

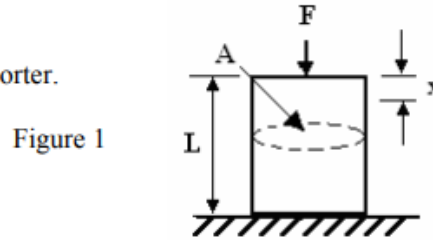
Stress and Strain

1. DIRECT STRESS σ

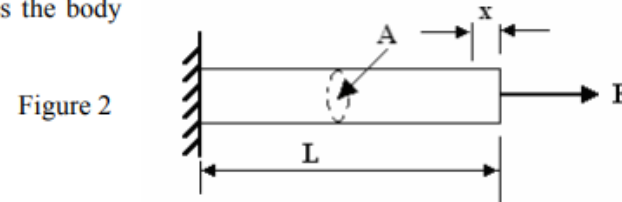
When a force is applied to an elastic body, the body deforms.

The way in which the body deforms depends upon the type of force applied to it.

A compression force makes the body shorter.



A tensile force makes the body longer.



Tensile and compressive forces are called **DIRECT FORCES**.

Stress is the force per unit area upon which it acts.

$$\text{Stress} = \sigma = \text{Force/Area} \text{ N/m}^2 \text{ or Pascals.}$$

The symbol σ is called SIGMA

NOTE ON UNITS The fundamental unit of stress is 1 N/m^2 and this is called a Pascal. This is a small quantity in most fields of engineering so we use the multiples kPa, MPa and GPa.

Areas may be calculated in mm^2 and units of stress in N/mm^2 are quite acceptable. Since 1 N/mm^2 converts to $1\,000\,000 \text{ N/m}^2$ then it follows that the N/mm^2 is the same as a MPa

2. DIRECT STRAIN ϵ

In each case, a force F produces a deformation x . In engineering we usually change this force into stress and the deformation into strain and we define these as follows.

Strain is the deformation per unit of the original length

$$\text{Strain} = \epsilon = x/L$$

The symbol ϵ is called EPSILON

Strain has no units since it is a ratio of length to length. Most engineering materials do not stretch very much before they become damaged so strain values are very small figures. It is quite normal to change small numbers in to the exponent for of 10^{-6} . Engineers use the abbreviation $\mu\epsilon$ (micro strain) to denote this multiple.

For example a strain of 0.000068 could be written as 68×10^{-6} but engineers would write $68 \mu\epsilon$.

Note that when conducting a British Standard tensile test the symbols for original area are S_0 and for Length is L_0 .

WORKED EXAMPLE No.1

A metal wire is 2.5 mm diameter and 2 m long. A force of 12 N is applied to it and it stretches 0.3 mm. Assume the material is elastic. Determine the following.

- i. The stress in the wire σ .
- ii. The strain in the wire ϵ .

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A metal wire is 2.5 mm diameter and 2 m long. A force of 12 N is applied to it and it stretches 0.3 mm. Assume the material is elastic. Determine the following.

- i. The stress in the wire σ .
- ii. The strain in the wire ϵ .

SOLUTION

$$A = \frac{\pi d^2}{4} = \frac{\pi \times 2.5^2}{4} = 4.909 \text{ mm}^2$$

$$\sigma = \frac{F}{A} = \frac{12}{4.909} = 2.44 \text{ N/mm}^2$$

Answer (i) is hence 2.44 MPa

$$\epsilon = \frac{x}{L} = \frac{0.3 \text{ mm}}{2000} = 0.00015 \text{ or } 150 \mu\epsilon$$

SELF ASSESSMENT EXERCISE No.1

1. A steel bar is 10 mm diameter and 2 m long. It is stretched with a force of 20 kN and extends by 0.2 mm. Calculate the stress and strain.

2. A rod is 0.5 m long and 5 mm diameter. It is stretched 0.06 mm by a force of 3 kN. Calculate the stress and strain.

SELF ASSESSMENT EXERCISE No.1

1. A steel bar is 10 mm diameter and 2 m long. It is stretched with a force of 20 kN and extends by 0.2 mm. Calculate the stress and strain.

(Answers 254.6 MPa and 100 $\mu\epsilon$)

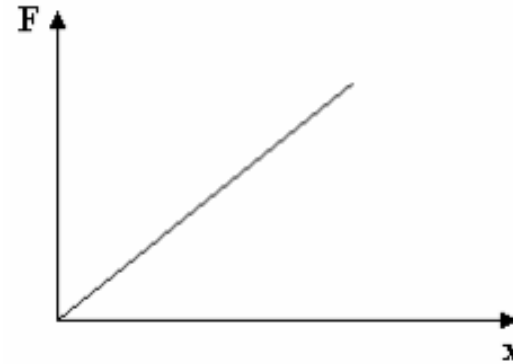
2. A rod is 0.5 m long and 5 mm diameter. It is stretched 0.06 mm by a force of 3 kN. Calculate the stress and strain.

(Answers 152.8 MPa and 120 $\mu\epsilon$)

3. MODULUS OF ELASTICITY E

Elastic materials always spring back into shape when released. They also obey HOOKE'S LAW. This is the law of a spring which states that deformation is directly proportional to the force. $F/x = \text{stiffness} = k \text{ N/m}$

Figure



3

The stiffness is different for different materials and different sizes of the material. We may eliminate the size by using stress and strain instead of force and deformation as follows.

If F and x refer to direct stress and strain then

$$F = \sigma A \quad x = \epsilon L \quad \text{hence} \quad \frac{F}{x} = \frac{\sigma A}{\epsilon L} \quad \text{and} \quad \frac{FL}{Ax} = \frac{\sigma}{\epsilon}$$

The stiffness is now in terms of stress and strain only and this constant is called the **MODULUS of ELASTICITY** and it has a symbol E .

$$E = \frac{FL}{Ax} = \frac{\sigma}{\epsilon}$$

A graph of stress against strain will be a straight line with a gradient of E . The units of E are the same as the units of stress.

4. ULTIMATE TENSILE STRESS

If a material is stretched until it breaks, the tensile stress has reached the absolute limit and this stress level is called the ultimate tensile stress. Values for different materials may be found in various sources such as the web site Matweb.

WORKED EXAMPLE No.2

A steel tensile test specimen has a cross sectional area of 100 mm^2 and a gauge length of 50 mm, the gradient of the elastic section is $410 \times 10^3 \text{ N/mm}$. Determine the modulus of elasticity.

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The gradient gives the ratio $F/A =$ and this may be used to find E .

$$E = \frac{\sigma}{\epsilon} = \frac{F}{x} \times \frac{L}{A} = 410 \times 10^3 \times \frac{50}{100} = 205\,000 \text{ N/mm}^2 \text{ or } 205\,000 \text{ MPa or } 205 \text{ GPa}$$

WORKED EXAMPLE No.3

A Steel column is 3 m long and 0.4 m diameter. It carries a load of 50 MN. Given that the modulus of elasticity is 200 GPa, calculate the compressive stress and strain and determine how much the column is compressed.

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SOLUTION

$$A = \frac{\pi d^2}{4} = \frac{\pi \times 0.4^2}{4} = 0.126 \text{ m}^2$$

$$\sigma = \frac{F}{A} = \frac{50 \times 10^6}{0.126} = 397.9 \times 10^6 \text{ Pa}$$

$$E = \frac{\sigma}{\epsilon} \quad \text{so} \quad \epsilon = \frac{\sigma}{E} = \frac{397.9 \times 10^6}{200 \times 10^9} = 0.001989$$

$$\epsilon = \frac{x}{L} \quad \text{so} \quad x = \epsilon L = 0.001989 \times 3000 \text{ mm} = 5.97 \text{ mm}$$

SELF ASSESSMENT EXERCISE No.2

1. A bar is 500 mm long and is stretched to 505 mm with a force of 50 kN. The bar is 10 mm diameter. Calculate the stress and strain.

The material has remained within the elastic limit. Determine the modulus of elasticity.

2. A steel bar is stressed to 280 MPa. The modulus of elasticity is 205 GPa. The bar is 80 mm diameter and 240 mm long.

Determine the following.

i. The strain.

ii. The force.

3. A circular metal column is to support a load of 500 Tonne and it must not compress more than 0.1 mm. The modulus of elasticity is 210 GPa. the column is 2 m long.

Calculate the cross sectional area and the diameter.

Note 1 Tonne is 1000 kg.

SELF ASSESSMENT EXERCISE No.2

1. A bar is 500 mm long and is stretched to 505 mm with a force of 50 kN. The bar is 10 mm diameter. Calculate the stress and strain.

The material has remained within the elastic limit. Determine the modulus of elasticity.

(Answers 636.6 MPa, 0.01 and 63.66 GPa.

2. A steel bar is stressed to 280 MPa. The modulus of elasticity is 205 GPa. The bar is 80 mm diameter and 240 mm long.

Determine the following.

- i. The strain. (0.00136)
- ii. The force. (1.407 MN)

3. A circular metal column is to support a load of 500 Tonne and it must not compress more than 0.1 mm. The modulus of elasticity is 210 GPa. the column is 2 m long.

Calculate the cross sectional area and the diameter. (0.467 m^2 and 0.771 m)

Note 1 Tonne is 1000 kg.

Hooke's Law – The force needed to extend or compress a spring by a distance is proportional to that distance

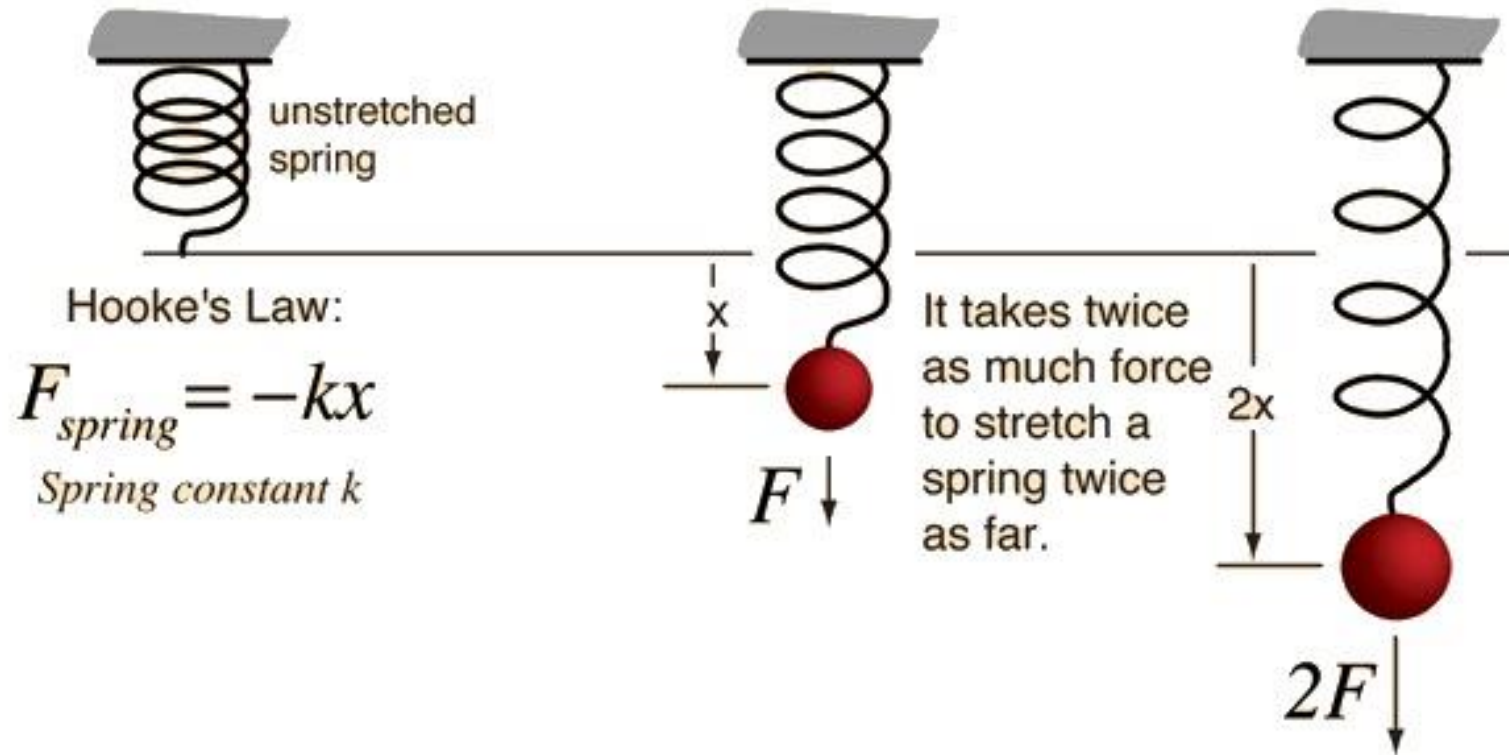
The law is named after 17th century British physicist Robert Hooke, who sought to demonstrate the relationship between the forces applied to a spring and its elasticity. "the extension is proportional to the force" or

$$F = -kx$$

Where F is the force applied to the spring (in the form of strain or stress), x is the displacement of the spring, and k is the spring constant – a measure of stiffness

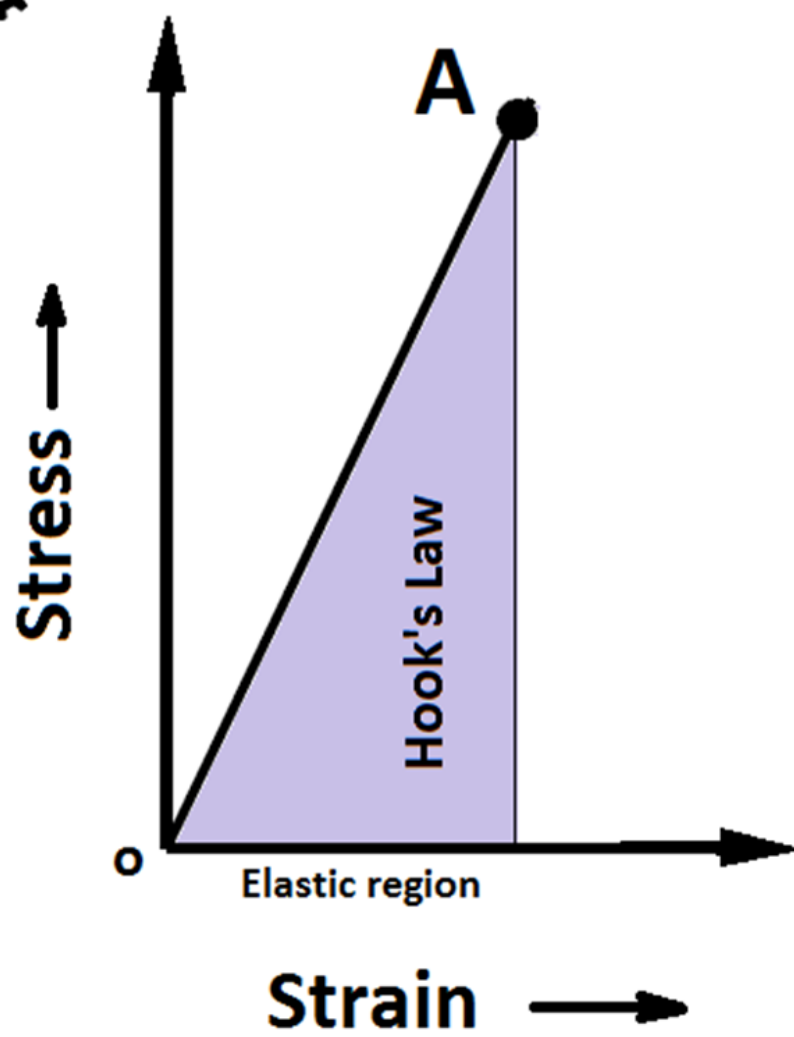
Read more at: <https://phys.org/news/2015-02-law.html#jCp>

Illustration of Hooke's Law



Hooke's Law is compatible with Newton's laws of static equilibrium (an object at rest is in equilibrium or in static equilibrium. An object at rest is described by Newton's First Law of Motion. An object in static equilibrium has zero net force acting upon it).

Hooke's Law is a perfect example of the First Law of Thermodynamics – Any spring when compressed or extended almost perfectly conserves the energy applied to it. The only energy lost is due to natural friction. In addition, Hooke's law contains within it a wave-like periodic function. A spring released from a deformed position will return to its original position with proportional force repeatedly in a periodic function. The wavelength and frequency of the motion can also be observed and calculated.



Stress $\propto$ Strain

$$\sigma \propto \varepsilon$$

$$\sigma = E \varepsilon$$

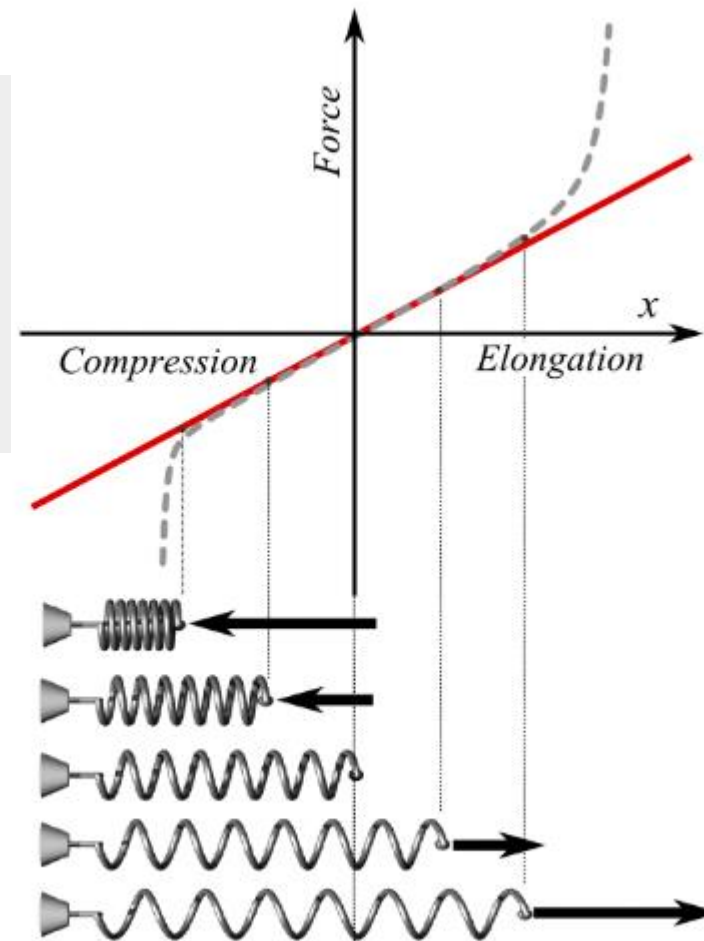
$$E = \frac{\sigma}{\varepsilon}$$

* E = Young's modulus of elasticity

Limitation's of Hooke's law

Many materials obey this law as long as the load does not exceed the material's elastic limit or the limit of proportionality
Use SI units N/m or kg/s².

Hooke's Law: The red line in this graph illustrates how force, F , varies with position according to Hooke's law. The slope of this line corresponds to the spring constant k . The dotted line shows what the actual (experimental) plot of force might look like. The pictures of spring states at the bottom of the graph correspond to some points of the plot; the middle one is in the relaxed state (no force applied).



Elastic Potential Energy

If a force results in only deformation, with no thermal, sound, or kinetic energy, the work done is stored as elastic potential energy.

Express elastic energy stored in a spring in a mathematical form

Elastic Potential Energy

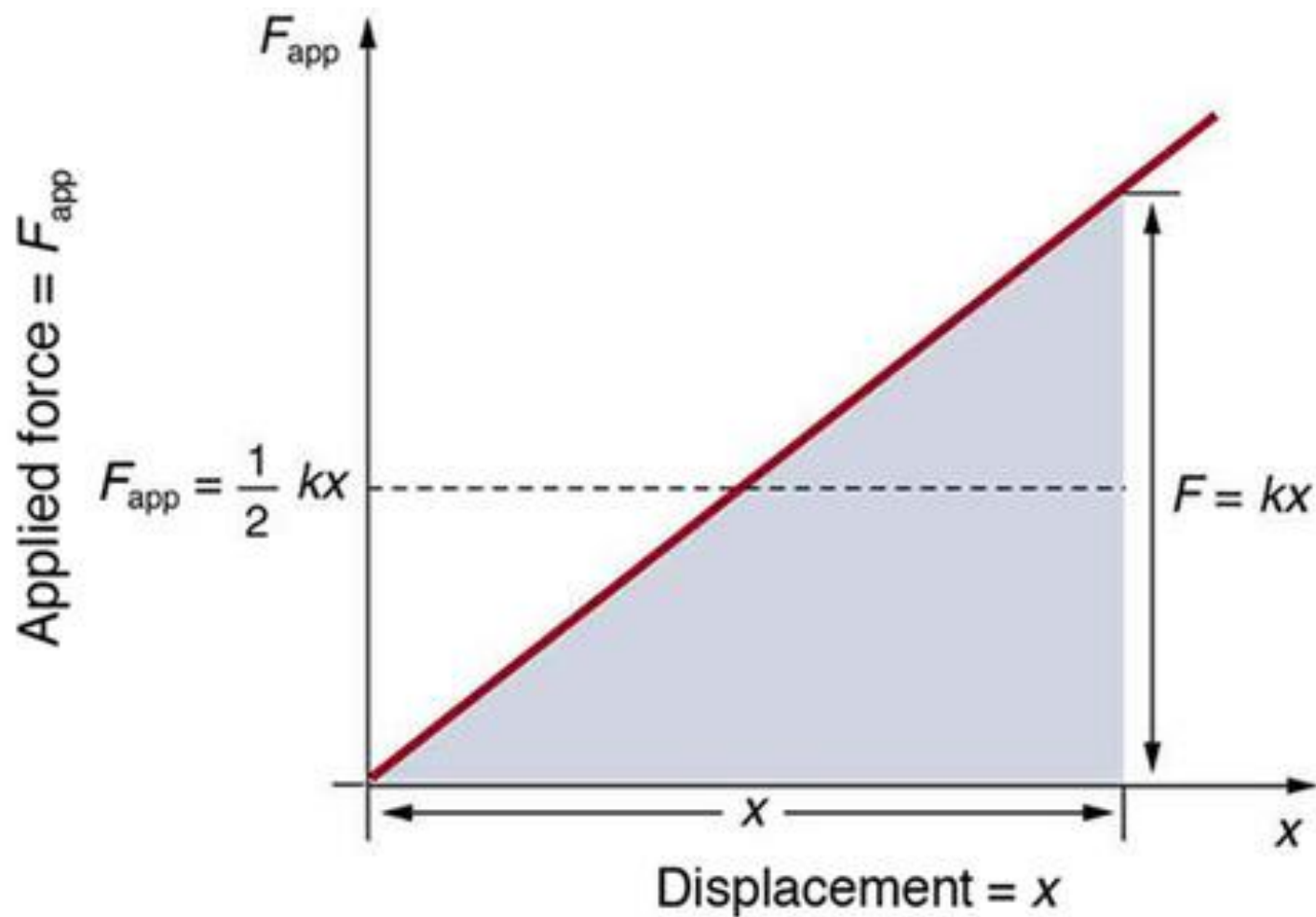
If a force results in only deformation, with no thermal, sound, or kinetic energy, the work done is stored as elastic potential energy.

Express elastic energy stored in a spring in a mathematical form

In order to produce a deformation, work must be done. That is, a force must be exerted through a distance, whether you pluck a guitar string or compress a car spring. If the only result is deformation and no work goes into thermal, sound, or kinetic energy, then all the work is initially stored in the deformed object as some form of potential energy. Elastic energy is the potential mechanical energy stored in the configuration of a material or physical system when work is performed to distort its volume or shape.

A graph shows the applied force versus deformation x for a system that can be described by Hooke's law. Work done on the system is force multiplied by distance, which equals the area under the curve, (Method A in the figure).

Another way to determine the work is to note that the force increases linearly from 0 to kx , so that the average force is $\frac{1}{2} kx$, the distance moved is x , and thus $W = F_{\text{app}} dx = (12kx)(x) = 12kx^2$ (Method B in the figure).



Method A

$$W = \frac{1}{2} bh = \frac{1}{2} kxx$$

$$W = \frac{1}{2} kx^2$$

Method B

$$W = f \cdot x = \left(\frac{1}{2} kx \right) (x)$$

$$W = \frac{1}{2} kx^2$$

- In order to produce a deformation, work must be done.
- The potential energy stored in a spring is given by $PE_{el} = \frac{1}{2}kx^2$ $PE_{el} = \frac{1}{2}kx^2$, where k is the spring constant and x is the displacement.
- Deformation can also be converted into thermal energy or cause an object to begin oscillating.

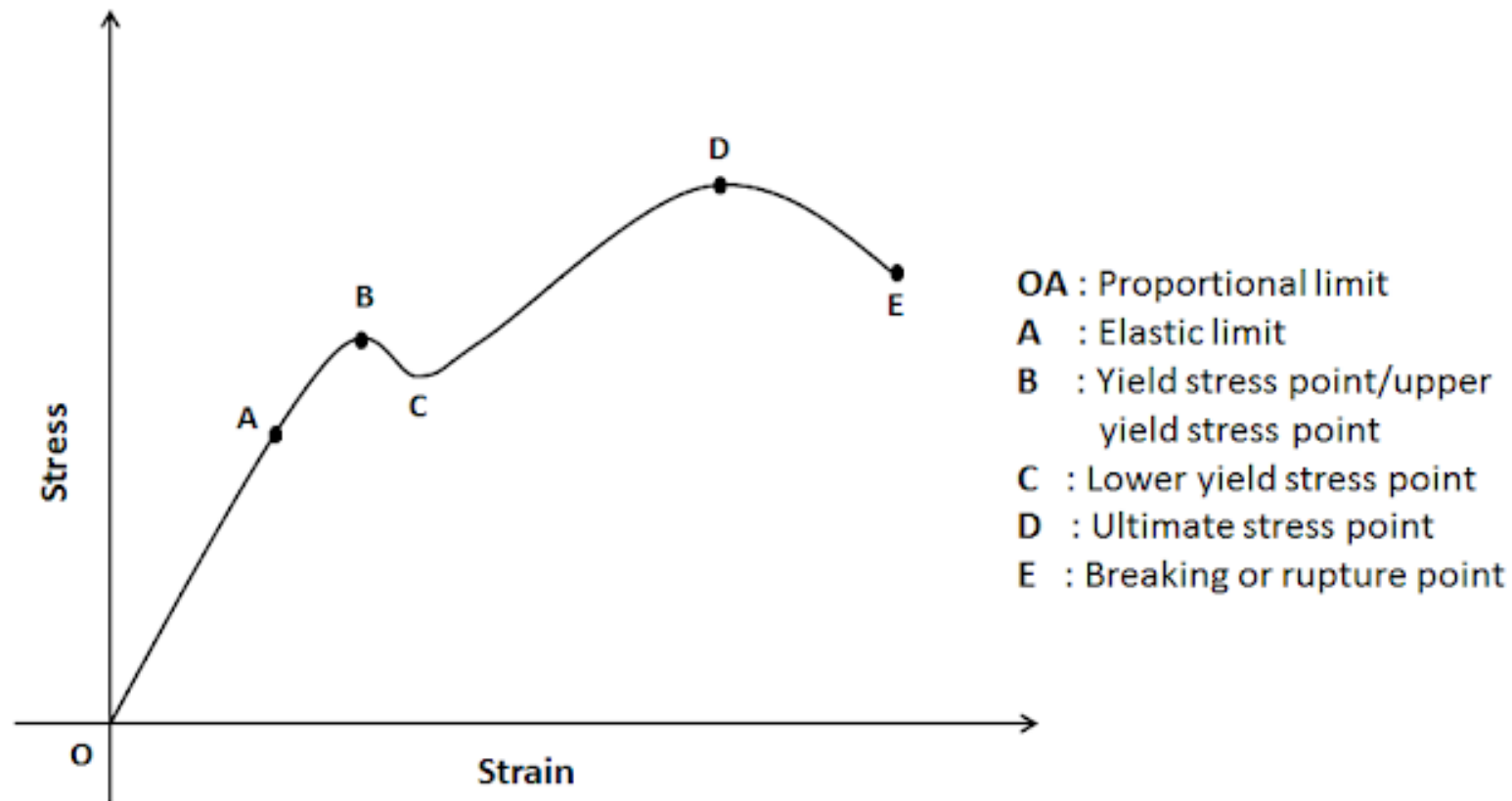
Key Terms

- **deformation**: A transformation; change of shape.
- **kinetic energy**: The energy possessed by an object because of its motion, equal to one half the mass of the body times the square of its velocity.
- **oscillating**: Moving in a repeated back-and-forth motion.

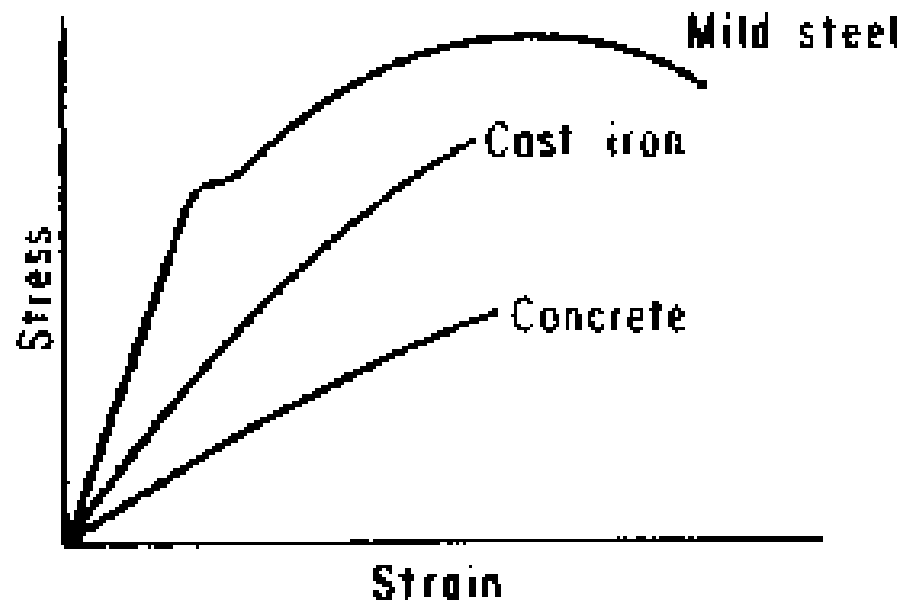
Tensile testing

Tensile testing involves stretching a test piece of the material in a testing machine. The machine indicates the tensile load being applied. The test piece is a circular or rectangular cross sectional bar which has a constant cross section over the gauge length, the length for which the extension is measured. Measurements are made of the extension as the load is gradually increased, Hence the strain is determined at different stresses.

Typical stress strain trace for mild steel, a ductile material

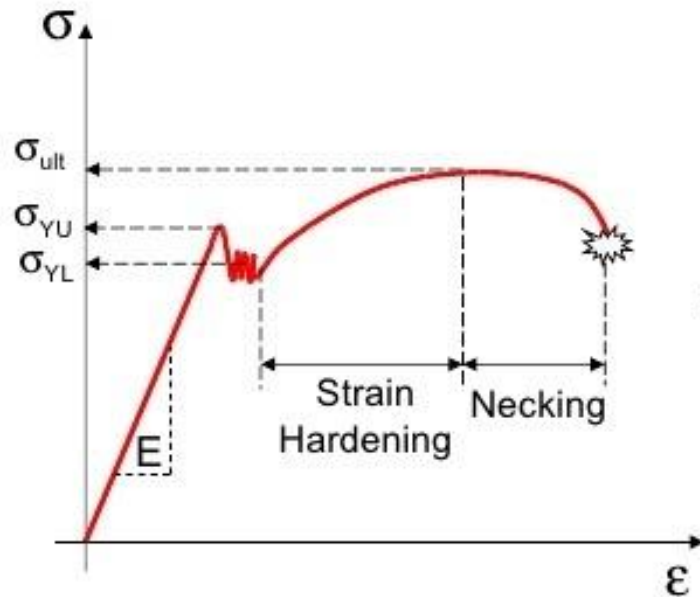


Cast iron and concrete are brittle materials



Elastic-Plastic Stress-Strain Curves

Low / Medium Carbon Steels

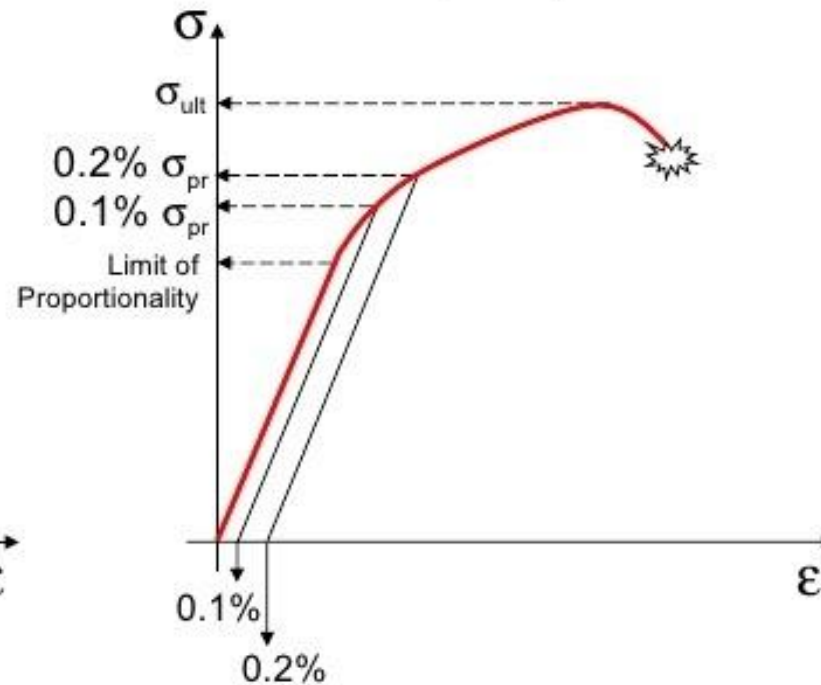


σ_{ult} = Ultimate Strength

σ_{YU} = Upper yield point

σ_{YL} = Lower yield point

Aluminium Alloys and Alloy Steels



σ_{pr} = Proof stress

Proof stress is used for aluminium alloys which do not have a well defined yield stress. 0.2% proof stress is the stress that gives a 0.2% offset. i.e. a line parallel to the linear part of the stress strain graph that cuts the x axis at 0.2%

Factor of safety

In design a factor of safety is added to keep working stresses in safe limits. For a brittle material, the factor of safety is defined as the ratio of tensile strength to the maximum working stress.

For ductile materials it is defined as the ratio of yield or proof stress to the maximum working stress.

For dead loads (unchanging) a factor of safety of 4 or more is used.

12. a) Consider an 95kg climber hangs from a nylon rope and stretches it by 35cm. If the rope was originally 34m long and its diameter was 10mm. (*take $g = 9.81ms^{-2}$*)

Calculate;

(i) the tensile strain

(ii) the tensile stress

(iii) the Young's Modulus.

[6]

b) Sketch a diagram of stress and strain and label with the following letters:

P, Y_1 , Y_2 , B, UTS and E

[6]

P – The limit of proportionality

Y_1 – Yield point 1

Y_2 – Yield point 2

B – Breaking point

UTS – Ultimate tensile stress

E – Elastic limit

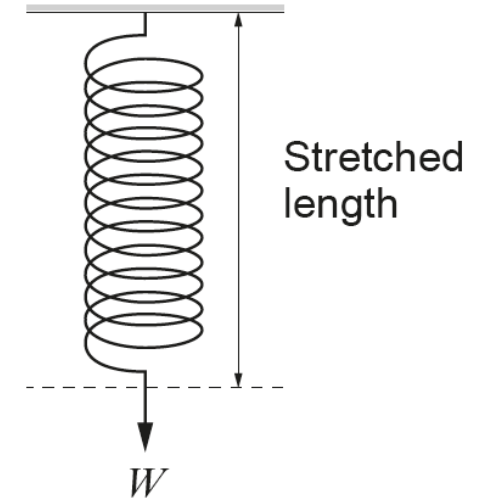
c) i) A second rope of cross sectional area $0.53cm^2$ breaks when the tensile stress exceeds $7 \times 10^6Nm^{-2}$. What is the maximum load that can be lifted by the rope?

ii) Given that the strain experienced by the rope in part c)i) above at a stress of $7 \times 10^6Nm^{-2}$ is 1.5, calculate the Young's modulus of the rope.

Question

- 1a)i) State what is meant by the spring constant, k
ii) Show that the unit of k may be written as kg s^{-2}
1b) A brief experiment is carried out in order to determine k for a spring. The following results are obtained.

Weight, W , attached to spring /N	Stretched length of spring/m
1.0	0.25
5.0	0.45



- b)i) Determine the value of k , stating any assumptions you make. [3]
ii) Determine the unstretched length of the spring. Show your reasoning [2]
iii) Calculate the elastic potential energy stored in the spring when $W = 5.0\text{N}$ [3]

(i) Force per unit extension. Accept $\frac{F}{e}$ (or x) only if e (or x) defined as extension. [1]

(ii) Base units of F shown: kg m s^{-2} (1) [2]

$$\frac{\text{kg m s}^{-2}}{\text{m}} \text{ seen (1)}$$

Alternative:

Base units of W shown: $\text{kg m}^2 \text{s}^{-2}$ (1)

$$\frac{\text{kg m s}^{-2}}{\text{m}} \text{ seen (1)}$$

Alternative:

Rearrangement of $T = 2\pi\sqrt{\frac{m}{k}}$ to show that $k = 4\pi^2 \frac{m}{T^2}$ (1)

Units of k shown to be kg s^{-2} (1)

(b)	(i) 4.0 N extends the spring by 0.20 m seen (1) award extension mark if 0.45 – 0.25 is seen (or 0.20) Or $k = \frac{4.0}{0.20}$ (gains first 2 marks) $k = 20 \text{ [N m}^{-1}\text{]}$ (1) Assumption: Hooke's law obeyed or force proportional to extension or elastic limit not reached (1)	[3]
	(ii) 1.0 N produces an extension of 0.05 m (or any other well-reasoned comment) (1) Original length = 0.20 [m] (1) Award 1 mark only for unsupported answer	[2]

(iii) Extension = 0.25 [m] **ecf** from (ii) (1)

Either:

Correct substitution into $W = \frac{1}{2} Fx$ ($\frac{1}{2} \times 5.0 \times 0.25$) (1)

$W = 0.625$ [J] (1)

Or:

Correct substitution into $W = \frac{1}{2} kx^2$ ($\frac{1}{2} \times 20$ **ecf** $\times 0.25^2$) (1)

$W = 0.625$ [J] (1)

Use of 0.45 m award 0 marks

[3]

Problems

- (1) What is the height to which a vertical concrete wall can be built if the compressive stress must not exceed 4 MPa? Concrete has a density of 2000 kg/m³.
- (2) A lamp of mass 50 kg is supported by two rods from the ceiling. One rod, of diameter 10 mm, is at an angle of 30° to the vertical and the other, of diameter 12 mm, is at 45° to the vertical. What are the direct stresses in the rods?
- (3) A uniform horizontal beam of length 4 m is supported at its ends by two vertical rods. If the rods have cross-sectional areas of 10 mm² and 15 mm², what will be the position of a load of 8 kN along the beam for the stresses in the two rods to be identical?
- (4) A rectangular cross-section bar has a width 100 mm and a thickness 10 mm. An axial tensile load of 100 kN is transmitted to the bar through pins passing through 25 mm diameter holes located near each end of the bar. What is the average stress at a section through the bar at (a) its midpoint and (b) through a hole?
- (5) A yoke and pin joint (as in figure 2.4) has a pin of diameter 20 mm. What will be the maximum force that can be supplied to the rods if the maximum permissible shear stress for the pin is 150 MPa?
- (6) A lap joint has two smooth plates connected by four identical bolts. What will be the maximum permissible tensile load that the joint can withstand if the maximum shear stress a bolt can withstand is 250 MPa and they each have a diameter of 25 mm?
- (7) A plate of mild steel with a shear strength of 200 MPa is to be cropped by a guillotine, i.e. sheared. What force will need to be applied by the guillotine if the plate has a width of 1.0 m and a thickness of 5 mm?
- (8) Two wires are to be used to suspend a lamp from the ceiling. Each wire has a length of 300 mm and makes an angle of 30° with the vertical. When the lamp is hung from the wires it is found that the point of suspension sinks by 3 mm under the weight of the lamp. What is the direct strain in each wire?
- (9) A tie bar has a length of 2.0 m and a diameter of 4 cm. What is the extension of the bar when subject to a tensile axial loading of 100 kN? The material has a tensile modulus of 205 GPa.
- (10) A circular metal rod of diameter 10 mm and length 1.0 m is loaded in tension with an axial tensile load of 5 kN. If the tensile modulus is 70 GPa, what will be the extension of the rod?
- (11) A surveyor's steel tape has a rectangular cross-section of 5 mm by 1 mm and a total length of 30 m. What will be the elongation of the tape when it is fully out and held taut by a force of 40 N? The

- tensile modulus of the tape material may be taken as 200 GPa.
- (12) Show that the total elongation produced by a vertical bar of constant cross-section subject to its own weight is the same as that which would have been produced by a load of half its weight applied to the lower end.
 - (13) A shaft of length L tapers from a radius of r_2 to r_1 . Show that the extension of the shaft when subject to an axial tensile load P is $FL/\pi E r_1 r_2$.
 - (14) A brass rod of diameter 10 mm is heated to 40°C and its ends then clamped to rigid supports. What forces will be exerted on the supports when the rod cools to 20°C ? The linear coefficient of expansion of brass is $16 \times 10^{-6} \text{C}^{-1}$ and the tensile modulus 90 GPa.
 - (15) A steam pipe of length 2.0 m connects two turbines. The pipe has an outer diameter of 100 mm and a wall thickness of 5 mm. If the connection was made at 20°C with no allowance for thermal expansion, what will be the force exerted on the turbines when the pipe reaches a temperature of 100°C and the points of attachment to the turbines are assumed to be rigid? Tensile modulus of steel = 200 GPa, coefficient of linear expansion $11 \times 10^{-6} \text{C}^{-1}$.
 - (16) A bar has a total length of 1.0 m and a constant diameter of 50 mm. Three quarters of the length is steel and a quarter is aluminium. What will be the total extension of the bar when subject to an axial tensile load of 100 kN? The tensile modulus of steel is 200 GPa, and for the aluminium 70 GPa.
 - (17) A plastic bar has a total length of 1.2 m and is made of two parts. The first part has a length of 0.6 m and a diameter of 100 mm and the second part a length of 0.6 m and a diameter of 60 mm. What will be the shortening of the bar when it is subject to an axial force of 100 kN? The compressive modulus of the plastic is 4.0 GPa.
 - (18) A reinforced concrete pillar has a cross-sectional area of $250\,000 \text{ mm}^2$. It contains 10 axial steel reinforcing bars, each having a diameter of 25 mm. What is the maximum axial compressive load that can be applied to the pillar if the maximum stress allowed for the steel is 80 MPa or for the concrete 8 MPa? The modulus of elasticity for the steel is 200 GPa and for the concrete 20 GPa. Ignore the weight of the pillar.
 - (19) A concrete pillar has a cross-sectional area of $160\,000 \text{ mm}^2$ and is reinforced with axial steel rods of diameter 20 mm. How many reinforcement rods are required if, when the compressive load on the column is 4 MN, the stress on the concrete is not to exceed 20 MPa? The modulus of elasticity for the steel is 200 GPa and for the concrete 20 GPa. Ignore the weight of the pillar.

(20)

(21)

(22)

- (20) A steel pipe with an outer diameter of 80 mm and a wall thickness of 10 mm is filled with concrete and the resulting column is subject to an axial compressive load of 60 kN. What are the stresses in the concrete and the steel? The modulus of elasticity for the steel is 200 GPa and for the concrete 20 GPa.
- (21) An aluminium cylinder of diameter 75 mm is located inside a steel cylinder of internal diameter 75 mm and wall thickness 15 mm. The assembly is compressed between two rigid cover plates by an axial force of 200 kN. What are the stresses in the steel and the aluminium? Modulus of elasticity for steel is 200 GPa, for aluminium 25 GPa.
- (22) A steel bolt of diameter 15 mm is enclosed in a copper sleeve of external diameter 32 mm and wall thickness 6 mm. The bolt is used between rigid end plates at both ends of the sleeve. A nut is screwed on to the bolt until it is just a snug fit and there are no stresses in the assembly. The nut is then rotated through 45° . What will be the stresses in the bolt and the sleeve if the screw thread has a pitch of 2 mm? The modulus of elasticity for the steel is 200 GPa and for the copper 100 GPa.
- (23) A composite bar of length 200 mm has 120 mm of its length as a steel bar of diameter 30 mm and the remaining 80 mm as a copper bar of the same diameter. At 0°C the bar is fixed between two rigid end supports and is stress free. What will be the stresses in the two materials when the temperature rises to 100°C ? The modulus of elasticity for the copper is 100 GPa and the coefficient of linear expansion is $20 \times 10^{-6}^\circ\text{C}^{-1}$, with the modulus for the steel being 200 GPa and the coefficient of linear expansion $11 \times 10^{-6}^\circ\text{C}^{-1}$.
- (24) A steel bolt with a cross-sectional area of 400 mm^2 is enclosed in a sleeve of an aluminium alloy tube with a cross-sectional area of 600 mm^2 and the assembly held rigidly together at both ends by the nut. At 20°C the nut is just snug and there are no stresses in the two materials, what will be the stresses when the temperature is raised to 85°C ? The modulus for the steel is 200 GPa and the coefficient of linear expansion $11 \times 10^{-6}^\circ\text{C}^{-1}$, the modulus for the aluminium being 70 GPa and the coefficient of linear expansion $22 \times 10^{-6}^\circ\text{C}^{-1}$.
- (25) An aluminium tube of external diameter 16 mm and wall thickness 2 mm has a brass rod of diameter 12 mm inside it and rigidly joined to it at both ends. At 15°C there are no stresses in the materials, what will the stresses become when the temperature is raised to 40°C ? The aluminium has a modulus of 70 GPa and a coefficient of linear expansion of $23 \times 10^{-6}^\circ\text{C}^{-1}$, the brass a modulus of 90 GPa and a coefficient of linear expansion of $15 \times 10^{-6}^\circ\text{C}^{-1}$.
- (26) An aluminium rod of diameter 22 mm is enclosed in a steel tube of external diameter 28 mm and internal diameter 25 mm. The rod has

- a screw thread at each end and rigid bolts are tightened up so that they just hold the rod in place inside the tube at 20°C . What will be the stresses produced in the materials when the temperature rises by 20°C ? The aluminium has a modulus of 70 GPa and a coefficient of linear expansion of $22 \times 10^{-6}^\circ\text{C}^{-1}$, the steel a modulus of 200 GPa and a coefficient of linear expansion of $11 \times 10^{-6}^\circ\text{C}^{-1}$.
- (27) A steel rod of diameter 30 mm is enclosed in a copper tube of external diameter 60 mm and internal diameter 35 mm . The rod and tube are attached at their ends to rigid cover plates. At 0°C there are no stresses in the materials. What will be the stresses in the materials if an axial compressive load of 50 kN is applied at a temperature of 250°C ? The steel has a modulus of 200 GPa and a coefficient of linear expansion of $11 \times 10^{-6}^\circ\text{C}^{-1}$, the copper a modulus of 100 GPa and a coefficient of linear expansion of $19 \times 10^{-6}^\circ\text{C}^{-1}$.
- (28) A steel bar of length 1.5 m has a rectangular cross-section of 100 mm by 50 mm . What will be the elongation of the bar and the changes in its cross-section when it is subject to an axial force of 120 kN ? The steel has a tensile modulus of 200 GPa and Poisson's ratio of 0.3 .
- (29) A steel bar of length 1.0 m has a square cross-section of side 50 mm . What will be the elongation of the bar and the changes in its cross-section when it is subject to an axial force of 250 kN ? The steel has a tensile modulus of 200 GPa and Poisson's ratio of 0.3 .
- (30) A pressure vessel of internal volume 2.0 m^3 is filled with water at atmospheric pressure. What is the reduction in volume of the water, bulk modulus 2.1 GPa , when the gauge pressure in the vessel is 2 MPa ?

- (17) 276 N
- (18) 208 N, 120 N
- (19) $R_x = 0$, $R_y = 200$ N, $M = 400$ N m
- (20) 660 N, 613 N at 24.1° to the bar
- (21) $R_A = 3.3$ kN, $R_{Fx} = 0$, $R_{Fy} = 1.7$ kN, $F_{BD} = 3.3$ kN, $F_{CD} = 2.4$ kN, $F_{CE} = 1.7$ kN, $F_{DF} = 2.4$ kN. Ties are BC, CD, AC, CE,
- (22) $R_{Ay} = 10$ kN, $R_{Cy} = 10$ kN, $F_{AB} = 14.1$ kN. struts are AB and BC, with AC a tie.
- (23) 2.9 kN
- (24) 11.5 kN
- (25) (a) 6.9 kN, (b) 10 kN
- (26) (a) 2.4 kN, (b) 0.5 kN

Chapter 2

- (1) 204 m
- (2) 4.6 MPa, 2.2 MPa
- (3) 2.4 m
- (4) (a) 10 MPa, (b) 13 MPa
- (5) 94.2 kN
- (6) 491 kN
- (7) 1.0 MN

- (8) 2.6 mm
- (9) 0.78 mm
- (10) 0.91 mm
- (11) 1.2 mm
- (12) Weight acts at centre of gravity, i.e. half bar length
- (13) See earlier in chapter for a similar example
- (14) 2.3 kN
- (15) 1.7 kN
- (16) 0.37 mm
- (17) 7.2 mm
- (18) 2.35 MN
- (19) 26
- (20) 3.84 MPa, 38.4 MPa
- (21) 80 MPa, 10 MPa
- (22) 109 MPa, 70 MPa
- (23) 208 MPa, 208 MPa
- (24) 49 MPa, 33 MPa
- (25) 6.8 MPa, 8.7 MPa
- (26) 62 MPa, 189 MPa
- (27) 197 MPa, 101 MPa
- (28) 0.18 mm, $-3.6 \mu\text{m}$, $-1.8 \mu\text{m}$
- (29) 0.50 mm, $-7.5 \mu\text{m}$
- (30) $1.9 \times 10^{-3} \text{ m}^3$

<https://www.youtube.com/watch?v=8wggwSZNxNUo>