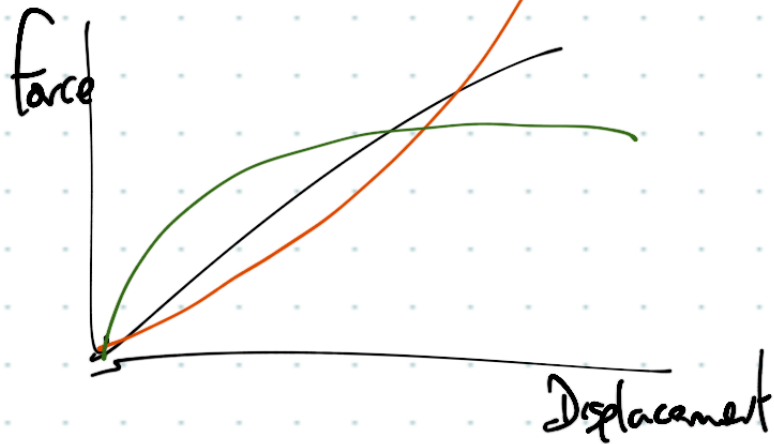


Suspension

Springs and Dampers

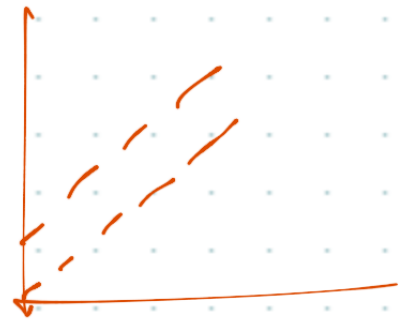
Springs.



Applications:

Dead rate?

parallel
springs
 k_1
 k_2



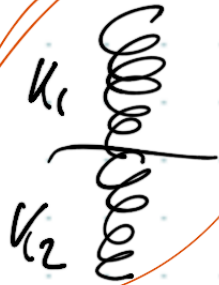
Spring rates?

k_1
 k_2

k_1 k_2

Springs in Series or parallel

Series



$$K = ?$$

$$\frac{1}{K} = \frac{1}{K_1} + \frac{1}{K_2}$$

So if $K_1 = 10 \text{ N/m}$ and $K_2 = 20 \text{ N/m}$

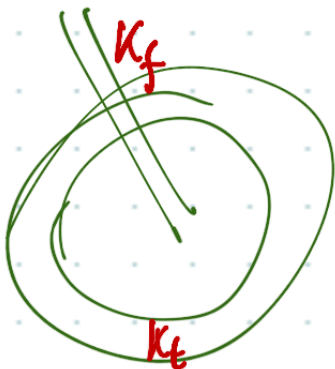
$$K = ? \quad 6.67 \text{ N/m}$$

Parallel?



$$K = K_1 + K_2$$

Where else do we see Series Springs on a bike?



K_f = Spring stiffness

K_t = Tyre stiffness.

What sort of spring rates do we see?

fork springs $\sim 8 \text{ N/mm}$
(2 forks $\rightarrow 16 \text{ N/mm}$)

Rear shock?

$80 \rightarrow 100 \text{ N/mm}$

Type? $\sim 100 \text{ N/mm} \rightarrow 200 \text{ N/mm}$

Front or rear are vastly different! why

twin shocks

single shock

all different configurations

The front does not change vastly since the forks are almost vertical. The rear change a lot based on the link ratio.

$$K_{\text{shock}} = K_{\text{vertical}} \times L \cdot R^2$$

$$\cos \theta^2 \neq \cos^2 \theta$$

$$\cos^2 \theta = \cos \theta \times \cos \theta$$

How do we find a way to find spring rates for ANY Bike?

We need a common factor between bikes and riders, regardless of weight.

We use the concept of Natural frequency

Is there a common natural frequency,

We call this 'Ride frequency'

Ride frequency.

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

For racing applications, we found the same answers

$$f_r = \underline{2.1 \text{ Hz}}$$

$\Rightarrow$ Start with 2.1 Hz

Find the Spring rate $k = (2\pi f)^2 \times m$

where the m is the mass on the front or rear

$\Rightarrow$ Spring mass

Try an example

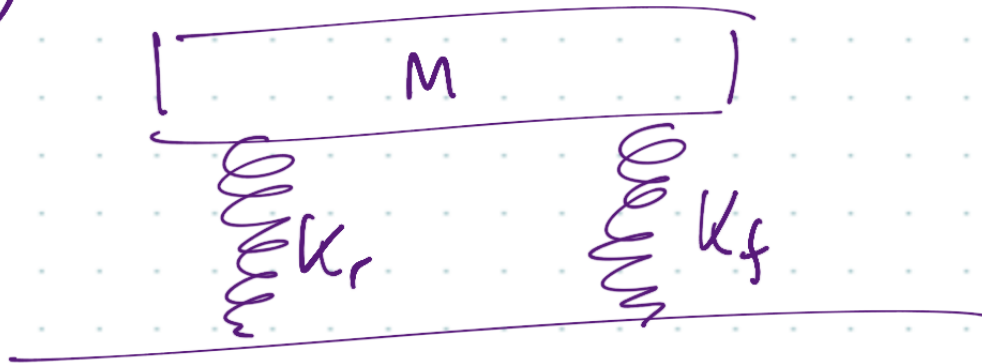
$$m_{\text{front}} \sim 80 \text{ kg}$$

$$f_r = 2.1 \text{ Hz}$$

what is k ?

$$k = 14,000 \text{ N/m}$$

This gives us the vertical stiffness



So we calculate 'vertical stiffness' then later work out the required Spring rates based on rake angle and link Ratio

So what about Damping?

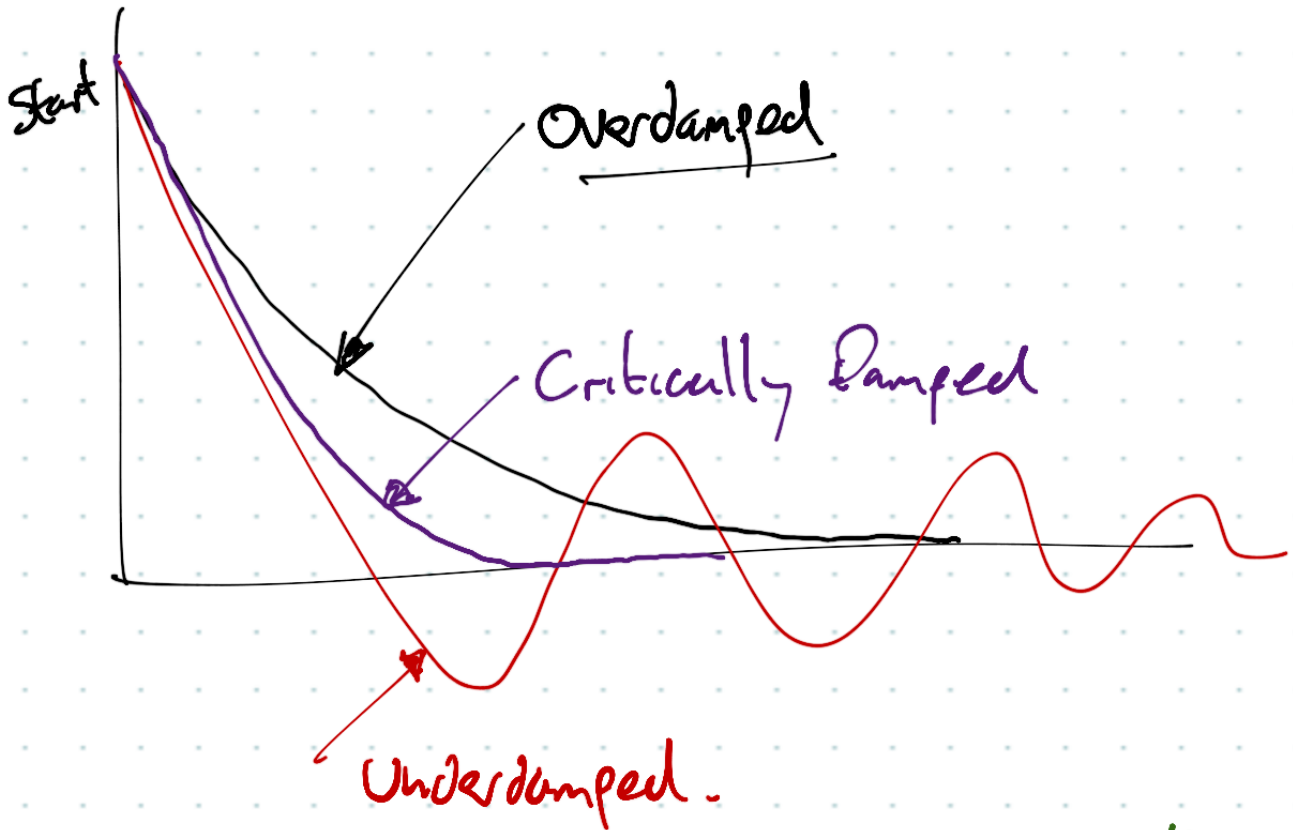
Damping is there to control the spring

what do we want to actually happen

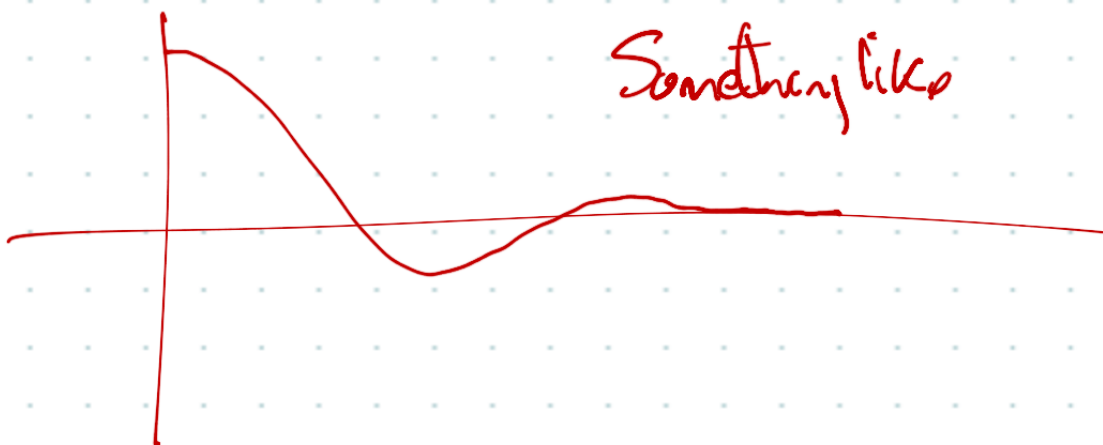


Mass, Spring, Damper system

What do we want the damper to do?



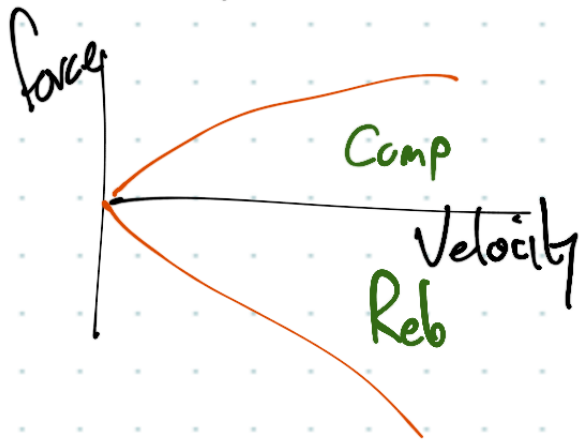
What do we want from a door? Maybe Critically
What about a motorcycle? Underdamped



How do we deal with damping mathematically?
What is damping? units?

$$\text{Damping Value} = \frac{\text{Force}}{\text{Velocity}}$$

Damper Dyno



C is the gradient

$$C = \frac{\text{Force}}{\text{Velocity}}$$

$$C \equiv \frac{\text{N}}{\text{m/s}} = \frac{\text{Ns}}{\text{m}}$$

Damping (C) is Newton Seconds per Meter

So we have

$$\text{Spring force} = kx$$

$$\text{Damping force} = CV$$

$$\text{Dynamic force} = M\ddot{x}$$



$$\text{Spring force} = Kx$$

$$\text{Damping force} = C \frac{dx}{dt} = C\dot{x}$$

$$\text{Dynamic force} = M \frac{d^2x}{dt^2} = M\ddot{x}$$

x = position

$\dot{x}$ = velocity

$\ddot{x}$ = acceleration.

So we get

$$M \frac{d^2x}{dt^2} + C \frac{dx}{dt} + Kx = \text{force}$$

$$M \frac{d^2x}{dt^2} + C \frac{dx}{dt} + Kx = 0$$

$\Rightarrow$ Second order differential equation

Equations

Critical Damping $C_c = 2\sqrt{KM}$

Damping ratio

$\zeta = \frac{C}{C_{crit}}$ (zeta)