

21X6493

ACDA4008

Maths Test 2.

1
a

i) $y = \frac{x^2}{2x+1}$

$$y = x^2 (2x+1)^{-1}$$

Product rule.

$$\frac{dy}{dx} \text{ or } f'(x) = g'(x)h(x) + g(x)h'(x)$$

first chain rule.

guess

$$y = 2x(-2) \frac{dy}{dx} = -4x$$

ii) $y = \sqrt{x^2+1}$

$$y = \sqrt{u}, y = u^{\frac{1}{2}} \quad u = x^2+1$$

$$\frac{du}{dx} = 2x \quad \frac{du}{2} = x dx \quad \frac{du}{dx} = dx$$

$$\frac{dy}{dx} = u^{\frac{1}{2}} \frac{du}{2x} \quad \frac{dy}{dx} = 2x u^{\frac{1}{2}} \frac{du}{2x}$$

$$\begin{aligned} \frac{dy}{dx} &= 2(\sqrt{u})^{\frac{3}{2}} \\ &= 2(x^2+1)^{\frac{3}{2}} \end{aligned}$$

a) iii) $y = 3x^{\frac{3}{2}} + 3x^{-2} - 6x.$

$$\frac{dy}{dx} = 4\frac{1}{2}x^{\frac{1}{2}} + (-6)x^{-3} - 6$$

$$\frac{dy}{dx} = \frac{9}{2}x^{\frac{1}{2}} - 6x^{-3} - 6.$$

iv) $y = \sec^3 4x.$ $u = 4x.$



$y = \sec^3 u$ $u = 4x.$

$$\frac{du}{dx} = 4 \quad dx = \frac{du}{4}.$$

v) NO

vi) $y = \frac{1-10x}{(2x-1)^2}$

$u = 2x-1.$

$$y = \frac{1-10x}{u^2}$$

$$\frac{du}{dx} = 2. \quad \frac{du}{2} = dx.$$

$$\frac{dy}{dx} = \frac{1-10x}{u^2} dx =$$

b) no

c) no

Q2.

a

$$i) \int \frac{2x}{4+3x^2} dx \quad u = 4+3x^2$$

$$\int u = 4x + x^3 + C.$$

ii) no

iii) no

b) no

c) no.

Q3

a

$$i) \left(\frac{1+i}{1-i} \right)^3$$

$$\frac{1+i}{1-i} \times \frac{1+i}{1+i} = \frac{(1+i)^2}{(1-i)(1+i)}$$

$$= \frac{1+i+i^2}{1-i+i-i^2} = \frac{1+2i+(-1)}{1-(-1)}$$

$$= \frac{2i}{2} = i \quad \text{therefore}$$

$$(i)^3 = -i$$

$$ii) \frac{3}{(1+i)(2+i)} = \frac{3}{2+2i+i^2} = \frac{3}{2+3i-1}$$

$$= \frac{3}{1+3i}$$

$$\frac{3}{1+3i} \times \frac{1-3i}{1-3i} = \frac{3(1-3i)}{1-3i+3i-9i^2} = \frac{3(1-3i)}{1-9(-1)}$$

$$= \frac{3-9i}{-10}$$

$$\text{so } -\frac{3}{10} - \frac{9}{10}i$$

$$\text{so } -\frac{3}{10} + \frac{9}{10}i$$

$$\text{iii)} \cdot \frac{1+i}{1-i} - (1+2i)(2+2i) + \frac{3-i}{1+i}$$

$\textcircled{x}$
 $\textcircled{y}$
 $\textcircled{z}$

or

$$\cancel{x} - y + z.$$

x can be taken from Q3a i

$$i - y + z.$$

$$\begin{aligned} y &= (1+2i)(2+2i) \\ &= 2 + 4i + 2i + 4i^2 \\ &= 2 + 6i + 4(-1) \\ &= -2 + 6i \end{aligned}$$

$$z = \frac{3-i}{1+i}$$

$$\begin{aligned} &= \frac{3-i}{1+i} \times \frac{1-i}{1-i} = \frac{(3-i)(1-i)}{(1+i)(1-i)} = \frac{3-i-3i+i^2}{1+i-i-i^2} \\ &= \frac{3-4i+(-1)}{1-(-1)} = \frac{2-4i}{2} \end{aligned}$$

$$z = 1-2i.$$

Put together

$$\begin{aligned} i - (-2+6i) + (1-2i) &= i + 2 - 6i + 1 - 2i \\ &= 3 - 7i. \end{aligned}$$

Q3a) ✓ $z = (i-1) + (\sqrt{3}-i) + \underbrace{(1+i)(1+i)}_{u}$ Page 6.

$$= 2i^2 - 2i + \sqrt{3} - i + u.$$

$$u = (1+i)/(1+i)$$

$$= 1+i+i+i^2$$

$$= 1+2i+(-1)$$

$$= 2i$$

$$= 2i^2 - 2i + \sqrt{3} - i + 2i.$$

$$= \cancel{2}(-1) - i + \sqrt{3}$$

$$= \sqrt{3} - 2 - i$$

$$\cancel{(\sqrt{3} - 2 - i)}$$

b) $z = (4 + 7i)(3 - 2i)$

$$z = r(\cos \theta + i \sin \theta).$$

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}\left(\frac{|y|}{|x|}\right)$$

$$= (4 + 7i)(3 - 2i).$$

$$= 12 + 21i - 8i - 14i^2$$

$$= 12 + 13i - -14$$

$$= 26 + 13i$$

$$26^2 = 676$$

$$13^2 = 169$$

$$= 845$$

$$\sqrt{845}$$

$$= 29.07$$

$$Z = 29.07(\cos Q + i \sin Q)$$

$$Q = \tan^{-1} \left(\frac{|y|}{|x|} \right)$$

$$Q = \tan^{-1} \left(\right)$$

~~out of time~~ . out of time

I studied Logs and vedars etc.

Didn't realise they wouldn't be on the test. Probably mentioned in an email

~~but~~ so I do apologise. I'm also not very good at integration and differentiation

oh well. Too tired to deal with this, and wasted 50 minutes confused on the first 2 questions.