



## EXAMINATION SOLUTIONS

(to accompany the examination paper sent to the external examiner  
but NOT to be seen by students)

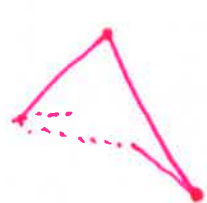
|            |                                           |        |   |
|------------|-------------------------------------------|--------|---|
| Programme: | BSc/HND Motorsport/Automotive Engineering | Level: | 5 |
| Module:    | Mechanical Principles                     |        |   |
| Author:    | T.Tudor                                   | Page:  | 1 |

| Q  | Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | Marks |
|----|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|
| Q1 | <p> <math>P = 20000 \text{ W}</math><br/> <math>N = 4000 \text{ RPM}</math><br/> <math>\mu = 0.25</math><br/> <math>d_i = 0.08 \text{ m}</math><br/> <math>d_o = 0.16 \text{ m}</math><br/> <math>W = 600 \text{ N}</math> </p> <p> <math display="block">a) T = \frac{60P}{2\pi N} = \frac{60 \times 20000}{2\pi \times 4000} = 47.75 \text{ Nm}</math> </p> <p> <math display="block">T = \frac{2}{3} \mu W \frac{(r_2^3 - r_1^3)}{(r_2^2 - r_1^2)} \times 2n</math> </p> <p> <math display="block">n = \frac{3T (r_2^2 - r_1^2)}{4\mu W (r_2^3 - r_1^3)} = \frac{3 \times 47.75 \times (0.16^2 - 0.08^2)}{4 \times 0.25 \times 600 \times (0.16^3 - 0.08^3)}</math> </p> <p> <math>n = 1.28 \quad \therefore 2 \text{ plates}</math> </p> <p> <math display="block">b) T = \frac{(r_2 + r_1) \mu W}{2} \times 2n</math> </p> <p> <math display="block">n = \frac{T}{(r_2 + r_1) \mu W} = \frac{47.75}{(0.16 + 0.08) \times 0.25 \times 600} = 1.33 \quad \therefore 2 \text{ plates.}</math> </p> |       |



| Q  | Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | Marks |
|----|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|
| Q2 | <p><math>P = 7500 \text{ W}</math><br/><math>N = 3800 \text{ rpm}</math><br/><math>d = 0.11 \text{ m}</math><br/><math>\mu = 0.3</math><br/><math>\theta = 120^\circ \rightarrow 2.0944.</math><br/><math>T_i = 200 \text{ N}.</math></p> <p>a) <math>V = \omega r = \frac{2\pi 3800}{60} \times \frac{0.11}{2} = 21.89 \text{ m/s}</math></p> <p>b) <math>F_1 + F_2 = \text{Constant} = 400 \text{ N}.</math><br/><math display="block">\frac{F_1}{F_2} = e^{\mu \theta} = e^{0.3 \times 2.0944}</math><math display="block">\frac{F_1}{F_2} = 1.874</math><math display="block">F_1 = 1.874 F_2</math><math display="block">F_1 = 1.874 (400 - F_1)</math><math display="block">F_1 = 749.78 - 1.874 F_1</math><math display="block">F_1 = 260.88 \text{ N}.</math><math display="block">F_2 = 139.12 \text{ N}.</math></p> <p>c) <math>P = V (F_1 - F_2) = 21.89 (260.88 - 139.12) = 2665 \text{ W}.</math></p> <p>d) <math>\eta = \frac{7500}{2665} = 2.81 \quad \therefore 3 \text{ belts required}.</math></p> |       |



| Q  | Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               | Marks |          |         |            |    |            |   |       |    |   |         |    |   |   |    |    |     |      |   |   |    |     |     |       |   |       |    |     |         |           |  |
|----|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|----------|---------|------------|----|------------|---|-------|----|---|---------|----|---|---|----|----|-----|------|---|---|----|-----|-----|-------|---|-------|----|-----|---------|-----------|--|
| Q3 |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |       |          |         |            |    |            |   |       |    |   |         |    |   |   |    |    |     |      |   |   |    |     |     |       |   |       |    |     |         |           |  |
| a) | <table><tr><th></th><th>m</th><th>r</th><th><math>\alpha</math></th><th>mr</th><th><math>mr\alpha</math></th></tr><tr><td>A</td><td><math>m_A</math></td><td>28</td><td>0</td><td><math>28m_A</math></td><td>0.</td></tr><tr><td>B</td><td>5</td><td>20</td><td>80</td><td>100</td><td>8000</td></tr><tr><td>C</td><td>3</td><td>50</td><td>150</td><td>150</td><td>22500</td></tr><tr><td>D</td><td><math>m_D</math></td><td>28</td><td>250</td><td><math>28m_D</math></td><td><math>7000m_D</math></td></tr></table> |       | m        | r       | $\alpha$   | mr | $mr\alpha$ | A | $m_A$ | 28 | 0 | $28m_A$ | 0. | B | 5 | 20 | 80 | 100 | 8000 | C | 3 | 50 | 150 | 150 | 22500 | D | $m_D$ | 28 | 250 | $28m_D$ | $7000m_D$ |  |
|    | m                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      | r     | $\alpha$ | mr      | $mr\alpha$ |    |            |   |       |    |   |         |    |   |   |    |    |     |      |   |   |    |     |     |       |   |       |    |     |         |           |  |
| A  | $m_A$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | 28    | 0        | $28m_A$ | 0.         |    |            |   |       |    |   |         |    |   |   |    |    |     |      |   |   |    |     |     |       |   |       |    |     |         |           |  |
| B  | 5                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      | 20    | 80       | 100     | 8000       |    |            |   |       |    |   |         |    |   |   |    |    |     |      |   |   |    |     |     |       |   |       |    |     |         |           |  |
| C  | 3                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      | 50    | 150      | 150     | 22500      |    |            |   |       |    |   |         |    |   |   |    |    |     |      |   |   |    |     |     |       |   |       |    |     |         |           |  |
| D  | $m_D$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | 28    | 250      | $28m_D$ | $7000m_D$  |    |            |   |       |    |   |         |    |   |   |    |    |     |      |   |   |    |     |     |       |   |       |    |     |         |           |  |
| b) | <p><math>mr\alpha</math> Scale (1 mark for appropriate scale)</p>  <p>Accuracy 2 marks.</p> <p><math>7000m_D = 21842.1</math></p> <p><math>m_D = 3.12 \text{ kg @ } 140.72^\circ</math></p>                                                                                                                                                                                                                                          |       |          |         |            |    |            |   |       |    |   |         |    |   |   |    |    |     |      |   |   |    |     |     |       |   |       |    |     |         |           |  |
| c) | <p>mr Scale</p> <p><math>28m_D = 87.36</math></p>  <p>Accuracy 2 marks</p> <p><math>28m_A = 78.19</math></p> <p><math>m_A = 2.793 \text{ kg @ } 177.15^\circ</math></p>                                                                                                                                                                                                                                                             |       |          |         |            |    |            |   |       |    |   |         |    |   |   |    |    |     |      |   |   |    |     |     |       |   |       |    |     |         |           |  |



| Q   | Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | Marks |
|-----|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|
| Q4. | <p>Energy</p> <p>A <math>E_A \leftarrow</math> min</p> <p>B <math>E_A + 500 \leftarrow</math> max</p> <p>C <math>E_A + 190</math></p> <p>D <math>E_A + 310</math></p> <p>E <math>E_A + 50</math></p> <p>F <math>E_A + 100</math></p> <p>G <math>E_A</math>.</p><br>$BW = 500 - 0 = 500 \text{ J.}$<br>$N_2 = 2500 \text{ RPM} \Rightarrow 261.8 \text{ rad s}^{-1}$ $N_1 = 2250 \text{ RPM} \Rightarrow 235.62 \text{ rad s}^{-1}$ $N_m = \frac{2500 + 2250}{2} \Rightarrow \frac{261.8 + 235.62}{2} = 248.72 \text{ rad s}^{-1}$ $\phi = \frac{261.8 - 235.62}{248.72} = 0.1052$<br>$I = \frac{BW}{\phi \omega_m^2} = \frac{500}{0.1052 \times 248.72^2} = 0.077 \text{ kg m}^2$<br>$I = m k^2$<br>$k = \sqrt{\frac{I}{m}} = \sqrt{\frac{0.077}{13}} = 0.077 \text{ m.}$ |       |



| Q  | Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | Marks |
|----|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|
| Q5 | <p><math>N_i = 1500 \text{ rpm}</math> anticlock</p> <p><math>T_i = 15 \text{ Nm}</math></p> <p><math>\eta = 0.85</math>.</p> <p>a) Output is clockwise</p> <p><math>GR = \frac{32}{20} \times \frac{36}{26} \times \frac{39}{16} = 5.4</math></p> <p><math>N_2 = \frac{N_1}{GR} = \frac{1500}{5.4} = 277.8 \text{ RPM.}</math></p> <p>b) <math>P_i = \frac{2\pi N}{60} \times T_i = \frac{2\pi \times 1500 \times 15}{60} = 2.36 \text{ kW.}</math></p> <p><math>P_o = \eta \times P_i = 2.36 \times 10^3 \times 0.85 = 2.00 \text{ kW.}</math></p> <p>c) <math>T_o = \frac{60 P_o}{2\pi N_o} = \frac{60 \times 2.00 \times 10^3}{2\pi \times 277.8} = 68.8 \text{ Nm.}</math></p> <p><math>T_i + T_o + T_h = 0</math></p> <p><math>T_h = -T_i - T_o = +15 - 68.8</math></p> <p><math>T_h = -53.8 \text{ Nm}</math><br/>↙ clockwise.</p> |       |