

Complete, fold and seal
(see note 2 below)

SURNAME:

Doyle

12 Pages

Instructions to Candidates

1. Please give the information requested in the boxes below.
2. Enter your name in the space provided to the right, fold over and seal down (using the label provided) to hide your name.
3. The answers must be legibly written. The Examiners can either ignore illegible script or have it typed and the expense of typing charged to the candidate.
4. Write your answers in ink.
5. Begin each answer on a fresh page. Do all rough work in the answer book and cross it through.
6. No part of this book is to be torn out; it must be handed in complete.
7. If more books than one are used, fasten them together using the tags available from the invigilator(s).
8. **Every book must be handed in. In no circumstances may answer books, used or unused, be taken out of the examination venue.**

1	Student Number	2237462
2	Title of Examination Paper	Mathematics Exam
3	Module Code	ACDBA003
4	Date and Time of Examination	9:00 - 11:00 1 st May 24
5	Examination Venue	IQ310
6	Seat Number	021

7	Number of Answer Books handed in	1
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[illegible]

97%
Gallant

(Gweler yr ochr arall am y fersiwn Cymraeg)

Q1

$$i) \begin{array}{l} 3x + 2x + 5x - 2z + 3z + 10z + 4y - 9y - 6y \\ 10x + 11z - 11y \end{array}$$

(2)

$$ii) \begin{array}{l} x(x+5) - (x-4) \\ x^2 + 5x - x + 4 \\ = x^2 + 4x + 4 \end{array}$$

= sign is important!

(2)

$$iii) \begin{array}{l} 3p^1r^1 \times 4p^3q^3 \times 5qr^2 \\ (3 \times 4 \times 5) \times (p^1 \times p^3 \times q^1) \times r^1 \times r^2 \\ = 60p^4q^3r^3 \end{array}$$

(2)

$$iv) \begin{array}{l} 2(p^2 \times p^{-3}) \times (q^3 \times q^{-3}) \times (r \times r^3) \\ = 2p^{-1}q^0r^4 \\ = \frac{2r^4}{p} \end{array}$$

if $a^0 = 1$ always.

(2)

$$v) \begin{array}{l} \frac{1}{8} f' g' h^{-1} \times 2 g h^2 h^{-1} f^{-1} \times \frac{1}{2} f' f^3 h^3 \\ \frac{1}{8} f' g' h^{-1} \times 2 g h^1 f^{-1} \times \frac{1}{2} f^{-2} h^3 \\ \left(\frac{1}{8} \times \frac{1}{2} \times 2 \right) \times (f' \times f^{-1} \times f^{-2}) \times (h^{-1} \times h^1 \times h^3) \times (g' \times g) \\ = \frac{1}{8} f^{-2} h^3 g^2 \\ = \frac{1}{8} h^3 g^2 f^{-2} h^3 g^2 \end{array}$$

(2)

$$Q2) \begin{array}{l} x^2 + 10x = -25 \\ i) x^2 + 10x + 25 = 0 \\ (x+5)(x+5) = 0 \\ x = -5 \text{ But twice!!} \end{array}$$

(4)

$$ii) \frac{x-3}{4} = x + \frac{2x-4}{9}$$

Q2

ii)

$$x-3 = 2(x-3) = 8x + 2x - 4$$

$$2x - 6 = 8x + 2x - 4$$

$$2x - 6 = 10x - 4$$

$$8x = 10x - 2x = 4 - 6$$

$$8x = -2$$

$$x = -\frac{2}{8} = -\frac{1}{4}$$

5

Q3

a) i)

$$4^{2x+1} = 3^{x+1}$$

$$2x - 1 \ln 4 = x + 1 \ln 3$$

$$x + 1 = \frac{2x - 1 \ln 4}{\ln 3}$$

$$\frac{x+1}{2x-1} = \frac{\ln 4}{\ln 3}$$

$$2x + 1 \log_3 4 = x + 1 \log_3 3$$

$$2x + 1 \log_3 4 = (x + 1) \cdot 1$$

$$\log_3 4 = \frac{x+1}{2x-1}$$

$$2x \log_3 4 - \log_3 4 = x + 1$$

$$1.26x - 1.26 = x + 1$$

Answered on page after
Q8

$$ii) \log(x^2 + 4) = \log x + \log(x+8) \quad \& \quad 2x - 1 \ln 4 =$$

$$ii) \log(x^2 + 4) = \log x + \log(x+8)$$

$$x^2 + 4 = x(x+8)$$

$$x^2 + 4 = x^2 + 8x$$

$$x^2 - x^2 + 4 = 8x$$

$$4 = 8x$$

$$x = \frac{4}{8} \quad x = \frac{1}{2}$$

5

Q3

$$b) i) \log(x^2 y z) = 2 \log x + \log y + \log z$$

$$ii) \log\left(\frac{x^4 y^2}{z}\right) = 4 \log x + 2 \log y - \log z$$

Q4

$$\underline{a} = 2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k} \quad \underline{b} = 1\mathbf{i} + 3\mathbf{j} - 1\mathbf{k}$$

$$i) \underline{a} \cdot \underline{b} = (2 \times 1) + (-3 \times 3) + (4 \times -1) \\ = 2 - 9 - 4 \\ = -11$$

$$ii) |\underline{a}| = \sqrt{2^2 + (-3)^2 + 4^2} = \sqrt{4 + 9 + 16} = \sqrt{29} \\ |\underline{b}| = \sqrt{1^2 + 3^2 + (-1)^2} = \sqrt{1 + 9 + 1} = \sqrt{11}$$

$$\theta = \cos^{-1}\left(\frac{-11}{\sqrt{29} \times \sqrt{11}}\right) = 128.01^\circ$$

$$iii) \underline{c} = -3\mathbf{i} + 1\mathbf{j} + 0\mathbf{k} \\ \underline{b} = 1\mathbf{i} + 3\mathbf{j} - 1\mathbf{k}$$

$$\underline{b} \cdot \underline{c} = (1 \times -3) + (1 \times 3) + (0 \times -1) \\ = -3 + 3 + 0 \\ = 0$$

as the dot product equals 0, $\underline{b}$ and $\underline{c}$ are perpendicular.

$$iv) \underline{d} = \underline{a} \times \underline{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -3 & 4 \\ 1 & 3 & -1 \end{vmatrix} = \begin{vmatrix} 3 & -1 \\ 1 & -1 \end{vmatrix} \mathbf{i} - \begin{vmatrix} 2 & 4 \\ 1 & -1 \end{vmatrix} \mathbf{j} + \begin{vmatrix} 2 & -3 \\ 1 & 3 \end{vmatrix} \mathbf{k} \\ = (3 - 1)\mathbf{i} - (-2 - 4)\mathbf{j} + (6 + 3)\mathbf{k} \\ = 2\mathbf{i} + 6\mathbf{j} + 9\mathbf{k}$$

$$|\underline{d}| = \sqrt{(-9)^2 + 6^2 + 9^2} \\ = \sqrt{81 + 36 + 81} \\ = \sqrt{198}$$

$$\hat{n} = \frac{-9\mathbf{i} + 6\mathbf{j} + 9\mathbf{k}}{\sqrt{198}}$$

Q5

$$\begin{aligned} \det A &= 1 \begin{vmatrix} -1 & 1 \\ 4 & -2 \end{vmatrix} - 2 \begin{vmatrix} 2 & 1 \\ 3 & -2 \end{vmatrix} + 3 \begin{vmatrix} 2 & -1 \\ 3 & 4 \end{vmatrix} \\ &= (2-4) - 2(-4-3) + 3(8+3) \\ &= -2 + 14 + 33 \\ &= 45 \end{aligned}$$

$$\begin{aligned} \text{Adj } A &= \begin{vmatrix} -1 & 1 & 2 \\ 4 & -2 & 3 \\ 2 & 3 & 1 \end{vmatrix} \begin{vmatrix} 2 & 1 & 3 \\ 1 & 3 & -2 \\ 1 & 2 & 3 \end{vmatrix} \begin{vmatrix} 2 & -1 & 1 \\ 3 & 4 & -2 \\ 1 & 2 & 3 \end{vmatrix} \\ &= \begin{vmatrix} 2-4 & -4-3 & 8+3 \\ -4-12 & -2-4 & 4-6 \\ 2+3 & 1-6 & -1-4 \end{vmatrix} \\ &= \begin{vmatrix} -2 & -7 & 11 \\ -16 & -6 & -2 \\ 5 & -5 & -5 \end{vmatrix} \begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix} = \begin{vmatrix} -2 & 7 & 11 \\ 16 & -11 & 2 \\ 5 & 5 & -5 \end{vmatrix} \end{aligned}$$

$$A^T = \begin{vmatrix} -2 & 16 & 5 \\ 7 & -11 & 5 \\ 11 & 2 & -5 \end{vmatrix} \quad A^T \times \frac{1}{\det A} = A^{-1}$$

$$\frac{1}{45} \times \begin{vmatrix} -2 & 16 & 5 \\ 7 & -11 & 5 \\ 11 & 2 & -5 \end{vmatrix} = \begin{vmatrix} \frac{-2}{45} & \frac{16}{45} & \frac{5}{45} \\ \frac{7}{45} & \frac{-11}{45} & \frac{5}{45} \\ \frac{11}{45} & \frac{2}{45} & \frac{-5}{45} \end{vmatrix}$$

$$= \begin{vmatrix} \frac{-2}{45} & \frac{16}{45} & \frac{1}{9} \\ \frac{7}{45} & \frac{-11}{45} & \frac{1}{9} \\ \frac{11}{45} & \frac{2}{45} & \frac{-1}{9} \end{vmatrix}$$

Q6

$$\begin{array}{c} \text{+} \\ \left| \begin{array}{ccc|c} 1 & 2 & 3 & x \\ 2 & -1 & 1 & y \\ 3 & 4 & -2 & z \end{array} \right| = \left| \begin{array}{c} 5 \\ 0 \\ 2 \end{array} \right| \end{array}$$

$A \quad \quad \quad x \quad \quad \quad b$

$$A^{-1}b = x$$

$$\begin{aligned} \det A &= \begin{vmatrix} 1 & -1 & 1 \\ 2 & -1 & 1 \\ 3 & 4 & -2 \end{vmatrix} = 1(2-4) - 2(-4-3) + 3(8+3) \\ &= -2 + 14 + 33 \\ &= 45 \end{aligned}$$

already done
previously

$$\text{Adj } A = \begin{vmatrix} 2 & -4 & 8 \\ -4 & -3 & -1 \\ 1 & -6 & -4 \end{vmatrix}$$

$$\begin{aligned} \text{Adj } A &= \begin{vmatrix} 2-4 & -4-3 & 8+3 \\ -4-12 & -2-9 & 4-6 \\ 2+3 & 1-6 & -1-4 \end{vmatrix} = \begin{vmatrix} -2 & -7 & 11 \\ -16 & -11 & -2 \\ 5 & -5 & -5 \end{vmatrix} \begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix} \\ &= \begin{vmatrix} -2 & 7 & 11 \\ 16 & -11 & 2 \\ 5 & 5 & -5 \end{vmatrix} \end{aligned}$$

$$A^T = \begin{vmatrix} -2 & 16 & 5 \\ 7 & -11 & 5 \\ 11 & 2 & -5 \end{vmatrix}$$

$$A^T \times \frac{1}{\det A} = A^{-1} \quad A^{-1} = \frac{1}{45} \times \begin{vmatrix} -2 & 16 & 5 \\ 7 & -11 & 5 \\ 11 & 2 & -5 \end{vmatrix}$$

$$x = A^{-1}b \quad x = \frac{1}{45} \times \begin{vmatrix} -2 & 16 & 5 \\ 7 & -11 & 5 \\ 11 & 2 & -5 \end{vmatrix} \begin{vmatrix} 5 \\ 0 \\ 2 \end{vmatrix}$$

$$\begin{vmatrix} x \\ y \\ z \end{vmatrix} = \begin{vmatrix} 0 \\ 1 \\ 1 \end{vmatrix}$$

$$= \frac{1}{45} \times \begin{vmatrix} -10+0+10 \\ 35+0+10 \\ 55+0-10 \end{vmatrix} = \frac{1}{45} \times \begin{vmatrix} 0 \\ 45 \\ 45 \end{vmatrix} = \begin{vmatrix} 0 \\ 1 \\ 1 \end{vmatrix}$$

$$x + 2y + 3z = 5$$

$$0 + 2 \times 1 + 3 \times 1 = 5 \quad 2 + 3 = 5$$

good

(10)

Q7

a) i) $y = 5x - 3$
 $\frac{dy}{dx} = 5$

(2)

ii) $y = 4x + 10x^{1/2} + \frac{5}{6}$
 $\frac{dy}{dx} = 4 + 5x^{-1/2} *$

(2)

iii) $y = 2x^{-5} + 9x^3$
 $\frac{dy}{dx} = -10x^{-6} + 27x^2$

(2)

iv) $y = \cos(20x)$
 $\frac{dy}{dx} = -20 \sin 20x$

(2)

v) $y = x^2 \sin x$
 $\frac{dy}{dx} = \overbrace{2x \sin x}^{\cancel{x^2 \cos x}} - \cancel{x^2 \cos x}$
 $\frac{dy}{dx} = x^2 \cos x + 2x \sin x$

(2)

vi) $y = (8x + 3)^5$
 $\frac{dy}{dx} = 5 \times 8 (8x + 3)^4$
 $\frac{dy}{dx} = 40 (8x + 3)^4$

(2)

vii) $y = 2e^x + \ln x$
 $\frac{dy}{dx} = 2e^x + \frac{1}{x}$

(2)

Q7

$$\begin{aligned}
 \text{b) } y &= \frac{x+3}{1+x^2} \quad \frac{dy}{dx} = \frac{(1+x^2) \times 1 - 2x(x+3)}{(1+x^2)^2} \\
 &= \frac{1+x^2 - 2x^2 - 6x}{(1+x^2)^2} \\
 &= \frac{1-x^2-6x}{(1+x^2)^2}
 \end{aligned}$$

When $x=2$:

$$\begin{aligned}
 \frac{1-2^2-6 \times 2}{(1+2^2)^2} &= \frac{1-4-12}{(1+4)^2} \\
 &= -\frac{15}{5^2} = -\frac{15}{25} = \boxed{-\frac{3}{5}}
 \end{aligned}$$

Q8

$$\text{i) } \int (4x + x^3) dx = \frac{4x^2}{2} + \frac{x^4}{4} + C = 2x^2 + \frac{1}{4}x^4 + C$$

$$\text{ii) } \int (2x + 3e^x) dx = \frac{2x^2}{2} + 3e^x + C = x^2 + 3e^x + C$$

$$\text{iii) } \int \left(\frac{1}{x} + 7\right) dx = \ln x + 7x + C$$

$$\begin{aligned}
 \text{iv) } \int (x^2 - 1)(3x + 4) dx &= \int (3x^3 + 4x^2 - 3x - 4) dx \\
 &= \frac{3x^4}{4} + \frac{4x^3}{3} - \frac{3x^2}{2} - 4x + C
 \end{aligned}$$

$$\begin{aligned}
 \text{v) } \int_3^6 (3x^2 + 2x) dx &= \left[\frac{3x^3}{3} + \frac{2x^2}{2} + C \right]_3^6 \\
 &= \left[x^3 + x^2 + C \right]_3^6 \\
 &= (6^3 + 6^2) - (3^3 + 3^2) \\
 &= 216
 \end{aligned}$$

NOO'S in
definite
integral!

Q3

i) $4^{2x-1} = 3^{x+1}$ ~~$2x-1 \log 4 = (x+1)$~~

$$(2x-1) \log_4 4 = (x+1) \log_4 3 \quad \text{Cancel here}$$

$$2x-1 = (x+1) \log_4 3$$

$$2x-1 = x \log_4 3 + \log_4 3$$

$$2x-1 - x \log_4 3 = \log_4 3$$

$$x(2 - \log_4 3) - 1 = \log_4 3$$

$$x = \frac{\log_4 3 + 1}{2 - \log_4 3}$$

$$x = \frac{\log_4 3 + 1}{2 - \log_4 3} = 1.48 \text{ approx.}$$

(5)

Kallum this is an excellent
Result, very well Done!