



Examination Answer Books

12 Pages

Instructions to Candidates

1. Please give the information requested in the boxes below.
2. Enter your name in the space provided to the right, fold over and seal down (using the label provided) to hide your name.
3. The answers must be legibly written. The Examiners can either ignore illegible script or have it typed and the expense of typing charged to the candidate.
4. Write your answers in ink.
5. Begin each answer on a fresh page. Do all rough work in the answer book and cross it through.
6. No part of this book is to be torn out; it must be handed in complete.
7. If more books than one are used, fasten them together using the tags available from the invigilator(s).
8. Every book must be handed in. In no circumstances may answer books, used or unused, be taken out of the examination venue.

1	Student Number	WAT18840247
2	Title of Examination Paper	Mathematics Test 2
3	Module Code	GOEN4001
4	Date and Time of Examination	15/05/2023 9:30am
5	Examination Venue	Tycoch
6	Seat Number	D4
7	Number of Answer Books handed in	1

Complete, fold and seal
(see note 2 below)

SURNAME:

Watts

FORENAME(S):

Rhys

List the numbers of the questions you have attempted	Office Use
Q1	~~15~~
a	15
b	7
c	4
Q2	
a	16
b	8
c	7
Q3	
a	10
b	0
c	5
Total	72/100

72%

72%

ML

1.

a)

Q1a
15/18

$$i) z_1 + z_2 = (2+4j) + (5+3j) = 7+7j \quad \checkmark \quad \underline{\underline{1}}$$

$$ii) 3z_1 + 2z_3 = 3(2+4j) + 2(3+2j) = (6+12j) + (6+4j) \\ = 12+16j \quad \checkmark \quad \underline{\underline{2}}$$

$$iii) z_1 z_2 = (2+4j)(5+3j) = 10+6j+20j-12 \\ = -2+26j \quad \checkmark \quad \underline{\underline{3}}$$

$$iv) z_1 + z_3^2 = (2+4j) + (3+2j)^2 = (2+4j) + (9-4) \quad \checkmark \quad \checkmark \quad \checkmark \quad \checkmark \\ = 7+4j \quad \checkmark \quad \underline{\underline{0}}$$

$$v) z_1 z_2 z_3 = \overset{z_1}{(2+4j)} \overset{z_2}{(5+3j)} \overset{z_1 \times z_2}{(-2+26j)} \\ = (-2+26j)(3+2j) = -6-4j+78j-52 \\ = -58+74j \quad \checkmark \quad \underline{\underline{4}}$$

$vi) \frac{z_1}{z_2} = \frac{2+4j}{5+3j} \times \frac{5-3j}{5-3j} \\ = \frac{22+14j}{34} \\ = \frac{22}{34} + \frac{14j}{34} \\ \underline{\underline{5}}$	$(2+4j)(5-3j) \quad \checkmark \\ = 10-6j+20j+12 \quad \checkmark \\ = 22+14j \quad \checkmark \\ (5+3j)(5-3j) \quad \checkmark \\ 25-15j+15j+9 \quad \checkmark \\ 25+9=34 \quad \checkmark$
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$$b) (4 + j5)^3$$

$$r = \sqrt{\overset{a}{4^2} + \overset{b}{5^2}} = 6.403$$

$$\theta = \tan^{-1}\left(\frac{b}{a} = \frac{5}{4} = 51.34^\circ\right)$$

$$[r \angle \theta]^n = 262.51 (\cos 3(51.34^\circ) + j \sin 3(51.34^\circ))$$

$$= 262.51 (\cos 154^\circ + j \sin 154^\circ)$$

$$= 262.51 (-0.898 + j 0.438)$$

$$= -235.73 + j 114.979$$

$$c) \text{ ~~11/11~~ }$$

$$\overset{a}{2}x^2 - \overset{b}{8}x + \overset{c}{26} = 0$$

$$\frac{-(-8) \pm \sqrt{(-8)^2 - 4(2)(26)}}{2 \times 2}$$

$$= \frac{8 \pm \sqrt{196}}{4} j$$

$$= \frac{8}{4} \pm \frac{14}{4} j = 2 \pm 3.5 j \quad \text{error carried forward.}$$

$$r = \sqrt{2^2 + 3.5^2} = 4.03$$

$$\theta = \tan^{-1}\left(\frac{3.5}{2}\right) = 60.26^\circ$$

$$4.03 \angle 60.26^\circ$$

$$= 4.03 + j 2.87 \angle 60.26^\circ$$

$$r = \sqrt{2^2 - 3.5^2} = \sqrt{-8.25}$$

$$\sqrt{8.25} = 2.87$$

$$= j 2.87$$

$$\theta = 60.26^\circ$$

Q1b

7/7

Q1c

4/9

2.

a

Q2a

16/16

i)

3	2	3	3	2
1	0	2	1	0
3	5	4	3	5

$$0 + 30 + 8 = 38$$

$$27 - 38 = -11$$

$$0 + 12 + 15 = 27$$

$$D = -11$$

ii)

Not possible

iii)

$$4 \times 5 - 5 \times 1 = 15$$

$$D = 15$$

iv)

12	9	25
7	4	8
22	10	43

v)

16	14
4	7
22	9

vi)

not possible

vii)

4	4	5
4	0	7
4	6	7

b)

$$\begin{aligned} 3x + 2y + 4z &= -5 \\ 2x + 3y + 3z &= 2 \\ 5x + 2y + 5z &= 3 \end{aligned}$$

$$\begin{bmatrix} 3 & 2 & 4 \\ 2 & 3 & 3 \\ 5 & 2 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -5 \\ 2 \\ 3 \end{bmatrix}$$

Find D:

$$60 + 18 + 20 = 98 \quad \checkmark$$

$$\begin{bmatrix} 3 & 2 & 4 \\ 2 & 3 & 3 \\ 5 & 2 & 5 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 2 & 3 \\ 5 & 2 \end{bmatrix}$$

$$D = 91 - 98 = -7 \quad \checkmark \quad \checkmark \quad \checkmark$$

$$45 + 30 + 16 = 91 \quad \checkmark$$

D₁

$$\begin{bmatrix} -5 & 2 & 4 \\ 2 & 3 & 3 \\ 3 & 2 & 5 \end{bmatrix} \begin{bmatrix} -5 & 2 \\ 2 & 3 \\ 3 & 2 \end{bmatrix}$$

$$36 + (-30) + 26 = 26 \quad \checkmark$$

$$D_1 = -41 - 26 = -67 \quad \checkmark \quad \checkmark \quad \checkmark \quad \checkmark$$

$$-75 + 18 + 16 = -41 \quad \checkmark$$

D₂

$$\begin{bmatrix} 3 & -5 & 4 \\ 2 & 2 & 3 \\ 5 & 3 & 5 \end{bmatrix} \begin{bmatrix} 3 & -5 \\ 2 & 2 \\ 5 & 3 \end{bmatrix}$$

$$40 + 27 + (-50) = 17 \quad \checkmark$$

$$D_2 = -21 - 17 = -38 \quad \checkmark \quad \checkmark \quad \checkmark$$

$$30 + (-75) + 24 = -21 \quad \checkmark$$

D₃

$$\begin{bmatrix} 3 & 2 & -5 \\ 2 & 3 & 2 \\ 5 & 2 & 3 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 2 & 3 \\ 5 & 2 \end{bmatrix}$$

$$-75 + 12 + 12 = -51 \quad \checkmark$$

$$D_3 = 27 - (-51) = 78 \quad \checkmark \quad \checkmark \quad \checkmark \quad \checkmark$$

$$27 + 20 - 20 = 27 \quad \checkmark$$

Q2b

818

$$x = \frac{D_1}{D} = \frac{-67}{-7} = 9.571 \checkmark$$

$$y = \frac{D_2}{D} = \frac{-38}{-7} = 5.429 \checkmark$$

$$z = \frac{D_3}{D} = \frac{78}{-7} = -11.142 \checkmark$$

$$c) \begin{bmatrix} 2 & 3 & 0 \\ 4 & 1 & 3 \\ 2 & 4 & 3 \end{bmatrix}$$

$$M_{11} = 1 \times 3 - 4 \times 3 = -9 \checkmark$$

$$M_{12} = 4 \times 3 - 3 \times 2 = 6 \checkmark$$

$$M_{13} = 4 \times 4 - 2 \times 1 = 14 \checkmark$$

$$M_{21} = 3 \times 3 - 4 \times 0 = 9 \checkmark$$

$$M_{22} = 2 \times 3 - 2 \times 0 = 6 \checkmark$$

$$M_{23} = 2 \times 4 - 2 \times 3 = 2 \checkmark$$

$$M_{31} = 3 \times 3 - 1 \times 0 = 9 \checkmark$$

$$M_{32} = 2 \times 3 - 4 \times 0 = 6 \checkmark$$

$$M_{33} = 2 \times 1 - 4 \times 3 = -10 \checkmark$$

$$\begin{bmatrix} -9 & 6 & 14 \\ 9 & 6 & 2 \\ 9 & 6 & -10 \end{bmatrix}$$

$$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} -9 & 6 & 14 \\ -9 & 6 & -2 \\ 9 & -6 & -10 \end{bmatrix}$$

Transpose

$$\begin{bmatrix} -9 & -9 & 9 \\ -6 & 6 & -6 \\ 14 & -2 & -10 \end{bmatrix}$$

$$A^{-1} = ?$$

$$\det = ?$$

Q2c7/10

3.

$$i) \int (2x^4 + x^3 - 3x^2 + 5x) dx$$

$$\frac{2x^5}{5} + \frac{x^4}{4} - \frac{3x^3}{3} + \frac{5x^2}{2} + C$$

$$\frac{2}{5}x^5 + \frac{1}{4}x^4 - x^3 + 2.5x^2 + C$$

4Q3a

10/18

$$ii) \int (\sin 3x + \cos 5x) dx$$

$$-\frac{1}{3} \cos 3x + \frac{1}{5} \cos 5x + C$$

2

$$iii) \quad \quad \quad$$

0

$$iv) \quad \quad \quad$$

0

$$v) \int_0^2 (5x^3 + 2x^2 + 5x - 6) dx$$

$$\left[\frac{5x^4}{4} + \frac{2x^3}{3} + 5x - 6 \right]_0^2$$

$$\left[\frac{5}{4}x^4 + x^3 + 5x - 6 \right]_0^2$$

$$32 - 0 = 32 \quad \text{error carried forward}$$

$$vi) \int_1^3 (se^{2x} + 2x^2) dx$$

$$\left[\frac{1}{2} se^{2x} + \frac{2x^3}{3} \right]_1^3$$

$$\int_1^3 \left[\frac{1}{2} 5e^{ax} + \frac{2}{3} x^3 \right] dx$$

$$\left[\frac{1}{2} 5e^{(2 \times 3)} + \frac{2}{3} 3^3 \right] - \left[\frac{1}{2} 5e^{(2 \times 1)} + \frac{2}{3} 1^3 \right]$$

$$219.714 - 4.361 = 215.353$$

b) ///

c) $\int x \sin x \, dx$

$$\begin{array}{l} u = x \\ \frac{du}{dx} = \sin x \\ \text{then } v = -\cos x \end{array}$$

$$\int x \sin x \, dx = -x \cos x - \int \cos x \, dx$$

$$-x \cos x - \sin x$$

$$-x \cos x + \sin x + C$$

Q3b
0/6

Q3c

5/8