

MATHEMATICS

FORMULAE BOOKLET

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GREEK ALPHABET

The letters of ancient Greece, which are frequently used in maths and science

A, α	alpha
B, β	beta
Γ , γ	gamma
Δ , δ	delta
E, ε	epsilon
Z, ζ	zeta
H, η	eta
Θ , θ	theta
I, ι	iota
K, κ	kappa
Λ , λ	lambda
M, μ	mu
N, ν	nu
Ξ , ξ	xi
O, $\omicron$	omicron
Π , π	pi
P, ρ	rho
Σ , σ	sigma
T, τ	tau
Y, υ	upsilon
Φ , ϕ	phi
X, χ	chi
Ψ , ψ	psi
Ω , ω	omega

LAWS OF INDICES

Definitions

$$a^n = a \times a \times a \times \cdots \times a \quad (n \text{ times})$$

$$a^0 = 1 \quad (a \neq 0)$$

$$a^{-n} = \frac{1}{a^n} \quad (a \neq 0)$$

$$a^{m/n} = \sqrt[n]{a^m} \text{ or } (\sqrt[n]{a})^m \quad (a \geq 0, m \geq 0, n > 0)$$

Combining

Multiplication:

$$a^x a^y = a^{x+y}$$

Division:

$$\frac{a^x}{a^y} = a^{x-y} \quad (a \neq 0)$$

Powers:

$$(a^x)^y = a^{x \cdot y}$$

Distributing $(a \geq 0, b \geq 0)$

$$(a \cdot b)^x = a^x \cdot b^x$$

$$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x} \quad (b \neq 0)$$

Careful!!

$$(a + b)^n \neq a^n + b^n$$

$$(a - b)^n \neq a^n - b^n$$

LOGARITHM RULES

For the following, assume that x, y, a , and b are all positive. Also assume that $a \neq 1, b \neq 1$.

Definitions

1. $\log_a x = N$ means that $a^N = x$.
2. $\log x$ means $\log_{10} x$. All $\log_a$ rules apply for $\log$. When a logarithm is written without a base it means common logarithm.
3. $\ln x$ means $\log_e x$, where e is about 2.718. All $\log_a$ rules apply for $\ln$. When a logarithm is written "ln" it means natural logarithm.

Rules

1. Inverse properties:

$$\log_a a^x = x \quad \text{and} \quad a^{(\log_a x)} = x$$

2. Product:

$$\log_a(x \cdot y) = \log_a x + \log_a y$$

3. Quotient:

$$\log_a \left(\frac{x}{y} \right) = \log_a x - \log_a y$$

4. Power:

$$\log_a(x^p) = p \cdot \log_a x$$

5. Change of base formula:

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Careful!!

$$\log_a(x + y) \neq \log_a x + \log_a y$$

$$\log_a(x - y) \neq \log_a x - \log_a y$$

FACTORING RULES

DIFFERENCE OF SQUARES

$$a^2 - b^2 = (a + b)(a - b)$$

SUM OF SQUARES

$$a^2 + b^2 = (a + b + \sqrt{2ab})(a + b - \sqrt{2ab})$$

This identity is always true, but it's useful for factoring only **when 2AB is a perfect square**.

FORM $a^2 - (r + s)a + r.s$

$$a^2 - (r + s)a + r.s = (a - r)(a - s)$$

FORM $a^2 \pm 2ab + b^2$

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

SUM AND DIFFERENCE OF CUBES

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

QUADRATIC FORMULA

If we consider a typical quadratic equation:

$$ax^2 + bx + c = 0$$

The solution of such an equation is given by the following **quadratic formula**:

$$x = \frac{-b \pm \sqrt{\text{discriminant}}}{2a} \quad \text{where the discriminant } \Delta = b^2 - 4ac$$

❖ If the discriminant is positive there are two real roots (i.e. solutions) x_1 and x_2 :

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$
$$x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Thus the quadratic equation can be factorised as follows:

$$ax^2 + bx + c = a \cdot \left(x - \frac{-b + \sqrt{b^2 - 4ac}}{2a} \right) \cdot \left(x - \frac{-b - \sqrt{b^2 - 4ac}}{2a} \right)$$

❖ If the discriminant is zero, there is exactly one distinct root:

$$x = \frac{-b}{2a}$$

Thus the quadratic equation can be factorised as follows:

$$ax^2 + bx + c = a \cdot \left(x + \frac{b}{2a} \right)^2$$

❖ And if the discriminant is negative, there are no real roots.

PARTIAL FRACTIONS

Provided that the **numerator** $f(x)$ is of **less degree than the denominator**, the following illustrate the form of partial fractions.

Linear Factors:

$$\frac{f(x)}{(x+a)(x+b)} = \frac{A}{x+a} + \frac{B}{x+b}$$

Repeated Linear Factors:

$$\frac{f(x)}{(x+a)^n} = \sum_{i=1}^n \frac{A_i}{(x+a)^i}$$

Quadratic Factor:

$$\frac{f(x)}{ax^2 + bx + c} = \frac{Ax + B}{ax^2 + bx + c}$$

Example:

$$\frac{10x-11}{(x-2)(2x+1)} = \frac{A}{(x-2)} + \frac{B}{(2x+1)}$$

$$10x - 11 = A(2x + 1) + B(x - 2)$$

Let $x = 2$. Then $20 - 11 = A(4 + 1) + B \cdot 0$ and so $9 = 5A$ and **$A = 9/5$**

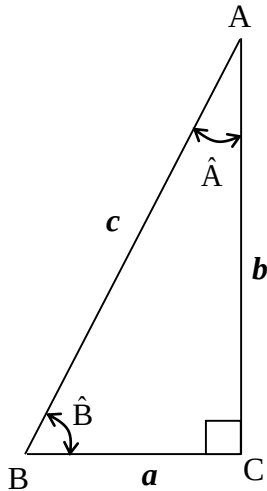
Let $x = -1/2$. Then $-5 - 11 = A \cdot 0 + B(-5/2)$ and so **$B = 32/5$**

Therefore,
$$\frac{10x-11}{(x-2)(2x+1)} = \frac{9}{5(x-2)} + \frac{32}{5(2x+1)}$$

TRIGONOMETRY

Trigonometric Ratios in Right-Angled Triangles

Given the right-angled triangle ABC as shown in the opposite figure, we define the trigonometric ratios as:



S O C A T O
H H A

$$\sin \hat{A} = \frac{\text{side opposite to angle } \hat{A}}{\text{hypotenuse}} = \frac{a}{c}$$

$$\cos \hat{A} = \frac{\text{side adjacent to angle } \hat{A}}{\text{hypotenuse}} = \frac{b}{c}$$

$$\tan \hat{A} = \frac{\text{side opposite to angle } \hat{A}}{\text{side adjacent to angle } \hat{A}} = \frac{a}{b}$$

$$\sin \hat{B} = \frac{\text{side opposite to angle } \hat{B}}{\text{hypotenuse}} = \frac{b}{c}$$

$$\cos \hat{B} = \frac{\text{side adjacent to angle } \hat{B}}{\text{hypotenuse}} = \frac{a}{c}$$

$$\tan \hat{B} = \frac{\text{side opposite to angle } \hat{B}}{\text{side adjacent to angle } \hat{B}} = \frac{b}{a}$$

$$\sin \hat{A} = \sin(180^\circ - \hat{A})$$

and

$$\cos \hat{A} = \cos(360^\circ - \hat{A})$$

Solution of Non-Right-Angled Triangles

The Sine Rule:

The sine rule is used when given:

1. Two angles and any side.

THE AMBIGUOUS CASE → 2. Two sides and an angle opposite to one of the given sides.

The sine rule states:

$$\frac{a}{\sin \hat{A}} = \frac{b}{\sin \hat{B}} = \frac{c}{\sin \hat{C}}$$

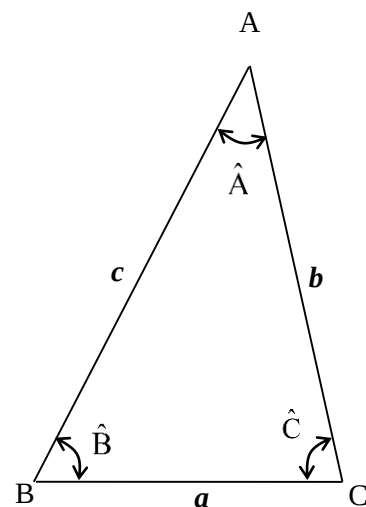
The Cosine Rule:

The cosine rule is used when given:

1. Two sides and the angle between them.
2. All three sides.

The cosine rule states:

$$\begin{aligned} a^2 &= b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos \hat{A} \\ b^2 &= a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos \hat{B} \\ c^2 &= a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos \hat{C} \end{aligned}$$



Area of any triangle

- **Basic Formula:**

$$\text{Area of triangle } ABC = \frac{1}{2} \times \text{base} \times \text{vertical height}$$

- **Formula using 2 sides and the angle between them:**

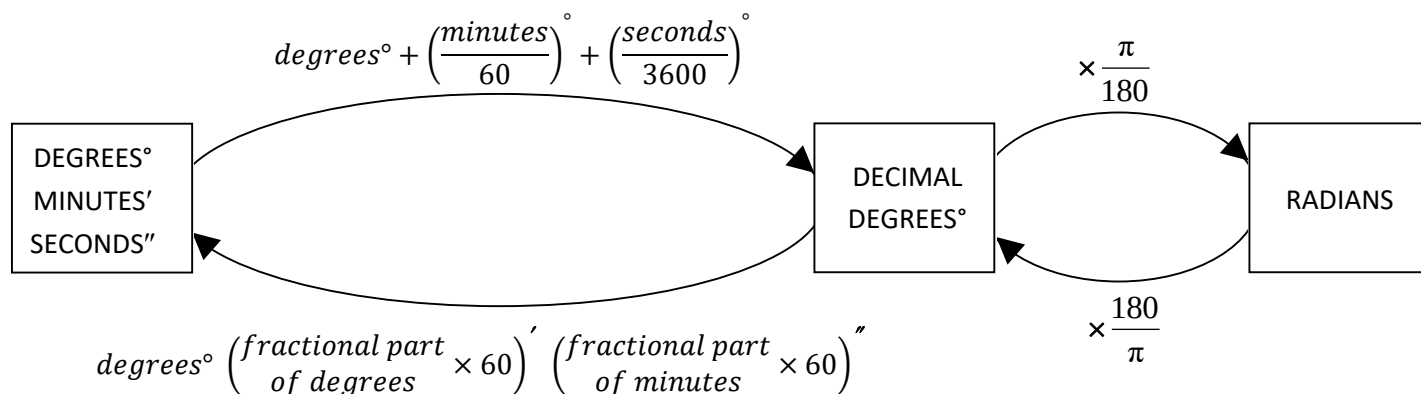
$$\text{Area of triangle } ABC = \frac{1}{2} \times a \times b \times \sin \hat{C}$$

- **Heron's formula using the 3 sides:**

$$\text{Area of triangle } ABC = \sqrt{s \cdot (s - a) \cdot (s - b) \cdot (s - c)}$$

$$\text{where } s = \frac{a + b + c}{2}$$

Angle units



Arc length

$$\text{circumference of a circle} = 2\pi \times \text{radius}$$

$$\text{Length of an arc} = \text{angle in radians} \times \text{radius}$$

TRIGONOMETRIC IDENTITIES

<u>Pythagorean Identities</u>	<u>Reciprocal Identities</u>	<u>Ratio Identities</u>
$\sin^2 x + \cos^2 x = 1$	$\sin x = \frac{1}{\csc x}$	$\tan x = \frac{\sin x}{\cos x}$
$\tan^2 x + 1 = \sec^2 x$	$\cos x = \frac{1}{\sec x}$	$\cot x = \frac{\cos x}{\sin x}$
$\cot^2 x + 1 = \csc^2 x$	$\tan x = \frac{1}{\cot x}$	
	$\cot x = \frac{1}{\tan x}$	

<u>Odd/Even Identities</u>	
$\sin(-x) = -\sin x$	$\csc(-x) = -\csc x$
$\cos(-x) = \cos x$	$\sec(-x) = \sec x$
$\tan(-x) = -\tan x$	$\cot(-x) = -\cot x$

<u>Cofunction Identities, radians</u>		<u>Cofunction Identities, degrees</u>	
$\sin\left(\frac{\pi}{2} - x\right) = \cos x$	$\cos\left(\frac{\pi}{2} - x\right) = \sin x$	$\sin(90^\circ - x) = \cos x$	$\cos(90^\circ - x) = \sin x$
$\tan\left(\frac{\pi}{2} - x\right) = \cot x$	$\cot\left(\frac{\pi}{2} - x\right) = \tan x$	$\tan(90^\circ - x) = \cot x$	$\cot(90^\circ - x) = \tan x$
$\sec\left(\frac{\pi}{2} - x\right) = \csc x$	$\csc\left(\frac{\pi}{2} - x\right) = \sec x$	$\sec(90^\circ - x) = \csc x$	$\csc(90^\circ - x) = \sec x$

<u>Periodicity Identities, radians</u>	
$\sin(x + 2\pi) = \sin x$	$\csc(x + 2\pi) = \csc x$
$\cos(x + 2\pi) = \cos x$	$\sec(x + 2\pi) = \sec x$
$\tan(x + \pi) = \tan x$	$\cot(x + \pi) = \cot x$

<u>Periodicity Identities, degrees</u>	
$\sin(x + 360^\circ) = \sin x$	$\csc(x + 360^\circ) = \csc x$
$\cos(x + 360^\circ) = \cos x$	$\sec(x + 360^\circ) = \sec x$
$\tan(x + 180^\circ) = \tan x$	$\cot(x + 180^\circ) = \cot x$

<u>Sum/Difference Identities</u>		<u>Double Angle Identities</u>
$\sin(x + y) = \sin x \cos y + \cos x \sin y$ $\sin(x - y) = \sin x \cos y - \cos x \sin y$ $\cos(x + y) = \cos x \cos y - \sin x \sin y$ $\cos(x - y) = \cos x \cos y + \sin x \sin y$ $\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$ $\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$		$\sin 2x = 2 \sin x \cos x$ $\cos 2x = \begin{cases} \cos^2 x - \sin^2 x \\ 1 - 2 \sin^2 x \\ 2 \cos^2 x - 1 \end{cases}$ $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$

<u>Half Angle Identities</u>
$\sin \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{2}} \quad \text{or} \quad \sin^2 x = \frac{1 - \cos 2x}{2}$ $\cos \frac{x}{2} = \pm \sqrt{\frac{1 + \cos x}{2}} \quad \text{or} \quad \cos^2 x = \frac{1 + \cos 2x}{2}$ $\tan \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{1 + \cos x}} \quad \text{or} \quad \frac{\sin x}{1 + \cos x} \quad \text{or} \quad \frac{1 - \cos x}{\sin x}$

<u>Product to Sum Identities</u>		<u>Sum to Product Identities</u>
$\cos x \cos y = \frac{1}{2} [\cos(x + y) + \cos(x - y)]$ $\sin x \sin y = \frac{1}{2} [\cos(x - y) - \cos(x + y)]$ $\sin x \cos y = \frac{1}{2} [\sin(x + y) + \sin(x - y)]$		$\sin x + \sin y = 2 \sin \left(\frac{x + y}{2} \right) \cos \left(\frac{x - y}{2} \right)$ $\sin x - \sin y = 2 \cos \left(\frac{x + y}{2} \right) \sin \left(\frac{x - y}{2} \right)$ $\cos x + \cos y = 2 \cos \left(\frac{x + y}{2} \right) \cos \left(\frac{x - y}{2} \right)$ $\cos x - \cos y = -2 \sin \left(\frac{x + y}{2} \right) \sin \left(\frac{x - y}{2} \right)$

DIFFERENTIATION FORMULAE

For the following, assume that f and g are functions of x

PROCEDURAL RULES

Constant multiple

$$\frac{d}{dx}(cf) = c \frac{df}{dx}$$

Difference rule

$$\frac{d}{dx}(f - g) = \frac{df}{dx} - \frac{dg}{dx}$$

Product rule

$$\frac{d}{dx}(fg) = f \frac{dg}{dx} + \frac{df}{dx} g$$

Chain rule

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

Sum rule

$$\frac{d}{dx}(f + g) = \frac{df}{dx} + \frac{dg}{dx}$$

Linearity rule

$$\frac{d}{dx}(af + bg) = a \frac{df}{dx} + b \frac{dg}{dx}$$

Quotient rule

$$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{\frac{df}{dx}g - f\frac{dg}{dx}}{g^2}$$

BASIC FORMULAE

Extended power rule

$$\frac{d}{dx}f^n = nf^{n-1} \frac{df}{dx}$$

Trigonometric rules

$$\frac{d}{dx}\cos f = -\sin f \frac{df}{dx}$$

$$\frac{d}{dx}\tan f = \sec^2 f \frac{df}{dx}$$

$$\frac{d}{dx}\sec f = \sec f \tan f \frac{df}{dx}$$

$$\frac{d}{dx}\sin f = \cos f \frac{df}{dx}$$

$$\frac{d}{dx}\cot f = -\csc^2 f \frac{df}{dx}$$

$$\frac{d}{dx}\csc f = -\csc f \cot f \frac{df}{dx}$$

Inverse trigonometric rules

$$\frac{d}{dx}\cos^{-1} f = \frac{-1}{\sqrt{1-f^2}} \frac{df}{dx}$$

$$\frac{d}{dx}\tan^{-1} f = \frac{1}{1+f^2} \frac{df}{dx}$$

$$\frac{d}{dx}\sec^{-1} f = \frac{1}{|f|\sqrt{f^2-1}} \frac{df}{dx}$$

$$\frac{d}{dx}\sin^{-1} f = \frac{1}{\sqrt{1-f^2}} \frac{df}{dx}$$

$$\frac{d}{dx}\cot^{-1} f = \frac{-1}{1+f^2} \frac{df}{dx}$$

$$\frac{d}{dx}\csc^{-1} f = \frac{-1}{|f|\sqrt{f^2-1}} \frac{df}{dx}$$

Logarithmic rules

$$\frac{d}{dx}\ln f = \frac{1}{f} \frac{df}{dx}$$

$$\frac{d}{dx}\log_b|f| = \frac{\log_b e}{f} \frac{df}{dx} = \frac{1}{f \ln b} \frac{df}{dx}$$

Exponential rules

$$\frac{d}{dx}e^f = e^f \frac{df}{dx}$$

$$\frac{d}{dx}b^f = b^f \ln b \frac{df}{dx}$$

INTEGRATION FORMULAE

PROCEDURAL RULES

Constant multiple	$\int cf(x)dx = c \int f(x)dx$
Sum rule	$\int [f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx$
Difference rule	$\int [f(x) - g(x)]dx = \int f(x)dx - \int g(x)dx$
Linearity rule	$\int [af(x) + bg(x)]dx = a \int f(x)dx + b \int g(x)dx$

BASIC FORMULAE

Constant rule

$$\int 0dx = C$$

Power rules

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C; n \neq -1$$

$$\int x^{-1} dx = \ln|x| + C$$

Exponential rules

$$\int e^x dx = e^x + C$$

$$\int a^x dx = \frac{a^x}{\ln a} + C; a > 0, a \neq 1$$

Logarithmic rule

$$\int \ln x dx = x \ln x - x + C; x > 0$$

Trigonometric rules

$$\int \sin x dx = -\cos x + C$$

$$\int \cos x dx = \sin x + C$$

$$\int \tan x dx = -\ln|\cos x| + C$$

$$\int \cot x dx = \ln|\sin x| + C$$

$$\int \sec x dx = -\ln|\sec x + \tan x| + C$$

$$\int \csc x dx = -\ln|\csc x + \cot x| + C$$

$$\int \sec^2 x dx = \tan x + C$$

$$\int \csc^2 x dx = -\cot x + C$$

$$\int \sec x \tan x dx = \sec x + C$$

$$\int \csc x \cot x dx = -\csc x + C$$

Cosine squared formula

$$\int \cos^2 x dx = \frac{1}{2}x + \frac{1}{4}\sin 2x + C$$

Sine squared formula

$$\int \sin^2 x dx = \frac{1}{2}x - \frac{1}{4}\sin 2x + C$$

Inverse rules

$$\int \frac{1}{\sqrt{a^2 + x^2}} dx = \sin^{-1} \frac{x}{a} + C$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$$

$$\int \frac{1}{x\sqrt{x^2 - a^2}} dx = \frac{1}{a} \sec^{-1} \frac{|x|}{a} + C$$

MATRICES

Basic Operations

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $B = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$, and k a constant number

Then:

$$A + B = \begin{bmatrix} a + e & b + f \\ c + g & d + h \end{bmatrix}$$

$$A - B = \begin{bmatrix} a - e & b - f \\ c - g & d - h \end{bmatrix}$$

$$k \times A = \begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix}$$

$$A \times B = \begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix}$$

Transpose

If M is the general $m \times n$ matrix then the $n \times m$ matrix obtained from M by interchanging the rows and columns is called the transpose of M , written M^T (in some books M').

Determinants

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

Inverse

$$A^{-1} = \left(\frac{1}{\det A} \right) \text{adj } A$$

Where the adjoint of A , denoted by $\text{adj } A$, is the transpose of the matrix of cofactors.

$$\therefore A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \text{ for a } 2 \times 2$$

Multiplication of a (4x4) matrix by another (4x4) is:-

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{pmatrix}$$

$$\begin{pmatrix} (a_{11}*b_{11}+a_{12}*b_{21}+a_{13}*b_{31}+a_{14}*b_{41}) & (a_{11}*b_{12}+a_{12}*b_{22}+a_{13}*b_{32}+a_{14}*b_{42}) & (a_{11}*b_{13}+a_{12}*b_{23}+a_{13}*b_{33}+a_{14}*b_{43}) & (a_{11}*b_{14}+a_{12}*b_{24}+a_{13}*b_{34}+a_{14}*b_{44}) \\ (a_{21}*b_{11}+a_{22}*b_{21}+a_{23}*b_{31}+a_{24}*b_{41}) & (a_{21}*b_{12}+a_{22}*b_{22}+a_{23}*b_{32}+a_{24}*b_{42}) & (a_{21}*b_{13}+a_{22}*b_{23}+a_{23}*b_{33}+a_{24}*b_{43}) & (a_{21}*b_{14}+a_{22}*b_{24}+a_{23}*b_{34}+a_{24}*b_{44}) \\ (a_{31}*b_{11}+a_{32}*b_{21}+a_{33}*b_{31}+a_{34}*b_{41}) & (a_{31}*b_{12}+a_{32}*b_{22}+a_{33}*b_{32}+a_{34}*b_{42}) & (a_{31}*b_{13}+a_{32}*b_{23}+a_{33}*b_{33}+a_{34}*b_{43}) & (a_{31}*b_{14}+a_{32}*b_{24}+a_{33}*b_{34}+a_{34}*b_{44}) \\ (a_{41}*b_{11}+a_{42}*b_{21}+a_{43}*b_{31}+a_{44}*b_{41}) & (a_{41}*b_{12}+a_{42}*b_{22}+a_{43}*b_{32}+a_{44}*b_{42}) & (a_{41}*b_{13}+a_{42}*b_{23}+a_{43}*b_{33}+a_{44}*b_{43}) & (a_{41}*b_{14}+a_{42}*b_{24}+a_{43}*b_{34}+a_{44}*b_{44}) \end{pmatrix}$$

Multiplication of a (4x4) matrix by a (4x1) matrix is:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} b_{11} \\ b_{21} \\ b_{31} \\ b_{41} \end{bmatrix} = \begin{bmatrix} (a_{11} * b_{11} + a_{12} * b_{21} + a_{13} * b_{31} + a_{14} * b_{41}) \\ (a_{21} * b_{11} + a_{22} * b_{21} + a_{23} * b_{31} + a_{24} * b_{41}) \\ (a_{31} * b_{11} + a_{32} * b_{21} + a_{33} * b_{31} + a_{34} * b_{41}) \\ (a_{41} * b_{11} + a_{42} * b_{21} + a_{43} * b_{31} + a_{44} * b_{41}) \end{bmatrix}$$

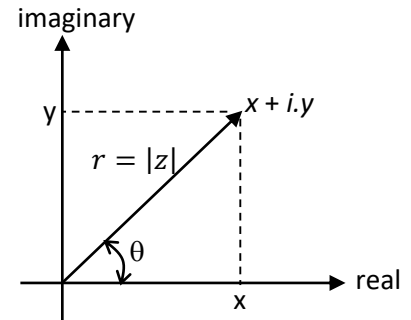
COMPLEX NUMBERS

The complex number z has the form

$$z = x + i.y$$

Where:

- $i^2 = -1$
- x is called the real part Re
- y is called the imaginary part Im



In terms of polar coordinates:

- $x = r \cos \theta$
- $y = r \sin \theta$

Therefore:

$$z = x + i.y = r(\cos \theta + i.\sin \theta)$$

Using Euler's formula

- $e^{i\theta} = \cos \theta + i.\sin \theta$
- $e^{-i\theta} = \cos \theta - i.\sin \theta$

The complex number z can be written:

$$z = x + i.y = r(\cos \theta + i.\sin \theta) = re^{i\theta}$$

Where:

- Modulus $r = |z| = \sqrt{x^2 + y^2}$
- Argument $\theta = \arg(z) = \tan^{-1} \frac{y}{x}$

Multiplication

$$z_1 z_2 = r_1 r_2 e^{i(\theta_1 + \theta_2)}$$

Division

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} e^{i(\theta_1 - \theta_2)}$$