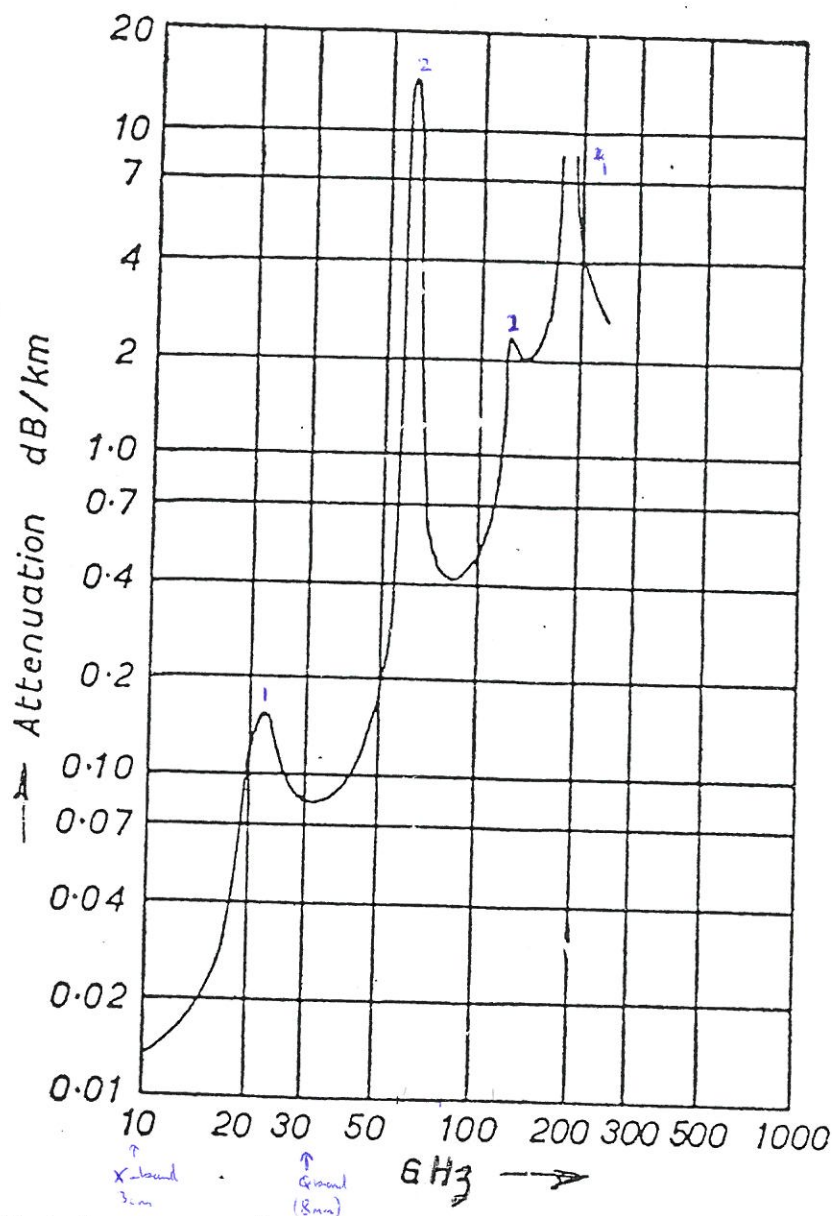


Again, the absorption produced depends on the breadth of the resonance lines, and measurements [15] show that the original figures used by van Vleck may give attenuations lower than those found in practice, Fig. 3.7. The measured and theoretical results for oxygen and water vapour may be combined to give a curve for total atmospheric absorption shown in Fig. 3.8. For a water vapour



Taken from
Matthews P.A.

1 - H₂O resonance 22,186 GHz
electric dipole moment
2 - O₂ resonance
60, 120 GHz
magnetic

Fig. 3.8 Total atmospheric attenuation in a standard atmosphere at sea level

content below the saturation point, the absorption is directly proportional to the amount of water in the atmosphere. The curves shown are for an atmosphere containing 1% of water vapour molecules, 7.5 gm/m³. The ratio of the mass of water vapour to that of dry air in a given volume of moist air is called the humidity mixing ratio. From the gas equations and the molecular weights of

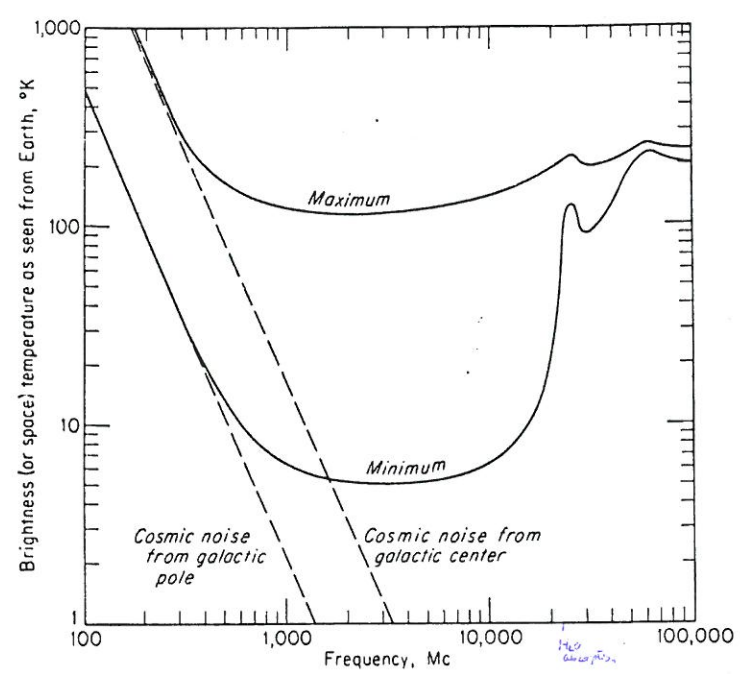


FIG. 8.6. Maximum and minimum brightness (or space) temperatures of the sky as seen by an ideal single-polarization antenna on earth. Dashed curves apply to cosmic noise. Cosmic noise predominates at lower frequencies; atmospheric absorption noise, at higher frequencies. Maximum and minimum cosmic noise correspond to the directions of the galactic center and the galactic pole, respectively. Maximum atmospheric absorption corresponds to the antenna beam pointing along the horizon, while minimum absorption corresponds to antenna pointed at the zenith. (After Greene and Lebenbaum,³⁷ *Microwave J.*)

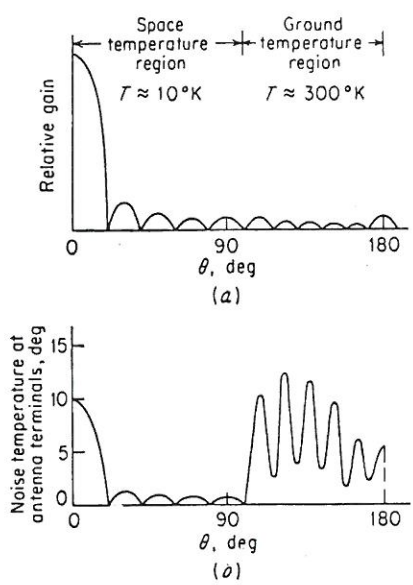


FIG. 8.9. (a) An antenna radiation pattern with the region in the vicinity of the main beam observing the space temperature and the region from about $\theta = 90^\circ$ to $\theta = 180^\circ$ observing the ground temperature; (b) profile of the noise temperature contributions weighed by the antenna pattern.

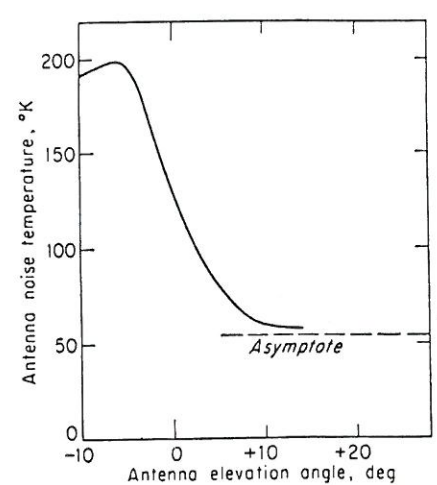


FIG. 8.10. Computed antenna noise temperature as a function of elevation angle for a 10-ft-diameter parabolic-reflector antenna operating at a frequency of 1,000 Mc. See text for assumptions. (After Greene,⁴³ courtesy Airborne Instruments Laboratory.)

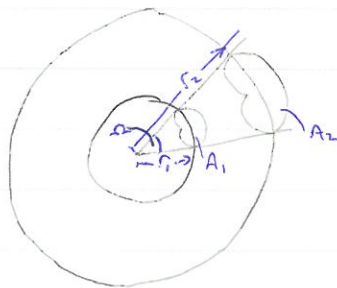
In these equations, the observer is at the source of the wave, looking out along its path.

Any polarisation can be formed from the weighted sum of the two pure circular polarisation (called "base vectors").

Power Flow

$$\underline{S} = \frac{1}{2} \operatorname{Re} [\underline{E} \wedge \underline{H}^*] = \underline{u}_r \frac{|e(\theta, \phi)|^2}{2Zr^2} = \underline{u}_r \frac{P_T G_T(\theta, \phi)}{4\pi r^2}$$

Rays; and the principle of geometrical optics: flow of power is confined to cones (or tubes) of rays. Rays are defined by the local direction of propagation of the wave



Consider the power flow through ~~the~~ A_1 and A_2 (both in the far field)

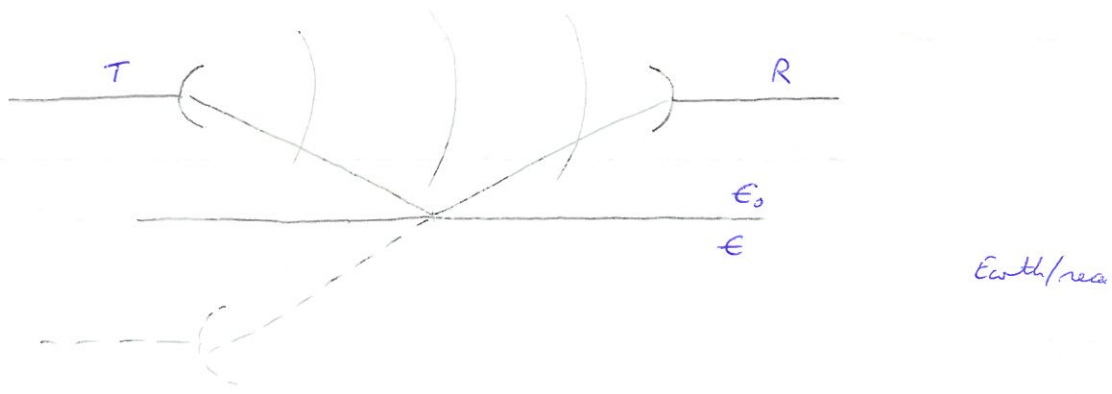
$$\left. \begin{aligned} S_1 &= \frac{P_T G_T}{4\pi r_1^2}, & A_1 &= \Omega r_1^2 \\ S_2 &= \frac{P_T G_T}{4\pi r_2^2}, & A_2 &= \Omega r_2^2 \end{aligned} \right\} \text{ i.e. } S_1 A_1 = S_2 A_2$$

$$\text{Then } \frac{S_1}{S_2} = \left(\frac{r_2}{r_1} \right)^2 \rightarrow \frac{E_1}{E_2} = \frac{r_2}{r_1}$$

and hence

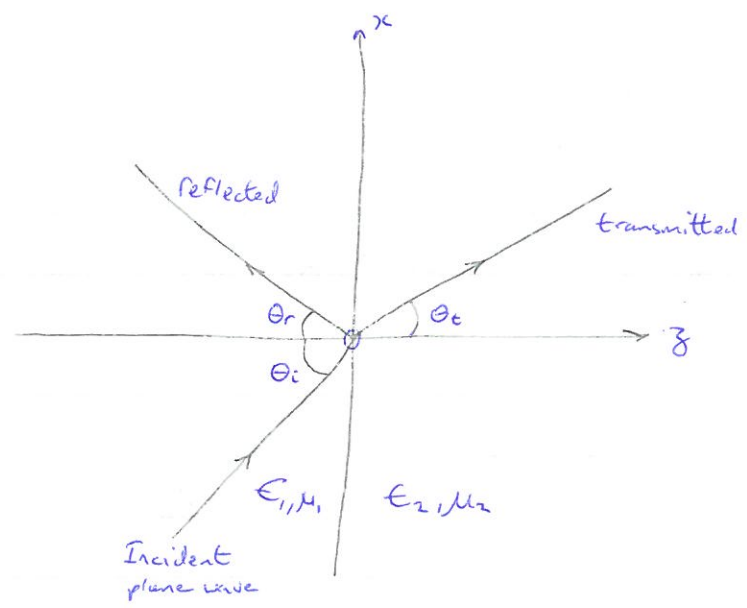
$$E_2 = \frac{r_1}{r_2} E_1 \exp[-jk(r_2 - r_1)]$$

Note have to be in the far field



Can consider spherical waves as superposition of plane waves. Now considering plane wave incident obliquely on a dielectric boundary

Reflection of Plane Waves at a Plane Interface (Oblique Incidence)



Incident

$$E_{inc}(x, y) = E_0 \exp\{-jk_1(x \sin \theta_i + z \cos \theta_i)\}$$

Transmission Coefficient

$$\tau = 1 + \rho = \frac{2z_2'}{z_2' + z_1'}$$

and $E_z \cos \theta_t = \tau E_0 \cos \theta_i$

Use Snell's law,

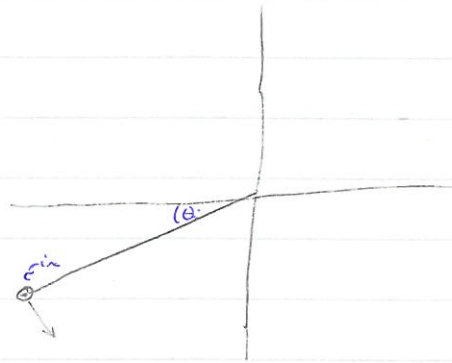
$$\cos \theta_t = \sqrt{1 - \sin^2 \theta_t}$$

→ can get in terms of $\sin^2 \theta_i$.

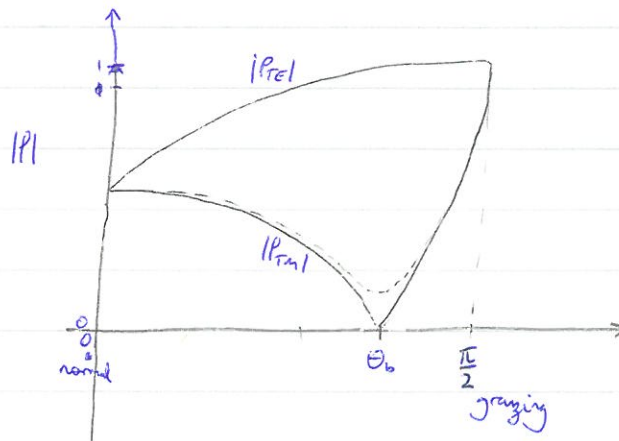
T.E. (H Plane) Case

Only difference is
the tangential
wave impedance

$$z_1'' = \frac{z_1}{\cos \theta_i}, \quad z_2'' = \frac{z_2}{\cos \theta_t}$$



Reflection coefficient as function of angle

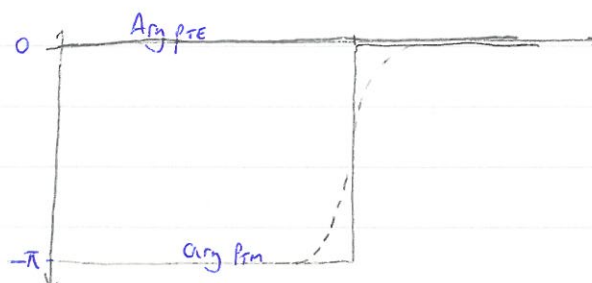


$$\rho_{\text{normal}} = \frac{1 - \sqrt{\epsilon_r}}{1 + \sqrt{\epsilon_r}}$$

$$\text{if } \epsilon_r = 2.57$$

$$\rho_{\text{normal}} = -0.23$$

$$\theta_0 = \text{Brewster angle}$$



Check these results
- may be incorrect.

Total Internal Reflection

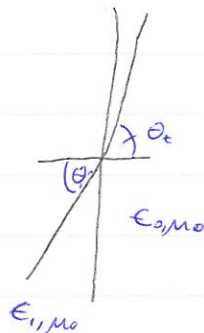
Assume $\epsilon_1 > \epsilon_0$, and $\epsilon_r > 1$

Snell's law

$$\sqrt{\epsilon_r} \sin \theta_i = \sin \theta_t$$

hence critical angle

$$(\theta_i)_{\text{crit}} = \sin^{-1} \frac{1}{\sqrt{\epsilon_r}}$$



$$\epsilon_r = \frac{\epsilon_1}{\epsilon_0}$$

Beyond critical angle, $\sin \theta_t > 1$,

OK if θ_t allowed to be complex.

We have

$$\cos \theta_t = \sqrt{1 - \sin^2 \theta_t} = \pm j \chi_{(\text{chi})}$$

χ is real + +ve

Then

$$E^{\text{trans}} = E_2 \exp \{-j k_0 (\chi \sin \theta_t \pm j \chi_z)\}$$

$$(j \chi_z = z \cos \theta_t)$$

$$= E_2 \exp \{-j k_0 \chi \sin \theta_t\} \exp \{\pm k_0 \chi_z\}$$

can reject +ve sign as contravenes conservation of energy

Surface Wave

Evanescent.

phase velocity

$$\frac{\omega}{k_0 \sin \theta_t} < \frac{\omega}{k_0}$$

slow wave

Terminating impedance pure imaginary; $|P| = 1$

"Reactive"

Boundary Conditions : at $z=0$

E_{tang} and H_{tang} continuous.

Reflected Field

$$\underline{E}^{\text{refl}}(x, z) = \underline{E}_1 \exp \{ -jk_1 (x \sin \theta_r - z \cos \theta_r) \}$$

hence $\sin \theta_r = \sin \theta_i$ — $\theta_i = \theta_r$ — law of reflection

Transmitted Field

$$\underline{E}^{\text{trans}}(x, z) = \underline{E}_2 \exp \{ -jk_2 (x \sin \theta_t + z \cos \theta_t) \}$$

hence $k_1 \sin \theta_i = k_2 \sin \theta_t$

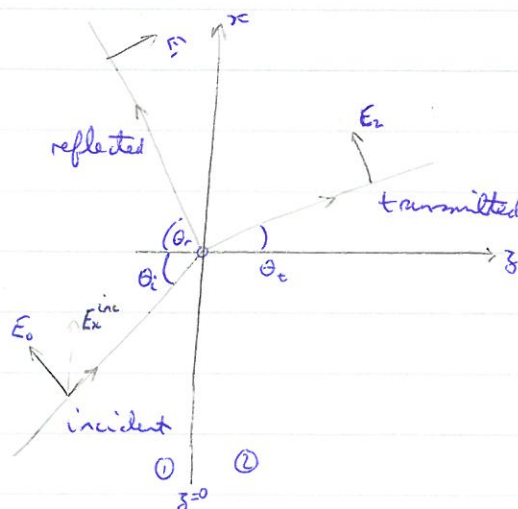
But refractive index

$$n = \frac{c_0}{c} = \sqrt{\frac{\mu \epsilon}{\mu_0 \epsilon_0}} = \frac{k}{k_0}$$

$$\therefore n_1 \sin \theta_i = n_2 \sin \theta_t$$

(Snell's law)

TM (E Plane) Case



$$\theta_r = \theta_i, \quad n_1 \sin \theta_i = n_2 \sin \theta_t$$

Interested in field components tangential to boundary plane $z=0$.
 To satisfy boundary conditions:

E_{tan} & H_{tan} must be continuous

$$E_x^{inc} = E_0 \cos \theta_i \quad \text{and} \quad H_y^{inc} = \frac{E_0}{Z_1} \quad \text{where } Z_1 = \sqrt{\frac{\mu_1}{\epsilon_1}}$$

These are analogous to V and I in equivalent transmission line (for propagation in the z -direction). Hence define wave impedances

$$Z_1' = \frac{E_x^{inc}}{H_y^{inc}} = Z_1 \cos \theta_i \quad \text{good for region ①}$$

$$Z_2' = Z_2 \cos \theta_t \quad \text{for region ②}$$

ie have

①	②
$k_1 \cos \theta_i$	$k_2 \cos \theta_t$
$Z_1 \cos \theta_i$	$Z_2 \cos \theta_t$

$z=0$

$$k_1 = \omega \sqrt{\mu_1 \epsilon_1}$$

etc

and no reflection coefficient

$$\rho = \frac{Z_2' - Z_1'}{Z_2' + Z_1'}$$

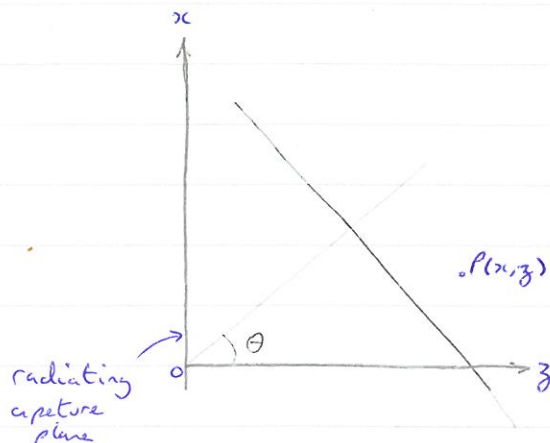
$$\text{ie } E_x^{refl}(0,0) = \rho E_x^{inc}(0,0)$$

$$\Rightarrow E_1 \cos \theta_r = \rho E_0 \cos \theta_i$$

Radiation from Aperture Fields

2D - TM

for typical plane wave,
amplitude E_0 ,



for typical plane wave, amplitude E_0 , fields at P are

$$E_x(x, z) = E_0 \cos \theta \exp \{-jK(sx + cz)\}$$

$$\begin{aligned} \text{where } s &= \sin \theta \\ c &= \cos \theta \\ &= \sqrt{1 - s^2} \end{aligned}$$

$$E_z(x, z) = -E_0 \sin \theta \exp \{-jK(sx + cz)\}$$

$$H_y(x, z) = \frac{E_0}{Z} \exp \{-jK(sx + cz)\}$$

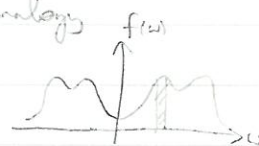
But the plane wave amplitude is given
in general by a spectrum function of angle i.e $A(\theta)$
Hence $E_0 = A(\theta) d\theta$. Then

$$E_x(x, z) = A(\theta) d\theta \cos \theta \exp \{-jK(sx + cz)\}$$

etc
for E_z, H_z

for any single plane wave.

Corresponding time - frequency
analogy



$f(\omega)$ frequency spectrum
(of some waveform $f(t)$) giving
 $f(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(t) e^{j\omega t} dt$

for which
 $f(t) = \int_{-\infty}^{\infty} f(\omega) e^{-j\omega t} d\omega$

If consider field at P as due to all direction plane waves coming from
origin,

$$E_x(x, z) = \int_{-\pi/2}^{\pi/2} A(\theta) d\theta \cos \theta \exp \{-jK(sx + cz)\}$$

etc.

for the region $z > 0$.

Now replace $A(\theta)$ by $F(s)$; $s = \sin \theta$
 Then ds replaces $\cos \theta d\theta$, and

$$E_x(x, z) = \int_{-\infty}^{\infty} F(s) \exp\{-jK(sx + cy)\} ds$$

This allows for complex angle (i.e. $\sin \theta > 1$)
 ($|s| > 1$)

Note that the spectrum function $F(s)$ includes non-propagating waves, for which $|s| > 1$.

$$E_z(x, z) = - \int_{-\infty}^{\infty} F(s) \cdot \frac{s}{c} \cdot \exp\{-jK(sx + cy)\} ds$$

note: get $\frac{1}{c}$ to make algebra hold.

$$H_y(x, z) = \frac{1}{Z} \int_{-\infty}^{\infty} \frac{F(s)}{c} \exp\{-jK(sx + cy)\} ds$$

Note field components given by $\underline{F(s)}$. Corresponding aperture field:
 (aperture field = field on a radiating plane $z=0$)

$$E_{ax}(x) = E_x(x, 0) \quad (= f(x))$$

so that

$$E_{ax}(x) = \int_{-\infty}^{\infty} F(s) \exp\{-jKsx\} ds$$

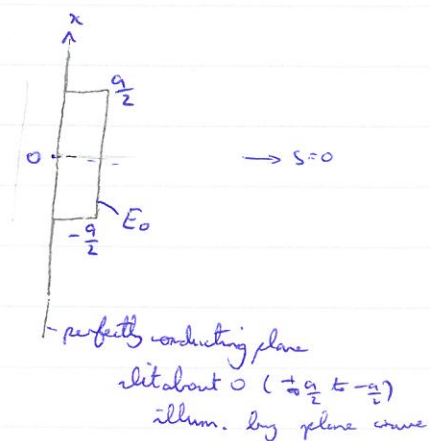
which means that $E_{ax}(x)$ and $F(s)$ are Fourier transforms, and the inverse yields

$$F(s) = \frac{1}{\lambda} \int_{-\infty}^{\infty} E_{ax}(x) \exp(+jKsx) dx$$

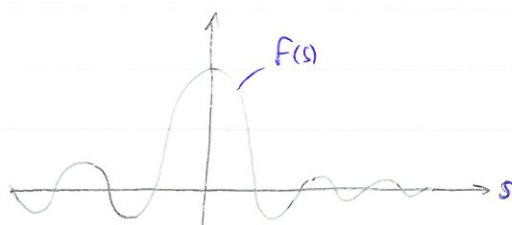
Example of a uniform aperture field,

$$E_{ax}(x) = E_0 \text{rect}_a(x)$$

ie



Where F.T. is the "sinc function"



for calculation of F.T. see overlap

TM 2-D in half space $z \geq 0$

$$\begin{vmatrix} E_x(x,z) \\ E_y(x,z) \\ H_y(x,z) \end{vmatrix} = \int_{-\infty}^{\infty} F(s) \begin{vmatrix} 1 \\ -\frac{s}{c} \\ \frac{1}{Z_0 c} \end{vmatrix} \exp \{ -jk(sx + cz) \} ds$$

Aperture field

$$E_{ax}(x) = \int_{-\infty}^{\infty} F(s) \exp(-jk_s x) ds$$

which give the angular spectrum as (the Fourier inverse)

$$F(s) = \frac{1}{\lambda} \int_{-\infty}^{\infty} E_{ax}(x) \exp(+jk_s x) dx$$

$$\boxed{E_{ax}(x) \xleftrightarrow{F.T.} F(s)}$$

$$F(s) = \frac{E_0}{\lambda} \int_{-\frac{a}{2}}^{\frac{a}{2}} e^{jksx} dx$$

$$e^{j\theta} - e^{-j\theta} = 2j \sin \theta$$

$$= \frac{E_0}{\lambda} \left[\frac{e^{jksx}}{jks} \right]_{-\frac{a}{2}}^{\frac{a}{2}} = \frac{E_0}{\lambda} \frac{2j \sin(ks \frac{a}{2})}{jks}$$

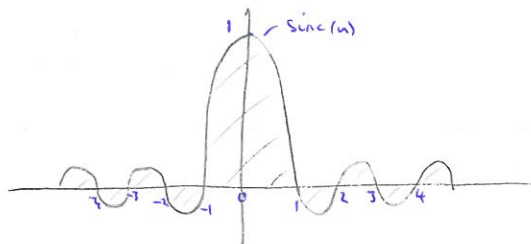
$$= \frac{E_0 a}{\lambda} \frac{\sin(\pi a s / \lambda)}{(\pi a s / \lambda)}$$

$$\text{where } k = \frac{2\pi}{\lambda}$$

$$= \frac{E_0 a}{\lambda} \text{sinc}\left(\frac{as}{\lambda}\right)$$

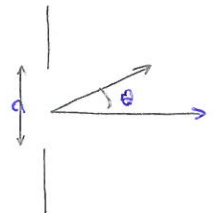
where

$$\text{sinc}(u) = \frac{\sin(\pi u)}{\pi u}$$



$$\text{area} \equiv 1 \quad (\text{i.e. } \int_{-\infty}^{\infty} \text{sinc}(u) du = 1)$$

$$\text{rect}_a(x) \leftrightarrow \text{sinc } u$$



$$\text{Maximum value} = F(0) = \frac{E_0 a}{\lambda} \quad : \text{ note increased gain as } \frac{a}{\lambda} \text{ increases}$$

note $u = \frac{as}{\lambda}$ in above.

$$u = \frac{as}{\lambda} = \frac{a \sin \theta}{\lambda}$$

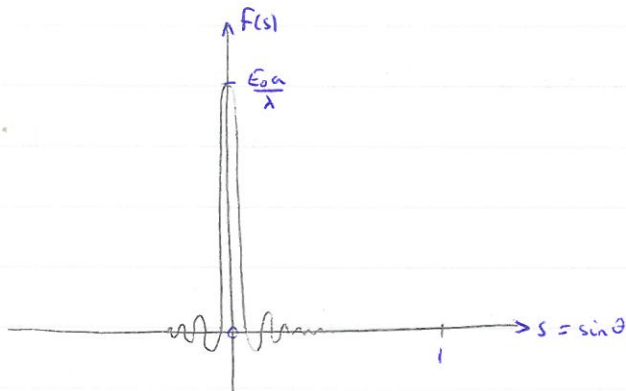
First zeroes occur at $u = \pm 1$, i.e. at $\sin \theta = \pm \frac{\lambda}{a}$

i.e. width of main lobe ($\approx \frac{2\lambda}{a}$) decreases as $\frac{a}{\lambda}$ increases.

Side lobe level (relative to main lobe level)

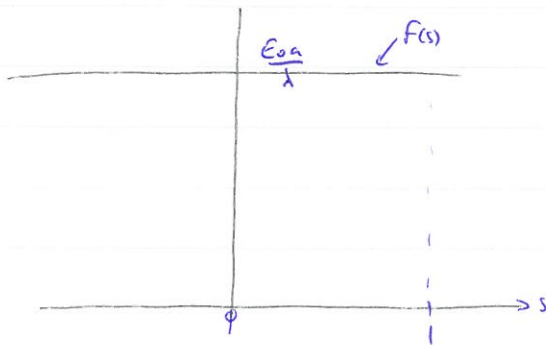
$$= \operatorname{sinc}\left(\frac{3}{2}\right) = \frac{1}{\pi \frac{3}{2}} = (-13\text{dB})$$

If $a \gg \lambda$



$\frac{E_0 a}{\lambda}$ is large

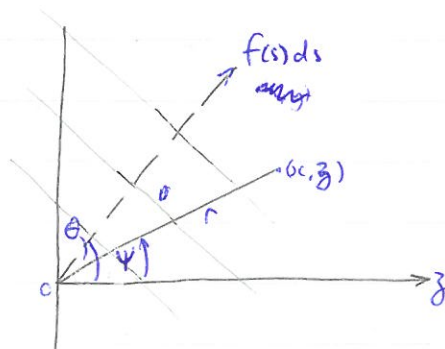
If $a \ll \lambda$



$\frac{E_0 a}{\lambda}$ is small

Note: significant amount of spectrum with $|s| > 1$ (reactive field)

Far field



$$\begin{aligned} \text{Put } x &= r \sin \psi \\ z &= r \cos \psi \end{aligned}$$

$$E_x(r, \psi) = \int_{-\infty}^{\infty} F(\sin \theta) \exp \left\{ -jkr (\sin \theta \sin \psi + \cos \theta \cos \psi) \right\} d(\sin \theta)$$

$$= \int_{-\infty}^{\infty} f(\sin \theta) \exp \left\{ -jkr \cos(\theta - \psi) \right\} d(\sin \theta)$$

Provided $kr \gg 1$ (i.e., $r \gg \lambda$), we can use the principle of "stationary phase" to calculate the integral as being asymptotically

$$E_x(r, \psi) \approx \text{constant} \times F(\sin \psi) e^{-jkr}$$

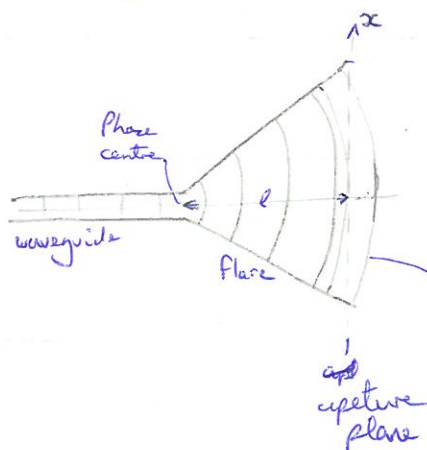
holds for $r > \frac{2a^2}{\lambda}$
(Rayleigh distance)

Note: Assumed that $F(s)$ is continuous

: kr is very large so that for adjacent values of θ , their phase differs by π and hence they cancel.

Some Aperture Antennas

Electromagnetic Horn



$$\text{Phase - spherical wave} = \frac{kx^2}{2l}$$

amplitude - ~~is~~ expanded form of waveguide field distribution



By Pythagoras

$$(l+s)^2 = l^2 + x^2$$

$$2ls + s^2 = x^2$$

By Pythagoras

then $\delta = \frac{x^2}{2l}$

- Note: This is a very robust construction - once welded on, is set permanently.
- Only has a modest gain. This is because flaring out more sharply causes more phase variation; hence lower gain.
 - Can rigorously calculate the field gain across the horn. Often used as a primary source in reflecting antennas.

Paraboloidal Reflector

From properties of parabola

$$FP + PQ = \text{const} = 2 OF = 2f \quad (1)$$

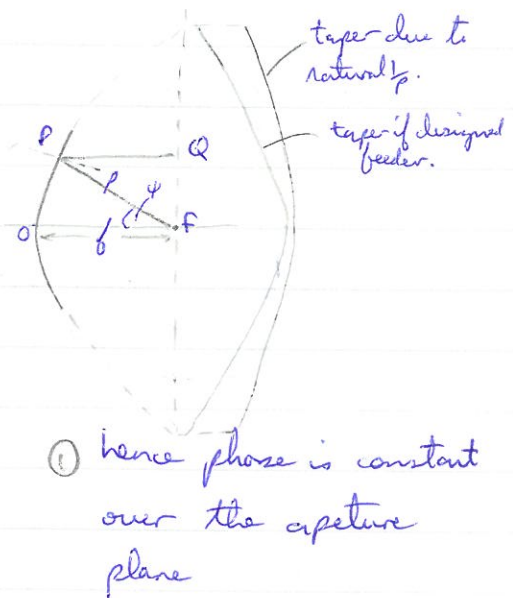
ie $p + p \cos \psi = p(1 + \cos \psi)$

$$= 2p \cos^2 \frac{\psi}{2}$$

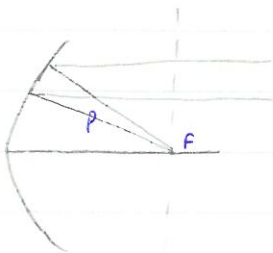
$$= 2f$$

$$\therefore p = f \sec^2 \frac{\psi}{2}$$

$$\left\{ \begin{array}{l} \text{note } p = f, \text{ when } \psi = 0^\circ \\ p = 2f \text{ when } \psi = 90^\circ \end{array} \right.$$



(1) hence phase is constant over the aperture plane



Suppose primary feed at F has a far field pattern $F(\psi)$. The amplitude of the aperture field $\propto \frac{1}{p} F(\psi)$; $x = p \sin \psi$

$\frac{1}{p}$ gives a natural tapering of the aperture field

amplitude. This can be enhanced by the feed pattern design.

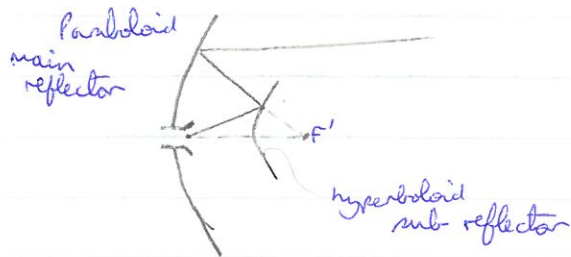
This reduces the side lobe level, and spill over loss, but also reduces the gain.

Cassegrain

Short wave guide
run (hence low loss).

long equivalent focal length
(good for polarization purity).

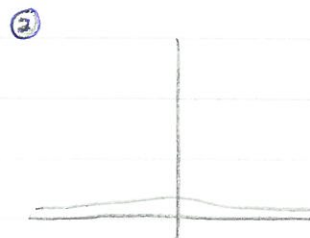
High gain.



Aperture Blocking

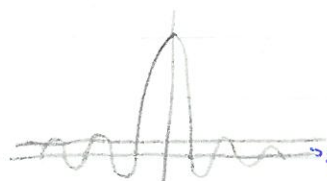


taking F.T.'s



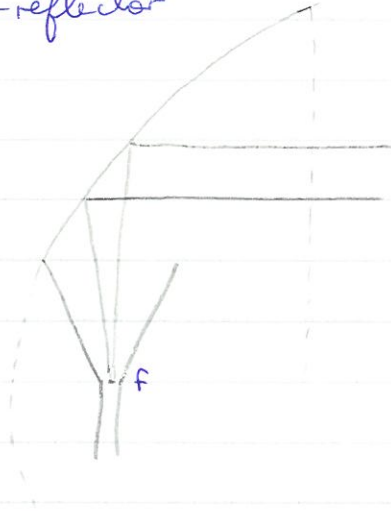
Periodic due to width & shape
of cut-off edges

The difference of these is



(ie just shift
axis up to level
of ①)

Horn-reflector

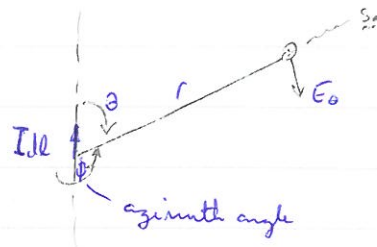


Extremely robust. Nothing can move if dropped. Everything except the aperture is blocked in with metal. Therefore it blocks out unwanted noise coming in from the side - you only get a inevitable noise.

Wire Antennas

Elemental (Hertzian) Dipole

Radiation from current element
Current is $I_0 dl$

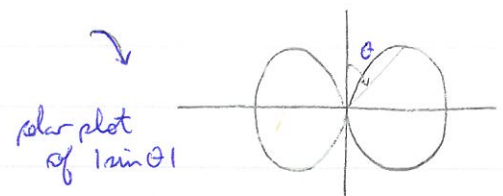


Far field

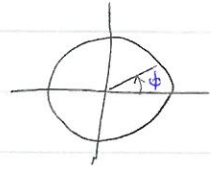
$$E_\theta = \frac{j\omega\mu I_0 dl}{4\pi r} e^{-kr} \sin\theta$$

$$H_\phi = \frac{1}{Z} E_\theta$$

- Note :
- : Spherical wave
 - : Local plane wave behaviour
 - : Vertical plane pattern is $\sin\theta$



: horizontal plane pattern is uniform



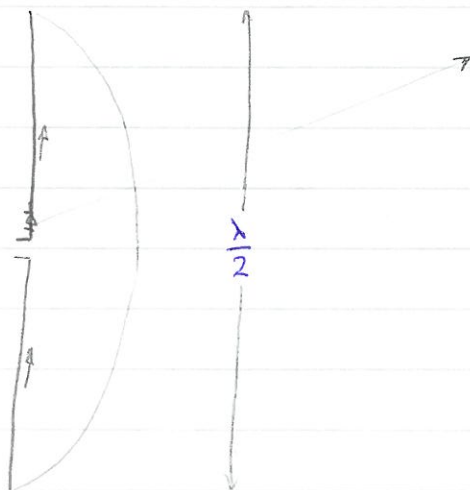
: Far field $\propto$ current

Radiation Resistance

Given by $\frac{1}{2} I^2 R_{\text{rad}} = P_{\text{rad}} = S_r$ integrated over a sphere.
Can show

$$R_{\text{rad}} = 80\pi^2 \left(\frac{dl}{\lambda}\right)^2 \quad (+ \text{a large reactive element})$$

Half-Wave Dipole



Can show vertical pattern : $\frac{\cos(\frac{\pi}{2} \cos \theta)}{\sin \theta}$

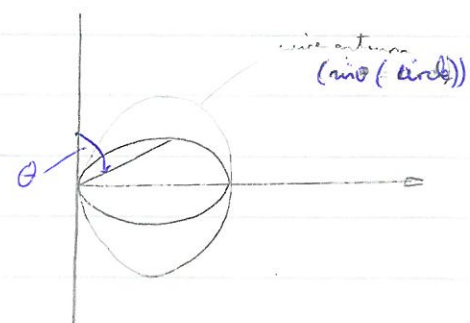
: Slightly improved pattern

: Important that

$$R_{\text{rad}} = 73 \Omega \quad \text{"at resonance"} \\ \text{is pure resistance.}$$

is efficient radiator

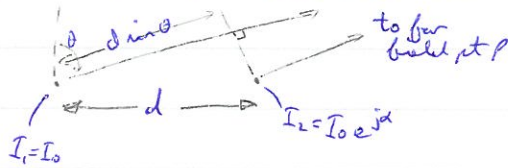
Used almost universally at UHF.



Arrays

Two Element Array

Assume identical elements same amplitude, but phase difference α



At large distances,

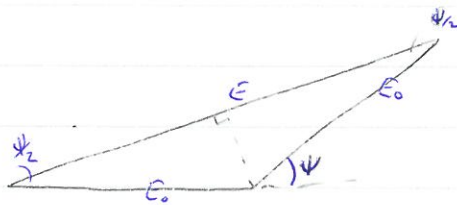
$$E = E_0 (1 + e^{j\psi})$$

$E_0 =$ field at P due to I_1

$$\text{where } \psi = \frac{2\pi d \sin \theta}{\lambda} + \alpha,$$

$$E = E_0 \left(1 + e^{j\left(\frac{2\pi d \sin \theta}{\lambda} + \alpha\right)} \right)$$

$E_0 =$ field at P due to I_1 .



Hence resultant field magnitude is

$$|E| = |E_0| 2 \left| \cos \frac{\psi}{2} \right|$$

$$= |E_0| 2 \left| \cos \left(\frac{\pi d \sin \theta}{\lambda} + \frac{\alpha}{2} \right) \right|$$

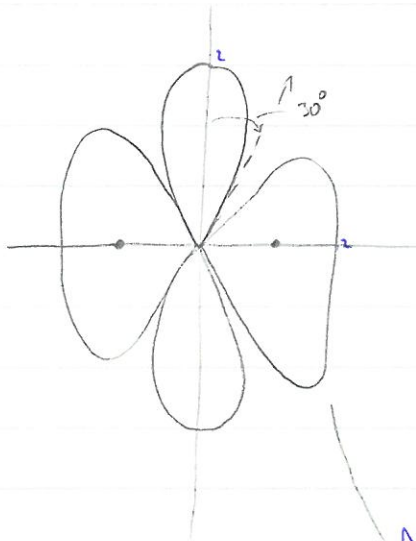
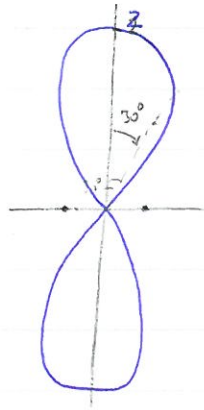
↑
element
pattern

array factor $A(\theta)$

if have

$$d = \frac{\lambda}{2}, \alpha = 0$$

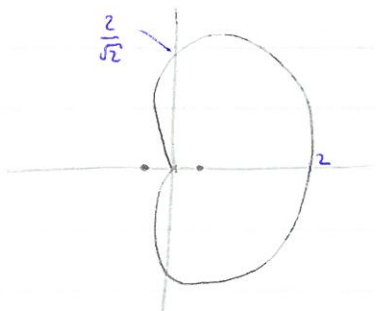
$$\text{then } A(\theta) = 2 \cos\left(\frac{\pi}{2} \sin \theta\right)$$



$$d = \lambda, \alpha = 0$$

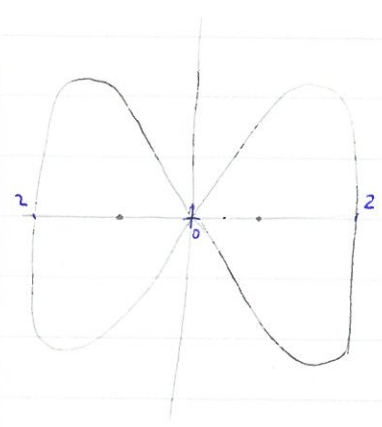
$$\therefore A(\theta) = 2 \cos(\pi \sin \theta)$$

Note "grating lobes" at $\theta = \pm \frac{\pi}{2}$



$$d = \frac{\lambda}{4}, \alpha = -\frac{\pi}{4}$$

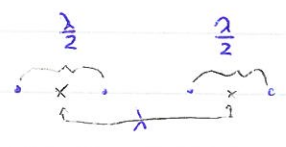
$$\therefore A(\theta) = 2 \cos\left(\frac{\pi}{4} \sin \theta - \frac{\pi}{4}\right)$$



$$d = \frac{\lambda}{2} \quad \alpha = \pm \pi$$

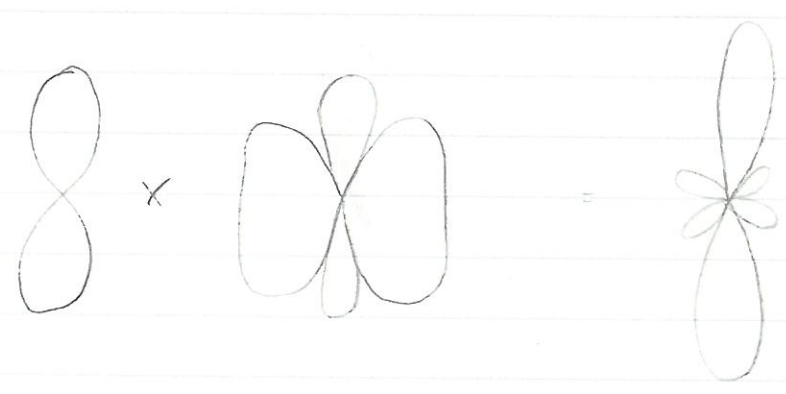
$$A(\theta) = 2 \cos\left(\frac{\pi}{2} \sin \theta + \frac{\pi}{2}\right)$$
$$= 2 \sin\left(\frac{\pi}{2} \sin \theta\right)$$

Uniform Broadside Array
4-element

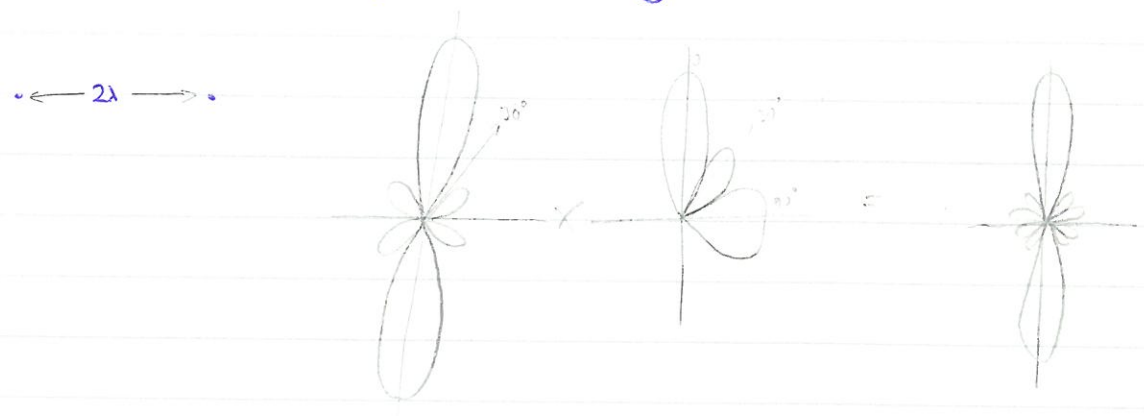


(see E.C. JORDAN)

Omni elements
In phase
 $\frac{\lambda}{2}$ spacing
uniform excitation



2 elements
Need 2-element array with 2λ spacing



Tapered Broadside Array



1 2 1



1 3 3 1

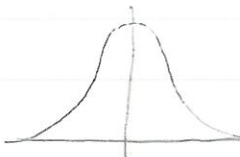
[1, 4, 6, 4, 1]

further addition may give

1 : 4 : 6 : 4 : 1

Called binomial array as current ratios have same ratios as binomial distribution.

Going continuous gives binomial distribution.

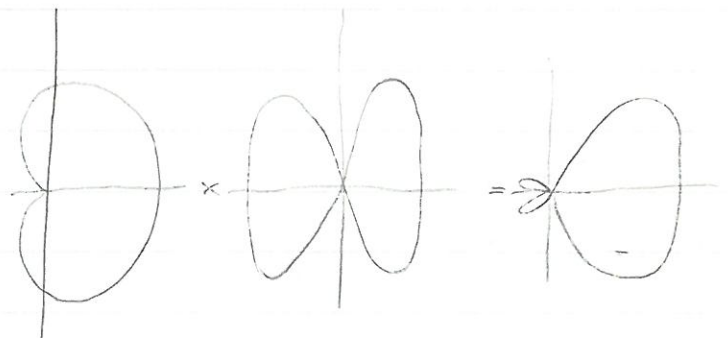
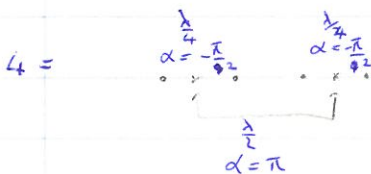


Gaussian electric field $\leftrightarrow$ gaussian spectrum

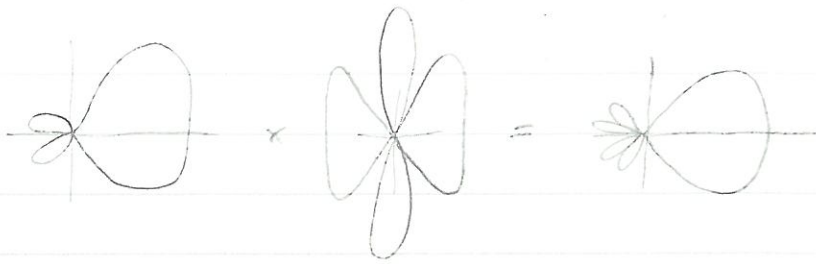
Interaction between elements ignored.

Uniform Endfire Array

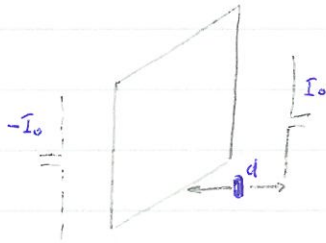
- progressive phase delay



8 =



Reflector Arrays



Principle of images

$$+z$$

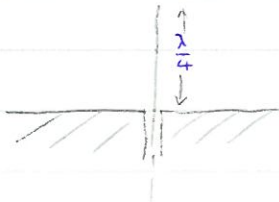
$$-z$$

$$-z \rightarrow -z$$

$$z \rightarrow -z$$

Separation of elements = $2d$, phasing = π

Quarter Wave Monopole



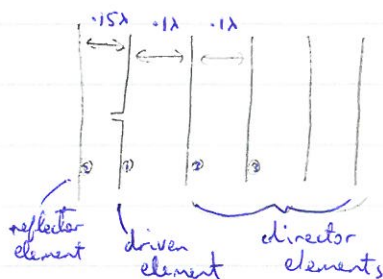
Yagi-Uda Array

(see Glazier & Lamant)

length
① = $\frac{\lambda}{2}$

② > $\frac{\lambda}{2}$

③ < $\frac{\lambda}{2}$



Gives up to 10dB (with max elements)

Beam width $\sim 45^\circ$