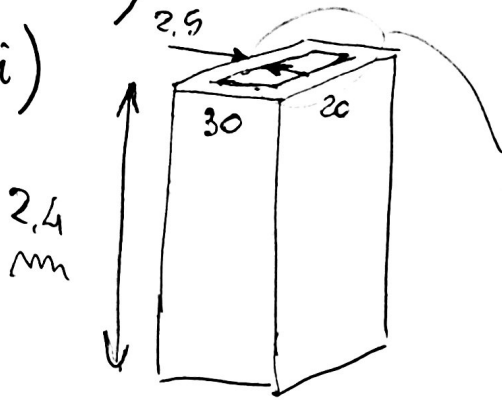
therefore, from $\gamma = \frac{Q}{AG}$, knowing that $U = \frac{1}{2} Q \int \gamma ds$

$$U = \frac{1}{2} Q \int \frac{Q}{AG} ds$$

$$U = \int_0^L \frac{Q^2}{2AG} ds = \boxed{U = \frac{Q^2 L}{2AG}}$$

Q2 b)

i)



$$B = 30 \text{ mm} \approx 0.030 \text{ m}$$

$$(20 - 2.5 - 2.5) \text{ mm} = 15 \text{ mm} \approx 0.015 \text{ m}$$

$$t = 2.5 \text{ mm} \approx 2.5 \cdot 10^{-3} \text{ m}$$

$$L = 2.4 \text{ mm}$$

$$U = ?$$

$$\sigma = 480 \text{ MPa}$$

$$E = 190 \text{ GPa}$$

$$U = \frac{\sigma^2 L}{2AG} \quad \text{ENERGY}$$

that can be written in stress form as:

$$U = \frac{\tau^2 AL}{2G} \approx \left(\frac{\tau^2}{2G} \cdot \text{VOLUME} \right) \quad \text{where } G = E \quad \text{ASSUMED}$$

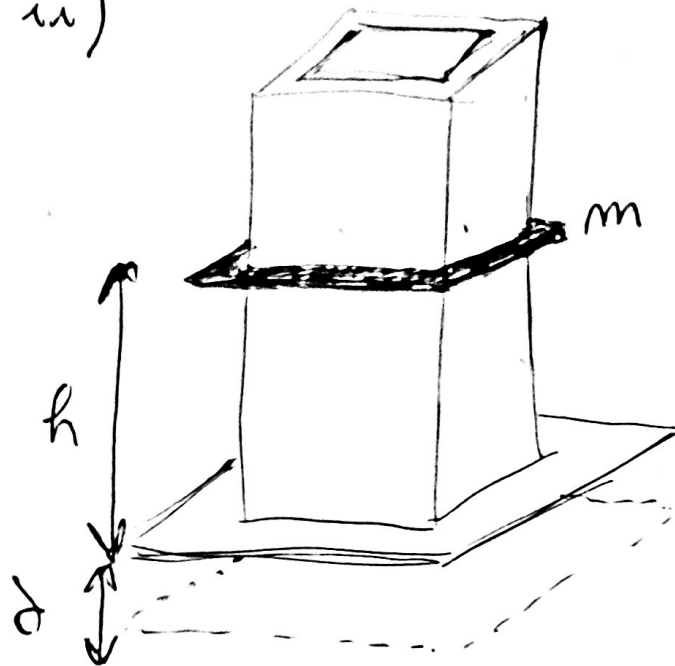
$$\therefore A = (0.03 \cdot 0.02) - (0.025 \cdot 0.015) = 6 \cdot 10^{-4} - 3.75 \cdot 10^{-4} =$$

$$A = 2.25 \cdot 10^{-4} \text{ m}^2$$

$$U = \frac{(480 \cdot 10^6)^2 \cdot (2.25 \cdot 10^{-4}) \cdot 2.4}{2 \cdot 190 \cdot 10^9} =$$

$$= \frac{1.24 \cdot 10^{14}}{3.8 \cdot 10^9} = 327.4 \text{ J}$$

Q2) ii)



show:

$$h = \frac{\sigma^2 V}{2EW}$$

where $W = F$

m has a potential energy of mgh .

where $mg = W$

$$\begin{aligned} \text{therefore POTENTIAL ENERGY} &= Wh \\ &= W(h + d) \end{aligned}$$

$$U = \frac{\sigma^2 AL}{2E} \Rightarrow W(h + d) = \frac{\sigma^2 AL}{2E}$$

if d is NEGLIGIBLE BECAUSE VERY SMALL:

$$Wh = \frac{\sigma^2 AL}{2E} \rightarrow h = \frac{\sigma^2 AL}{2EW}$$

where $AL = \text{Volume}$

$$\text{therefore, } h = \frac{\sigma^2 V}{2EW}$$

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MGX00420 ADVANCE STRESS ANALYSIS & FEA

EMANUELE DI CROSTA PIS1058

(4)

Q2) iii)

$$m = 20 \text{ kg}$$

$$h = 450 \text{ mm} = 0,45 \text{ m}$$

$$\sigma = ?$$

$$\sigma = \sqrt{\frac{2 W E h}{A L}}$$

$$W = mg = 20 \cdot 9,81 = 196,2 \text{ N}$$

$$E = 190 \text{ GPa}$$

$$A = (0,03 \cdot 0,02) - (0,025 \cdot 0,015) \\ = 2,25 \cdot 10^{-4} \text{ m}^2$$

$$L = 2,4 \text{ mm}$$

$$\sigma = \sqrt{\frac{2 \cdot 196,2 \cdot 190 \cdot 10^9 \cdot 0,45}{2,25 \cdot 10^{-4} \cdot 2,4}} = 12,58 \text{ MPa} \\ 12583057 \text{ Pa}$$

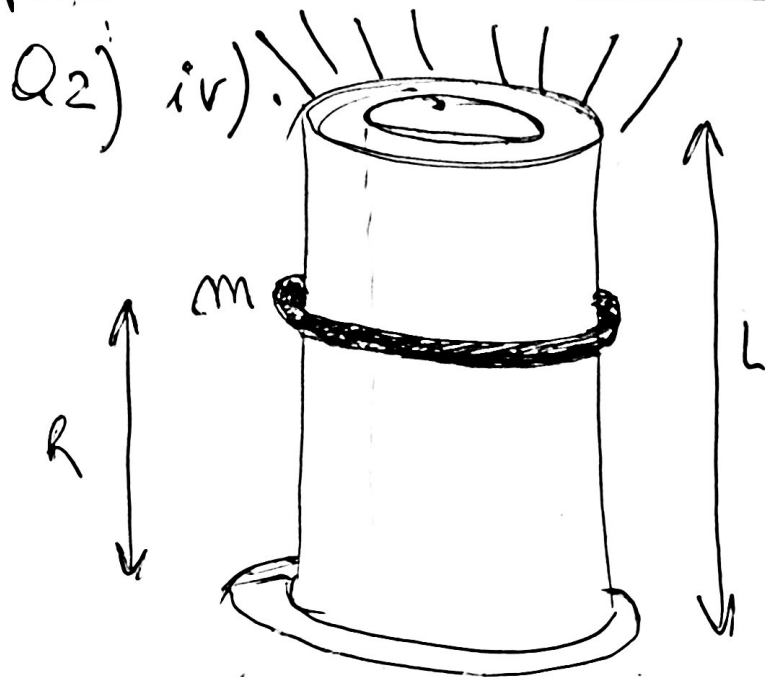
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EMANUELE DI CROSTA PISICOP

(5)



$$\phi_{\text{ext}} = 20 \text{ mm} = 0,02 \text{ m}$$

$$L = 2,4 \text{ m}$$

$$\sigma = 12,58 \text{ MPa}$$

$$t = ?$$

$$h = 450 \text{ mm} = 0,45 \text{ m}$$

$$m = 20 \text{ Kg}$$

$$E = 190 \text{ GPa}$$

$$\sigma^2 = \frac{2 \cdot m \cdot g \cdot E \cdot h}{A \cdot L}$$

$$A_T = [\pi r^2] \rightarrow A_T = A_{\text{ext}} - A_{\text{int}} = \left[\pi \cdot \left(\frac{0,02}{2} \right)^2 \right] - \left[\pi \cdot \left(\frac{\phi_{\text{ext}} - 2t}{2} \right)^2 \right]$$

$$= 3,14 \cdot 10^{-4} - \left(\frac{\phi_{\text{ext}} - 2t}{4} \right)^2 \pi$$

$$\frac{2 \cdot 20 \cdot 9,81 \cdot 190 \cdot 10^9 \cdot 0,45}{\left(2,4 \cdot \left(3,14 \cdot 10^{-4} - \left(\frac{0,02 - 2t}{4} \right)^2 \pi \right) \right)} = (12,58 \cdot 10^6)^2$$

$$\frac{3,355 \cdot 10^{13}}{2,4 \cdot \left(3,14 \cdot 10^{-4} - \left(\frac{0,02 - 2t}{4} \right)^2 \pi \right)} = (12,58 \cdot 10^6)^2$$

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M6 X 00420 ADVANCE STRESS ANALYSIS & FEAT
EVALUARE DI CROSTA - PISIORE

(6)

$$\frac{3,355 \cdot 10^{13}}{2,4 \cdot \left(3,14 \cdot 10^{-4} - \left(\frac{4 \cdot 10^{-4} - 0,04t - 0,04t + 4t^2}{4} \right) \pi \right)} = (12,58 \cdot 10^6)^2$$
$$\frac{1}{4} t^2 - 0,02t + 4 \cdot 10^{-4} \pi$$

(7)

e

h

ln

~~STUDENT~~

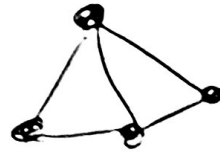
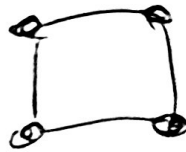
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MEX 60420 ADVANCE STRESS & FEA
EMILIO DI COSTA 191088

(7)

Q1) THE ELEMENT TYPE COULD BE
1D 2D OR 3D



1D is when one dimension is much greater than the other ~~2~~

2D is when two dimensions are larger than the other

3D is when all three have the same magnitude.

1D elements are used for ROD, BARS, BEAM
and it is necessary define
AREA, SECOND MOMENT OF AREA

2D are for PLATES ~~AND~~ SHEET STRUCTURES

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M6 x 00420 ADVANCE STRESS & 12A

EXAMPLES ON CROSSA P151088

b) THE ELEMENT SIZE

d) 1ST ORDER ELEMENTS HAS NODES ONLY AT CORNERS

2ND ORDER ELEMENTS HAS NODES AT MID SIDES.

THE LATTER ARE MORE ACCURATE

HOWEVER THEY AFFECT THE RUN TIME -

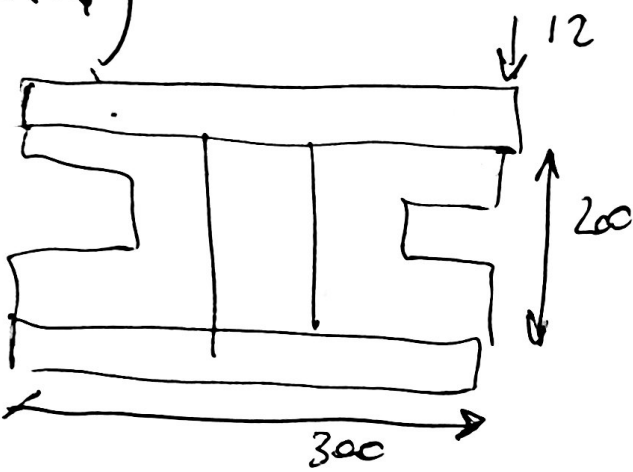
THEREFORE, WHICH ORDER TO USE IS DICTATED BY COMPUTATIONAL POWER & ACCURACY REQUIRED

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MG X 00420 ADVANCE STEEL

(9)

Q4)



EXAMPLE OF DESIGN PROBLEM

$$I_{xx} = 26,2 \cdot 10^6 \text{ mm}^4$$

$$I_{yy} = 1,57 \cdot 10^6$$

$$A = 4400 \text{ mm}^2$$

$$I = \frac{Bd^3}{12}$$

$$I_{xx} = 2 I_c + \left[\frac{300 \cdot 224^3}{12} - \frac{700 \cdot 200^3}{12} \right]$$

$$2 \cdot 26,2 \cdot 10^6 + \left[280,9 \cdot 10^6 - 200 \cdot 10^6 \right]$$

$$= 133,3 \cdot 10^6$$

$$I_{yy} = 2 \left(1,57 \cdot 10^6 + 4400 (79)^2 \right) + \left(2 \cdot \left(\frac{300^3 \cdot 12}{12} \right) \right)$$

$$58,06 \cdot 10^6 + 180 \cdot 10^3$$

$$I_{yy} = 180,05 \cdot 10^3$$

IT WILL BUCKLE AROUND Y

$$I_{yy} < I_{xx}$$

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$$K = \sqrt{\frac{I}{A}} = \sqrt{\frac{180,05 \cdot 10^3}{(4400 \cdot 2)(12 \cdot 300 \cdot 2)}}$$

$$\approx 3,35$$

$$\frac{L}{K} = \frac{8}{3,35} = 2,38$$

$$P_e = \frac{\pi^2 EI}{L^2}$$

$$P_R = \frac{\sigma_y \cdot A}{1 + a \left(\frac{L}{K}\right)^2}$$