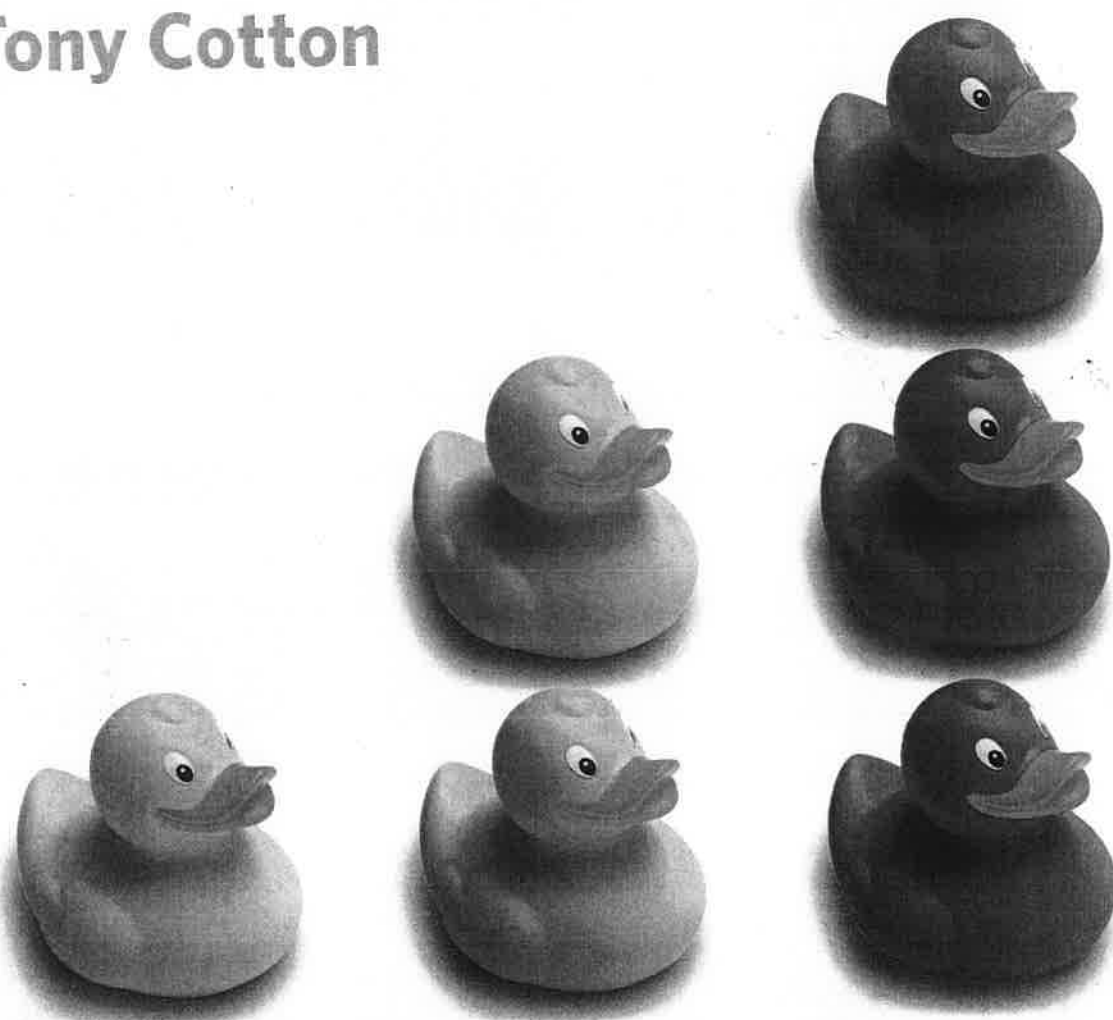


Understanding
and Teaching

Primary Mathematics

Tony Cotton

SECOND
EDITION

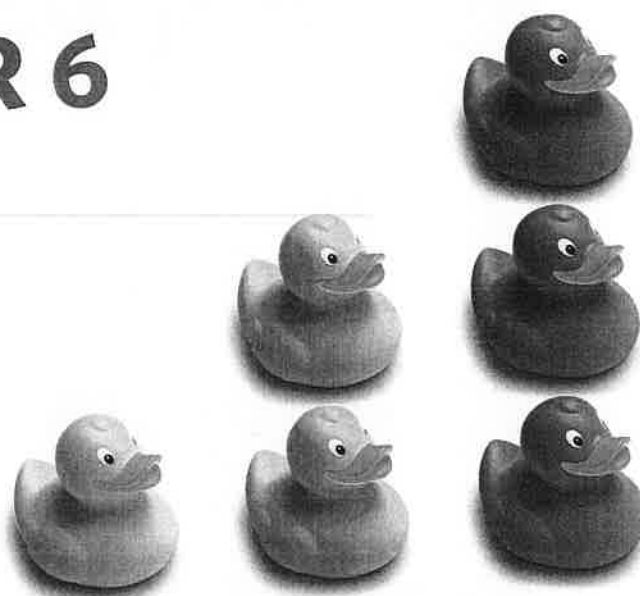


ALWAYS LEARNING

PEARSON

CHAPTER 6

CALCULATING



Introduction



I often ask children to think of someone they know who is 'good' at mathematics. Then I ask them what this person can do that makes them 'good' at mathematics. One child said recently, 'They can't just do additions, they can do subtractions too'; another said, 'They can do really hard things like long divisions.' An answer that I am regularly given is that 'they can work things out really quickly, in their head - and they don't even need to use a calculator.' Although the main thrust of this book is to illustrate that mathematics is much more than simply calculating, this suggests that for many of us our image of mathematics is linked predominantly to calculating.

This chapter will explore how you can develop your capacity to 'do it really quickly in your head'. Developing your own mental methods in order to help you teach your pupils to calculate 'in their head' can be challenging, as you were probably not encouraged to do this when learning mathematics. Some of my students also remember being taught a particular method that seemed to work most of the time but there were always times when the method didn't seem to work. Because they had only one method to fall back on, they were unable to carry out the calculation, particularly in high-pressure situations like tests or exams.

You will be introduced to the idea of 'appropriate methods'. This is a move away from teaching one method for all calculations. What is more efficient is reaching a stage where you have a range of methods to draw on and you can select the most

appropriate method for the particular calculation you are working on. You will also come to accept that the method which is most appropriate for one person may be different for another person. Through discussing our own methods we can expand the range of methods we have to draw on.

You will also see how addition, subtraction, multiplication and division all connect. If you can add, you can subtract; if you can multiply, you can divide. Hopefully by the end of the chapter you will feel confident that you are a 'good' calculator – perhaps this will go a long way to convincing you that you can be 'good' at mathematics too. This will give you the confidence to use your calculating skills in solving problems – an important functional skill.

Starting point

Work out the answers to the following calculations mentally. Avoid writing anything down at this point:

$$11 + 6 =$$

$$28 - 9 =$$

$$20 \times 5 =$$

$$57 \times 3 =$$

Now ask yourself, 'how did I carry out the calculation?' You may think 'I just did it', but let's think more deeply. Did you just know that 11 and 6 made 17, or did you add 6 and 1 to make 7 then add that to 10? Did you partition 9 into $8 + 1$ so you could take 8 away from 28 to get 20 and then take away another 1 to give 19? Did you multiply 10 by 5 and then double the answer to get 100, did you add $20 + 20 + 20 + 20 + 20$, or is this a fact you know? And, finally, did you work out 3×50 and 3×7 and then add $150 + 21$ to give you 171, or did you use the **algorithm**, or rule, you were taught in school? In your head you might have said something like ' $3 \times 7 = 21$; put the 1 down and carry 2; 3×5 is 15, add 2 is 17. So the answer is 171'. This models the written algorithm.

When you carried out these calculations you were making decisions about what was the most effective strategy for you. It is unlikely that you drew on one formal written method for any of these calculations. In Chapter 5 you were introduced to the idea of 'number sense' and it was this 'number sense' that you drew on to decide how to come to an answer. The only time this 'number sense' might have deserted you is if you drew on the formal algorithm to carry out the multiplication. You may have lost sight of the value of the numbers – saying something like 'add 2 to give me 17 and then 1 gives me 171'. This describes the patterns of the numbers rather than their values, which can lead to confusion.

It is significant that in November 2011 Ofsted chose to release a report called *Good Practice in Primary Mathematics: Evidence from 20 Successful Schools* which focused on calculation. The key findings in this report included the following:

- Children should focus on practical activities and developing mental methods in the early stages of their time in primary school.

- A good understanding of place value and the ability to recall multiplication tables and number bonds supports effective calculation. (This area has been covered in the previous chapter.)
- Your learners' confidence and 'fluency' in calculating will be improved if you place a strong emphasis on problem solving and make the most of any cross-curricular links.
- A focus on misconceptions will make sure that these do not develop and impede next steps in learning.

These schools did not teach 'chunking' as a method of division, nor did they teach the long-division algorithm. The teachers in these schools had realised that neither of these approaches help children become efficient at calculations which involve long division. (You will read more about chunking and the long-division algorithm later in this chapter.)

As you can see, there is much more to calculating than just knowing the written methods for adding, subtracting, multiplying and dividing – the message is clear: mental methods first, then a range of written methods to allow children to develop 'efficient, reliable and effective' methods when it is not possible to use mental methods. Chapter 12 looks at the use of calculators in detail and describes how you can introduce these to your learners so that they are supported in developing their mental methods and know when it is appropriate to use them. In this chapter you will read about the importance of developing a range of calculating methods so that your learners can make sensible choices over their methods when they are faced with a calculation to carry out.

Taking it further



In Chapter 4 you were briefly introduced to *Children and Number: Difficulties in Learning Mathematics* (1986, Blackwell) by Martin Hughes. It is worth revisiting this research at this point. There is an important section exploring calculation strategies and the way in which children invent their own forms of recording when carrying out calculations. Hughes suggests that, left to their own devices, children go through a series of stages in their responses. An initial stage is the 'pictographic' response. At this stage children literally 'draw' the calculation. So if they are adding three multilink cubes to four multilink cubes they will draw the cubes in two groups and then count them up to add them together. Another stage is described by Martin Hughes as the 'iconic' stage. Here children represent the cubes by 'icons'. So for the calculation above a child might draw three sticks in a circle and four sticks in a circle and then add them. Again there is a direct correspondence between the drawing and the physical act of counting. Both of these stages precede the use of formal symbols – here a child would write the symbol '3' and the symbol '4' to represent the calculation.

I have seen children using the number symbols as icons – for example, when working out

$$3 + 4 =$$

they will count each point of the 3 by tapping their pencil on the points and then tap each point of the 4 to carry out the calculation. They count the taps and reach 7. You may have seen children doing this – you may even remember doing it yourself!

Be sure to spend some time with early learners in your setting as they start to record their calculating process. Encourage them to use individual whiteboards to 'jot' down their 'working out'. It is likely that you will notice them developing in the way that Martin Hughes suggests.

Progression in 'Calculating'

Foundations for 'Calculating'

Counting is at the heart of calculating and initially you will be working with your learners on counting objects and using language such as 'more' and 'less' to introduce them to ideas of addition and subtraction. You should compare different groups of objects to find out how many more or less there are in each group and start to explore sharing by splitting groups of objects into equal parts.



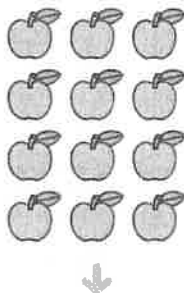
Beginning in 'Calculating'

The next step is to introduce your pupils to the idea of **counting on**. That is, that if you are adding 5 and 3 you point to 5 on a number line and count on saying, 6, 7, 8. You can teach that addition can be carried out in any order and that you can carry out subtraction by finding the difference between two numbers through 'counting up'. This also shows them how addition and subtraction are inverse or opposite operations, and the two operations should be taught together. You may introduce the number symbols to record practical activities but should ensure pupils are secure in their understanding before you do this.

You should develop children's understanding of division by combining groups of 2, 5 or 10 objects and by sharing into equal groups.

At this stage you can begin to introduce children to the four symbols $+$, $-$, $\times$ and $\div$ as well as $=$. This will help children find unknown numbers in number sentences such as $20 - 8 = 12$, which is the early stages of algebra. Mentally they will be able to add and subtract one-digit numbers or multiples of 10 from two-digit numbers and they could use practical or informal methods to add and subtract two-digit numbers. Children should understand that addition and subtraction are inverse operations, and should represent multiplication as an array – for example,

4×3 is the same as



Becoming confident in 'Calculating'

At this stage you can support your pupils to add or subtract mentally one- and two-digit numbers and introduce them to practical and informal methods to multiply and divide two-digit numbers (these informal methods are described later in the chapter), teaching them that multiplication and division are inverse operations. Your pupils will also start to find unit fractions. Unit fractions have 1 as the numerator. So they will be able to find $\frac{1}{2}$ of 5 metres or $\frac{1}{6}$ of 18 litres, for example.

You can develop your pupils' use of written methods to record and explain multiplication and division of two-digit numbers by single-digit numbers, and teach them to use written methods to add and subtract two- and three-digit whole numbers, including working in the context of money. You should expect your pupils to multiply and divide by 100 and 1000, and they will have developed their understanding of fractions to find fractions of quantities and shapes. You will explore the use of a calculator to help them carry out one- and two-step calculations involving all four operations, including understanding the meaning of negative numbers in the display.

The next stage is to develop calculation skills so they can use efficient written methods to add and subtract whole numbers and decimals up to two decimal places. You will have helped them extend their mental methods for whole number calculations, including multiplying one-digit numbers by two-digit numbers; multiplying by 25; and subtracting near multiples of 1000. You will ask them to draw on their understanding of place value to multiply by 10, 100 or 1000 and be able to draw on a range of written methods to multiply and divide three-digit numbers by single-digit numbers. They will be able to find fractions and percentages by division including using a calculator.

By the end of primary school, children will be calculating mentally when appropriate and will be able to use a range of written methods for those calculations which cannot be carried out mentally. You will expect them to use a calculator to solve multi-step problems and find fractions and percentages of whole-number quantities.



Extending learning in 'Calculating'

With these learners you can teach them to apply the commutative, associative and distributive laws that were described in Chapter 5, as well as using their understanding of inverses to calculate more efficiently. Look at these examples:

$$12 \times 13 = 156$$

$$13 \times 12 = 156$$

$$156 \div 12 = 13$$

$$156 \div 13 = 12$$

For example, multiplication and division are **inverse operations**; this means they are the opposites of each other. By knowing the answer to one of the above problems you can work out all the others.

The children will have developed their mental methods to include fractions and decimals and will be able to calculate percentage increases and decreases.

Big ideas

To introduce the big ideas in calculating try this activity:

Portfolio task 6.1

Work out $48 + 36$ and record your solution using a pencil and paper method.



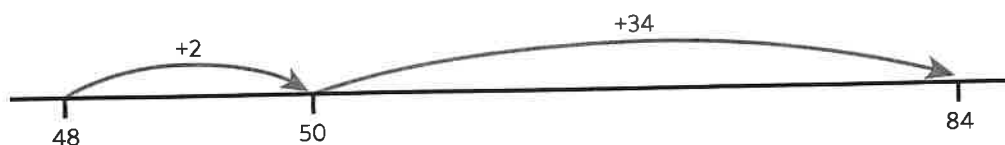
When I use this activity with teachers the most common response is

$$\begin{array}{r} 48 \\ + 36 \\ \hline 84 \end{array}$$

When I ask them 'how' they calculated the answer a common response is, 'I added 40 and 30 to give me 70 and then added on 14'. What is interesting here is that this calculation is not represented by the method they record. The written method records the question and the answer but does not 'represent' the method used. When I work with children they are more likely to use an empty number line to record the calculation. For example,



or



The big ideas in calculating are how children develop mental methods, which support them in understanding and developing formal written methods at an appropriate stage.

Children's development of mental methods

The reason why we encourage pupils to develop mental approaches to calculation is that it is much more efficient to work things out 'in our heads' if we can, rather than immediately reverting to a pencil and paper or a calculator. If we are fluent in mental methods we carry this confidence over into other areas of mathematics. The reason for developing rapid recall of number facts is to help us become fluent in mental calculation – not so that we can pass our weekly times-tables tests. It is interesting that the poet Ted Hughes and the

writer Jeanette Winterson both advocated the memorising of poems and other classic literature. Ted Hughes even put together poetry collections just for that purpose. They argue that having access to these memorised texts gives them easy access to a wealth of examples of great literature to support them in their writing. In the same way, having access to a wealth of number facts and key relationships between numbers means that we can quickly manipulate numbers in our heads. It is interesting that Ted Hughes describes how important it is to be able to draw on images in order to memorise text. This is just the same for number facts and mental calculations. The most appropriate strategy to support children in developing their mental methods is to use a wide range of images and models which children can draw on to enable them to 'see' calculations in their heads. Using number lines, number squares and empty number lines all support the development of mental methods.

Activities such as 'Guess my Number' or 'loop cards', discussed in the previous chapter, give you plenty of opportunity to ask individuals how they are calculating mentally and also support your pupils in developing effective mental methods. You should join in these activities too and make your own thinking overt; share it with your learners. In this way you will make sure that you develop your own mental capacity too.

Children's development of written methods

There has been a long-held belief that young children cannot carry out calculation activities until they reach what Piaget called 'concrete operational thinking' at around age 7. Martin Hughes, who carried out the research explored earlier in the chapter, challenged this view, suggesting that very young children can begin to explore the underlying processes of calculating. He developed a 'box task' – in this task he worked with children aged between 3 and 5. With some children he asked them to count a number of bricks into a box which he would then close – he would either add additional bricks to the box, or remove bricks, and ask the children how many were in the box now. The young children could understand and succeed on the task so long as the numbers were very small. When working with one child he counted five bricks into the box, then removed three and asked the child how many were left. The young boy told him 'two'. Martin Hughes then records the following conversation (see page 27 in his 1986 book *Children and Number*):

MH: I want to take three bricks out of the box now.

Richard: You can't can you?

MH: Why not?

Richard: You just have to put one in, don't you?

MH: Put one in?

Richard: Yeah, and then you can take three out.

This suggests that Richard was capable of carrying out two successive mental calculations before he was 5. Asking children to represent activities such as this early in their experience in school will help them see written methods as supportive of mental calculation rather than simply a record of the question and the answer. They should be given the opportunity to develop pictographic recording of their calculations when the activities are predominantly practical; they will move on to iconic representations during the early years, but it is unlikely that formal written methods will be helpful until Year 3.

The following sections of the chapter draw heavily on a series of articles written by Ian Thompson for the Association of Teachers of Mathematics (ATM). These appear in *Mathematics Teaching*, issues 202, 204, 206 and 208 and explore in detail an approach to calculation which has been drawn on by many effective schools. These are available from the ATM, which can be found on the internet at <http://www.atm.org.uk/>. The articles and many others are also available at Ian's own website www.ianthompson.pi.dsl.pipex.com.

Addition

An initial approach to written calculation is the empty number line which I introduced you to at the beginning of this 'Big ideas' section. The empty number line is a useful 'record' of a mental strategy as you can use it to support any addition or subtraction calculation.

The next stage which is introduced by the strategy is the use of 'partitioning'. So, for example,

$$39 + 52 = 39 + 50 + 2 = 89 + 2 = 91$$

or

$$39 + 52 = 30 + 50 + 9 + 2 = 80 + 11 = 91$$

Can you see that only the first example of partitioning builds on the use of an empty number line? If you draw a number line you can sketch the calculation on that line. The second example changes the order of the numbers so that an empty number line would not model the calculation, as you need to conserve the order of the calculation to sketch it in a number line.

In order to move towards formal written methods it is important to offer children practice in the second form of partitioning. This can then be represented in a more formal way as

$$\begin{array}{r} 39 = 30 + 9 \\ +52 \quad 50 + 2 \\ \hline 80 + 11 = 91 \end{array}$$

This leads children into the 'expanded column method' - this method would record the calculation above as either

$$\begin{array}{r} 39 \text{ (adding 10s first, that is } 30 + 50\text{)} \\ +52 \\ \hline 80 \\ 11 \\ \hline 91 \end{array}$$

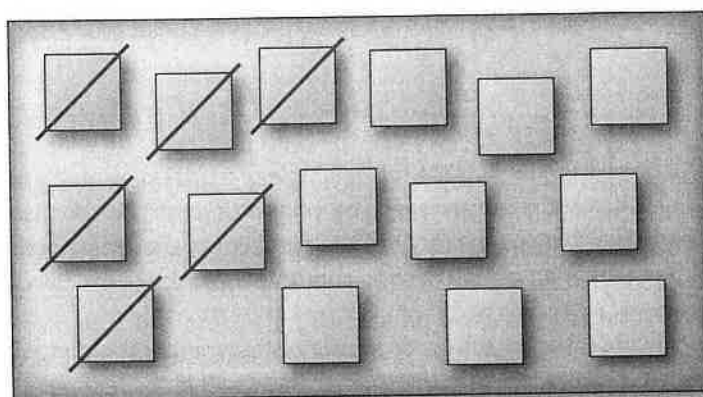
or

$$\begin{array}{r} 39 \text{ (adding 1s first, that is } 9 + 2\text{)} \\ +52 \\ \hline 11 \\ 80 \\ \hline 91 \end{array}$$

It is helpful to work your examples both ways so that your pupils can see there is no difference between adding 10s first or 1s first. The final strategy to be taught to children is the column method involving 'carrying' with the 'carry digits' recorded below the line. In order to make sure that children make the links back to place value it is suggested that they are encouraged to say 'carry 10' or 'carry 100' rather than carry 1' as may have become a habit.

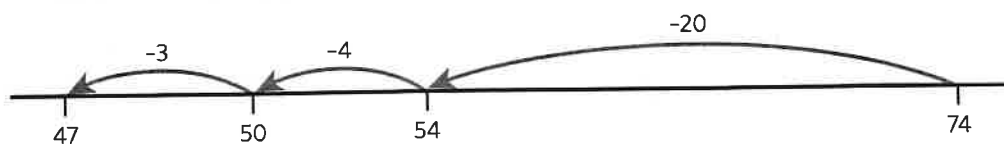
Subtraction

We can view subtraction in two ways: subtraction as take away or as difference. Sometimes children will model a subtraction calculation such as $15 - 6$ by placing 15 objects in a box, taking out 6 and then counting how many remain. This will give them 9. They may represent this by a jotting like this:

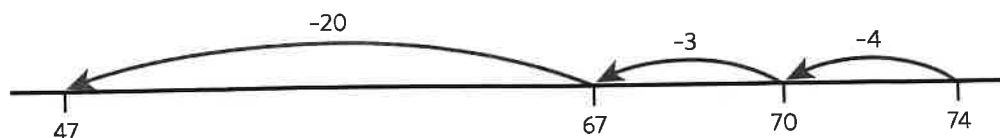


Subtractions as difference can be calculated on a number line. Here the pupil will start at 15 and count back 6 ending up on 9.

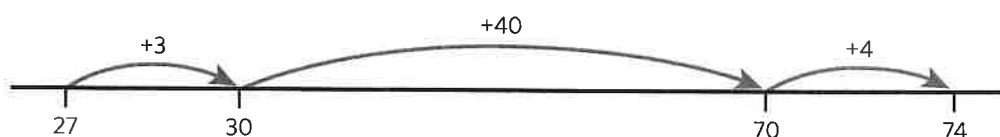
The stages for developing written methods to support subtraction also employ a progression through the empty line to partitioning before moving on to column methods. The first stage in the strategy involves counting back on an empty number line. So, for example, to calculate $74 - 27$ we might write:



or combine steps by initially subtracting 7 and then 20. It is important that children see the calculation can be carried out in any order:



An alternative is to begin with the 27, and ask the children how many they would have to count up to get to 74. This can be recorded on an empty number line as follows:



Ian Thompson suggests that if children can use the empty number line in this way there is no benefit in using partitioning to support subtraction. In fact children are effectively partitioning if they are using the empty number line efficiently.

The final stage in moving towards a formal algorithm is the expanded layout. An example of this is $653 - 289$:

$$\begin{array}{r} 600 + 50 + 3 \\ -200 + 80 + 9 \\ \hline 300 + 60 + 4 \end{array} \qquad \begin{array}{r} 500 + 140 + 13 \\ -200 + 80 + 9 \\ \hline 300 + 60 + 4 \end{array}$$

In moving from one calculation to the other I have partitioned:

$$\begin{array}{l} 600 = 500 + 100 \\ 50 = 40 + 10 \\ 3 = 3 \end{array}$$

So 653 can be written as $500 + 140 + 13$.

Taking it further



In his article 'The same difference' in *Mathematics Teaching* 202, published by the ATM in May 2007, Ian Sugarman describes a mental strategy for subtraction developed by children he was working with which does not appear in the National Strategy. He argues that this method is more appropriate for carrying out subtraction calculations than the decomposition method described by the strategy. He worked with Year 4, 5 and 6 children and the activity he used with them involved transforming numbers that would be difficult to work with using the column method to a pair of numbers that could be subtracted mentally. So, for example, rather than trying to calculate $563 - 278$ the pupils transformed this to $565 - 280$ and then $585 - 300$ by initially adding 2 to each number and then another 20. The children described this as the 'same difference' method. They realised that if you add the same number to each number in the calculation, the 'difference' remains the same, and so the answer to the calculation is unaltered. This can be extended to four-digit numbers: for example,

$$\begin{array}{l} 2038 - 1757 \\ 2040 - 1759 \text{ (adding 2)} \\ 2000 - 1719 \text{ (subtracting 40)} \\ 2081 - 1800 \text{ (adding 81)} \\ = 281 \end{array}$$

Multiplication

Ian Thompson suggests it is important to let children develop their own informal methods to record multiplication as with addition and subtraction. The first formal stage therefore would be the grid method. This approach is supported by the 2011 Ofsted report mentioned earlier in this chapter. The schools which were visited, all of which were successful in supporting their pupils in becoming effective at 'calculating', used the 'grid' method to introduce the pupils to the ideas of 'long multiplication'. It is worth spending some time becoming secure with this method yourself so that you are able to introduce it confidently to your learners. It will also help you see why the more traditional long multiplication method 'works'.

You record the grid method as follows. We would write 38×7 as

$\times$	7
30	210
8	56
	266

The next step would be recording this as:

$$\begin{array}{r}
 38 \\
 \times 7 \\
 \hline
 210 \\
 56 \\
 \hline
 266
 \end{array}$$

because it develops naturally from mental methods and the grid method shown above. The grid method can be extended to multiply a two-digit number by a two-digit number. For example,

$\times$	20	9	Total
30	600	180	780
7	140	42	182
			962

It is important to remember that by the end of primary school the most complex calculation that pupils would be expected to carry out is a two-digit by three-digit one. There are three ways to carry out this calculation. Firstly, consider:

$$286 \times 29$$

$\times$	20	9	
200	4000	1800	5800
80	1600	720	2320
6	120	54	174
			8294

or writing the same calculation in columns

286×29		
	200×20	4,000
	80×20	1,600
	6×20	120
	200×9	1,800
	80×9	720
	6×9	54
		8294

or finally

$$\begin{array}{r}
 286 \\
 \times 29 \\
 \hline
 5720 \quad (286 \times 20) \\
 2574 \quad (286 \times 9) \\
 \hline
 8294
 \end{array}$$

Division

When I talk to people about why they do not feel confident about teaching mathematics they often describe 'long division' as a hated memory. This is not something that many pupils should be exploring before Year 6. It is interesting that not a single school that Ofsted identified as being effective in teaching calculation expected their pupils to use the long-division algorithm which you probably remember.

It is important to allow children to explore mental methods and also their understanding of division as the inverse of multiplication. It is also important not to rush into any formal recording of division. The written algorithm or rule for short division will be recognisable as:

$$\begin{array}{r}
 27 \\
 3 \overline{)821}
 \end{array}$$

I still hear children in schools, or my students, describing this process in the following way: '81 divided by 3. So 8 divided by 3 goes 2 carry 2; 3 into 21 goes 7. So the answer is 27.' The problem with this form of calculation is that the actual place value of the numbers is forgotten: 80 becomes 8 and 20 becomes 2. This can lead to children not noticing if answers are incorrect as they do not return to think if 27 is a sensible answer to 81 divided by 3. Estimating first can help this process. I would expect the answer to be less than 90 because 90 divided by 3 is 30. I would also expect it to be more than 25 as 75 divided by 3 is 25. The answer should therefore be between 25 and 30, so 27 seems about right!

Towards the end of their time in primary school you can introduce your pupils to the short division method – for most children this will be early in Year 6. This is the method you will probably be familiar with. So, to calculate $291 \div 3$ we can write:

$$\begin{aligned} 291 \div 3 &= (270 + 21) \div 3 \\ &= (270 \div 3) + (21 \div 3) \\ &= 90 + 7 \\ &= 97 \end{aligned}$$

This same calculation can be shortened to:

$$\begin{array}{r} 97 \\ 3 \overline{)291} \end{array}$$

Teaching points

Teaching point 1: Misapplying algorithms

I often tell teachers that children very rarely make mistakes – they can come up with an answer that doesn't seem quite right but when you ask them what they have done, they have answered a question correctly – it just isn't the question you have asked them. For example, Sam, aged, 9 had been asked to calculate:

$$\begin{array}{r} 85 \\ -48 \\ \hline \end{array}$$

and gave the answer 43. He was irate when his teacher suggested this was 'wrong'. 'It isn't,' he said, 'I took away 8 from 5 by "finding the difference" so I put 3 down and 80 take away 40 is 40 so that gives me 43.' Whenever you are asking children to calculate using a formal algorithm it is useful to have them 'talk it through', sometimes at the front so that the rest of the group can see the approach, and sometimes in a one-to-one situation with you. This way you will begin to understand whether your learners understand the mathematics behind the algorithms or whether they are simply following, or misapplying, a rule. This is where having secure mental methods is helpful: this can help the pupils see when they have misapplied an algorithm, as they will have a good sense of what the answer should be. In this case Sam

could work out that the answer should be 37. (He counted on, $48 + 30$ is 78, $78 + 2$ is 80, and another 5 is 85. So the answer is $30 + 5 + 2 = 37$.)

Teaching point 2: Not checking calculations

We all remember putting our hands up to tell a teacher we had finished our calculations, or completed a test paper, only to be told by the teacher to check our answers carefully. Be honest with yourself – did you always check carefully? It is important to try to get children into the habit of checking as a first step. So when Helen came up to the board and wrote:

$$3 \times 142 = 126$$

I said, 'I don't know what the answer is but I know that it isn't 126.' She looked at me and then said, 'Oh no – it should be more than 300, shouldn't it?' She was then able to find her mistake and carry out the calculation. Sometimes when pupils are following algorithms they lose track of the numbers they are working with and so forget to estimate and use this estimate as a check. By encouraging pupils to estimate as a first step you will help them get into this habit.

In the activity described in the following Resource inspiration box the pupils are being encouraged to look for errors by drawing on their knowledge of odd and even numbers. It is important that the pupils are not asked to carry out the calculation.

Resource inspiration



Checking calculations

① Will the results of these calculations be odd or even?

a $4861 + 3758$

b $7052 - 507$

c 34×57

d $711 - 296$

e 72×28

f $461 + 836$

② Which of these are incorrect? Use rules for odd and even numbers to check.

a $30 \times 91 = 2730$

b $9072 - 978 = 8272$

c $826 + 7095 = 7922$

d $68 \times 47 = 3196$

③ Which of these are incorrect? Use rules for odd and even numbers to check.

a $76 + 128 = 204$

b $241 - 129 = 111$

c $97 - 38 = 58$

d $50 + 503 = 553$

④ Will the results of these calculations be odd or even?

a $419 + 282$

b $174 - 86$

c $347 + 63$

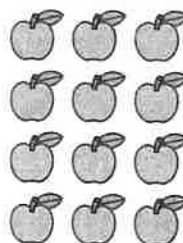
d $70 - 47$

Teaching point 3: Not understanding written symbolism

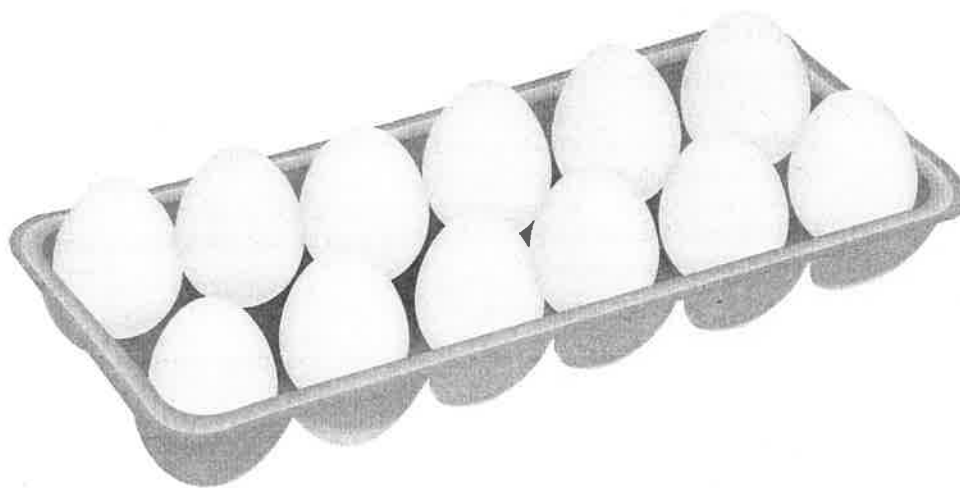
The research by Martin Hughes described in some detail earlier in the chapter suggests that often children's errors in calculations are not because they do not understand the mathematical concepts, rather they get confused with how the written symbolism relates to the calculations they are carrying out. It is a problem of 'translation' rather than a problem of mathematics.

The best way to support pupils in making this translation is to work with the pupils as they say number sentences and to write it down for them. In this way they will begin to see the + sign as simply shorthand for saying 'add'.

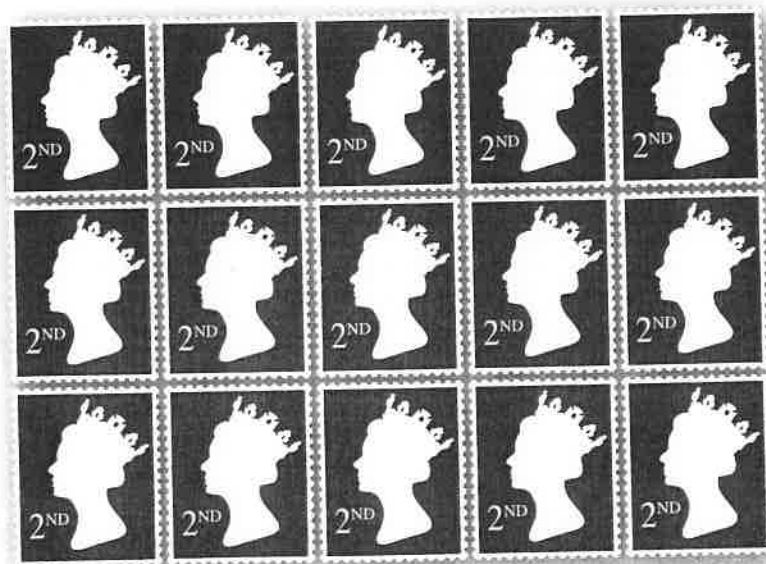
Other activities which can help children begin to understand the use of the multiplication sign '×' is to introduce them to **arrays**. An array is simply an 'orderly arrangement of objects'. Arrays can help us understand multiplication by lining up objects in rows and columns. So, for example, 4×3 can be shown as



When I look at a box of six eggs I can see both 6 and 3×2 . Similarly for a box of a dozen eggs I see 12 and 2×6 .



If I buy a book of stamps arranged in the way they are below I see both 15 and 3×5 .



This sense of multiplication as an array helps me both with multiplication and with rapid recall of number facts. It is worth drawing on the many examples of arrays that your pupils will see around them every day: the milk bottles in the crate; desks if you organise them in rows on occasions; lining up in pairs. Making physical arrays is a great activity. Get a group of 12 of your class, or 24, and see how many different arrays they can form. Ask the pupils to make sketches of these arrays.

Teaching point 4: Carrying out the wrong operation when solving number problems

I recently picked up a greeting card from my local farm shop. It is headed 'Rural Maths' and contains the following text:

'Jack,' said the teacher, 'you have 30 sheep in your field, two escape through a hole in the wall, how many are left?'

'None,' Jack replied instantly.

'No Jack, it's 28!' the teacher replied, sounding cross, 'I thought you knew your maths!'

'Yes Miss, I know my maths alright, it's you that doesn't know sheep.'

This is a great example of a child solving a mathematics problem in their 'actual', real world rather than the pseudo-reality of a maths problem. Children who are successful solvers of word problems realise that these are very rarely 'real' problems but are mathematics problems set in a context. The first skill for your learners to develop is to decode such problems so that they apply the correct operations to solve the problem and then check the answer. Many teachers use the acronym RUCSAC to help their learners solve such problems. This acronym stands for:

R - **Read** the question carefully

U - **Understand** the question - what do you have to find out?

C - **Choose** the correct operations to solve the problem

S - **Solve** the problem

A - Find the **Answer**

C - **Check** your answer - is it sensible?

It is helpful just to focus on a series of questions without solving them to teach your pupils to select the operation which will solve the problem. Spend some time simply underlining key pieces of information in questions in order to help pupils see through all the extraneous information that is contained in word problems. Focusing on selecting the correct operation without having to carry out the calculation allows your pupils to see the importance of this step.

Another useful classroom activity is to provide your pupils with a sketch or an image from which they can devise their own word problems. For example, an image of a pot containing six pencils with two pens lying by the side of the pot will allow learners in the early years to devise several different word problems. The answers might be $6 + 2 = 8$ or $8 - 2 = 6$ or even $2 + 2 = 4$. (There are two pencils outside my pot. I take another two out, how many are outside the pot now?) If you visit <http://www.oxfam.org.uk/education> you will find many images illustrating global issues which can be used for your learners to devise number problems with the added bonus of developing their global awareness.

Portfolio task 6.2**The grid method**

Calculate the following using the grid method. Before you carry out the calculation estimate the answer. Which two numbers do you think the answer should be between?

$$358 \times 34$$

	×	30	4	
300				
50				
8				

Then use the expanded method shown on page 115.

In practice

The following lesson plan and evaluation describe a lesson taught to a group of pupils who have been introduced to pencil and paper procedures. The particular focus was developing skills in multiplying and dividing using mental methods to support their use of pencil and paper methods for more complex procedures. The program 'Grid Multiplication' was used to introduce the idea of multiplying by partitioning to the whole class. This program is available on the companion website to this book.

Topic: Multiplying two-digit numbers

Age group: Upper primary

Objectives

Develop and use written methods to record, support and explain multiplication of two-digit numbers by one-digit numbers

Key vocabulary	Context	
Digit, multiply, product, estimate	This is the second lesson exploring the grid method for multiplication. The group worked on differentiated examples using the grid method in the last lesson. The children who needed extra support found the concept of the grid difficult although they were able to carry out the individual calculations	
Resources	Starter activity	
Set of 1-9 digit cards	Working in pairs use any operation with one two-digit number and a one-digit number to make 100	
Main activity	Teacher	Pupil activity
	Use the 'Grid Multiplication' program to explore multiplying through partitioning. Use the examples 13×6 ; 18×4 ; 24×7 and 32×5 . Partition using the 10s. So $24 \times 7 = 20 \times 7 + 4 \times 7$ and so on	Before carrying out the calculation ask pairs to estimate to the nearest 10 using individual whiteboards
Group activity Children operating above expected levels	This group can work with four digits and carry out $TU \times TU$ as well as $HTU \times U$	Pupils should estimate before carrying out the calculation. They should try to explain why a particular arrangement of digits gives the largest answer, drawing on their understanding of place value
Group activity Children operating at expected levels	The group should pick any three cards from the set of 1-9 digit cards and write down all the multiplication calculations that can be formed from these numbers. Working in pairs within the group they should carry out these calculations and see which gives the largest product	Pupils should estimate before carrying out the calculation. They should try to explain why a particular arrangement of digits gives the largest answer, drawing on their understanding of place value
Group activity Children operating below expected levels	This group should be supported by the teacher. The group should just work on two-digit multiplied by one-digit numbers. The teacher should model each calculation on a whiteboard	Estimate which calculation will give the smallest answer and which will give the largest. Pupils should 'tell' the teacher how to proceed throughout the calculation
Plenary		
Each group feed back one of their calculations - the core group and the extension group explain why the particular arrangement of digits gives the largest answer		

Rationale and evaluation

My aim was that, by presenting this as an investigation into which arrangement of digits would give the largest product, I would encourage the pupils to estimate first. This worked very well – it also encouraged them into checking as they went through the grid multiplication. The extension and the core groups didn't share out the different multiplications as I suggested – rather they took it in turns to calculate while the others in the group watched to see what the answer would be. This was actually a useful way of organising the activity as those who weren't carrying out the calculation were checking as they went along.

I worked alongside the support group so that I could model the process for them carefully. This group still aren't making the connection between the grid method and the numbers they are working with. So, for example, Parmjit carried out the calculation

$$23 \times 4$$

	×		4	
	2			
	3			

and Jay wanted to put a '2' instead of '20' and was quite happy with the answer 20 as a result: $(4 \times 2) + (4 \times 3) = 20$. This was despite the group agreeing that the answer should be bigger than 80. As a result I have suggested that she uses the 'Grid Multiplication' program with this group in the next lesson so that they can have an image of how partitioning works to support the grid method. I am also going to focus on multiplication of multiples of 10 by single-digit numbers with this group before coming back to the grid multiplication with them.

Troy seems to be understanding how the grid method can help, though – in the plenary he showed everyone how you can write it 'without the grid'. He wrote:

$$\begin{array}{r}
 27 \\
 \times 4 \\
 \hline
 80 \text{ (saying 'that's 4 times 20')} \\
 28 \text{ ('and that's 4 times 7')} \\
 \hline
 108 \text{ (so that's all together which is right cause it's a bit more than 100 and 4 times 25} \\
 \text{would be 100')}
 \end{array}$$

Observing 'Calculating'

On the companion website find the clip called 'How Many Peas Fill The Classroom'. This is a fascinating activity in which the teacher works with her class to try to calculate how many peas it would take to fill the classroom. As well as developing and practising children's calculation skills it also develops the idea of very large numbers in a real context. Children

often find it extremely difficult to visualise very large numbers. The task also requires the pupils to break down a complex problem into small steps.

Although this clip is from a Year 7 group I have carried out very similar activities with learners in primary schools. Focus particularly on the segment from four minutes onwards, which focuses on the teacher taking feedback from the groups. What might you do differently as a result of observing this section of the lesson? One concern might be that the large numbers they were working with did not carry very much meaning to the children as they didn't break the task down into chunks which they could visualise. Perhaps dividing the classroom up into more manageable 'chunks' to start with would have allowed the learners to make more sense of the answers they were coming up with.

Imagine you are feeding back to the teacher. What are three things that you feel were strengths in the lesson, and what is one key thing that you would suggest the teacher does differently? How would you adapt this activity for the group that you are currently working with?

Assessing 'Calculating'

The 2011 Ofsted report mentioned earlier in this chapter describes 'Problem Solving' as at the heart of learning arithmetic. This supports the argument presented to you in Chapter 3 and suggests that the ability to solve problems should also be central to assessing your pupils' calculating skills. This can be carried out in a wide range of contexts. It is helpful if these contexts are genuine 'real life' contexts rather than the pseudo-reality I referred to earlier in the chapter. Examples that you can use could include:

- Calculating the best value purchase. Bring in a range of cereals, or drinks in different sized containers. The pupils have to calculate which size offers the best value.
- Calculating the costs to redecorate the classroom. Include the costs of paint for the walls and carpet for the floor.
- Calculating costs of fruit for younger pupils. Make sure you let the pupils devise their own ways of recording their calculations at this stage. It is likely that these learners will begin with repeated addition and will sketch the fruit they want to buy and 'attach' a price to it.
- Ask very young learners to work out how many raisins there are in a packet. Observe how they group the raisins. Some may choose to count in 2s or 3s or even 5s.

Cross-curricular teaching of 'Calculating'

The idea of the extended project is to draw on the key ideas within the chapter to develop a cross-curricular project which you can explore with your learners over a series of lessons. This allows you and your class to develop your subject knowledge together. The examples given above in the assessment section all offer cross-curricular routes into calculating. I have found that developing the assessment activity around redecorating the classroom is a very fruitful cross-curricular activity which involves all learners in developing their calculation skills.

You need to begin this investigation in an open space. Ask 24 pupils to come to the centre of the space. The rest of the pupils stand in pairs. They take it in turns to tell the 24 pupils how to arrange themselves in an array, for example 2×12 or 3×8 . Keep going until you have exhausted all the possibilities. Repeat for a range of other numbers – some that give a large number of possibilities and some that only offer one.

Return to the classroom and ask the pupils to explore how many different 'arrays' you can make for each number from 1 to 36. At this stage you can ask pupils to identify other arrays that they notice around them, such as carpet tiles on a floor, or panes in a window.

This allows you to begin to think through different ways of organising the classroom. How many ways could the desks be arranged – what is appropriate in the space, and which arrangements work for which parts of the curriculum? Depending on the time you have you can develop scale models in either 2D or 3D.

Summary

The key focus for this chapter is the link between the mental methods we employ as a first choice and how these can be formalised into written algorithms when the calculations become too complex for us to hold in our head. You have read how the four operations link together and how you can use your knowledge of one of the operations to support calculations using another operation. Our aim for children leaving school is that they have a good understanding of the four operations and that they can carry out calculations mentally, when possible, and can employ a range of written methods, making sensible choices depending on the calculation they are carrying out.

I have emphasised the importance of children approaching calculation practically and through activity – this allows you to support them in developing their vocabulary of calculation. They should also be encouraged to develop their own informal records for calculations. The key to understanding the more formal algorithms is to build carefully on previous understanding so that your pupils understand the reasons why the algorithms 'work' and don't just apply them unthinkingly. Most of the misconceptions come from losing sight of the value of the numbers in the calculation so that children don't notice they have an answer that doesn't make sense.

Work your way carefully through the progression of written methods – the grid method may be an idea that you are not used to. If the written methods make sense to you, you will be able to share these with your learners.

Reflections on this chapter

My guess would be that many of you have found this chapter the most challenging in the book – not because the ideas are more complex but because the methods of calculation may well be different from those that you were taught. My hope is that you can see the rationale for teaching these methods of calculation and that pupils will be able to see

why the methods are effective, and will begin to move away from simply trying to remember how you carry out a calculation to understanding why a 'rule' works. If you understand how the calculation is carried out you can make effective decisions about which method to use, and are also more likely to notice when you have made a mistake.

I also hope that you have become able to notice the choices that you are making when you carry out calculations, whether mentally or on paper. In a sense you have 'learnt' the algorithms for the four operations, noticing, maybe for the first time, why you carry out calculations in a particular way. This process of revisiting your own learning should support you in explaining 'Calculating' to your learners.



Self-audit

Carry out this audit to explore how your learning has progressed as a result of working on the ideas in this chapter. Include the results in your portfolio.

1 Make a copy of the following table:

Just know it	Need to think a bit	Need to work it out on paper
--------------	---------------------	------------------------------

Using the digits 1, 3, 5, 7, 8, 9 make up a range of addition and subtraction calculations. Make sure you have some calculations that you would place in each column. Write the calculations in the appropriate column, together with a rationale for your choice.

2 Set yourself the following questions:

A column addition involving decimals.

An addition involving decimals you would calculate mentally.

A column subtraction involving decimals.

A subtraction involving decimals you would do mentally.

A short multiplication involving decimals you could do mentally.

A multiplication involving decimals you would do using pencil and paper.

A short division involving decimals you could do mentally.

A division involving decimals you would do using pencil and paper.

3 Spot the mistake: these three long multiplication calculations have got mistakes in them. Write down the mistake that the pupils have made, then complete the long multiplication correctly.

$$\begin{array}{r} 135 \\ \times 62 \\ \hline 270 \\ 710 \\ \hline 270 \end{array}$$

$$\begin{array}{r} 270 \\ \times 30 \\ \hline 710 \end{array}$$

$$\begin{array}{r} 452 \\ \times 18 \\ \hline 324016 \\ 452 \\ \hline 324468 \end{array}$$

Going further

Issues in Teaching Numeracy by Ian Thompson

This book contains a very useful section on calculation which outlines progression in written and mental calculation in much greater detail than there was space for in this chapter. There is also a very helpful chapter on using the empty number line which you will find invaluable in supporting your learners in developing mental methods.

Thompson, I. (2010) *Issues in Teaching Numeracy in Primary Schools*. Buckingham: Open University Press.

Teaching and Learning Early Number by Ian Thompson

This book is the forerunner of the text above. It is important for all teachers, not just those who teach learners in the early years, as he outlines in detail the ways in which early learners begin the progress of learning to calculate. These two books will expand the research base on which you can draw to develop your practice as well as offering lots of practical approaches to learning and teaching calculation skills.

Thompson, I. (2008) *Teaching and Learning Early Number*. Buckingham: Open University Press.

Good Practice in Primary Mathematics: Evidence from 20 Successful Schools by Ofsted

This offers useful small-scale case studies on how to teach calculation. There are several vignettes from classrooms which are enlightening as well as evidence from children. This is particularly helpful in seeing how learners can develop their own written methods for calculating.

Ofsted (2011) *Good Practice in Primary Mathematics: Evidence from 20 Successful Schools*. Manchester: Ofsted.