

Finite element analysis of the impact of liner thickness and hydrodynamic limit on the penetration depth of a shaped charge warhead[†]

Youngku Kang^{*} and Jincheol Jeon

Defense R&D Center, Hanwha Corporation, 10, Yuseong-daero 1366 beon-gil, Yuseong-gu, Daejeon 34101, Korea

(Manuscript Received March 23, 2018; Revised August 15, 2018; Accepted August 30, 2018)

Abstract

In this work, an identification method for the hydrodynamic limit of shaped charge jets (SCJs) is proposed using numerical analysis. To identify the hydrodynamic limit, we consider situations where two targets of the same density but different strengths are penetrated by the same SCJ. As a result, the SCJ corresponding with the hydrodynamic theory is a jet region with a velocity larger than 4 km/s. In addition, an investigation based on the hydrodynamic limit and liner thickness indicates that the penetration capability before and after the hydrodynamic limit improves as apex thickness decreases and base thickness increases, respectively. The simple and clear identification of the hydrodynamic limit is expected to be possible using the proposed method. Accordingly, a selective and organized liner thickness design can be developed.

Keywords: Shaped charge warhead; Hydrodynamic theory; Hydrodynamic limit; Penetration depth; Numerical analysis; Target density; Target strengths

1. Introduction

High stagnation pressure occurs at the point where a target collides with a hypervelocity penetrator. This pressure exceeds the strengths of the two materials. Hydrodynamic theory describes this phenomenon with the assumption of incompressible and steady state flow [1]. In this paper, V and U are the velocities of the penetrator and stagnation point in the fixed reference frame, respectively. If these velocities are represented by a moving frame attached to the stagnation point, then the moving frame translates at velocity U and the penetrator strikes the origin of the moving frame at velocity $V-U$. The momentum balance relationship at the stagnation point can be expressed as Eq. (1). Moreover, it can be replaced with Eq. (2) in terms of velocity and density.

$$\frac{1}{2} \rho_t U^2 = \frac{1}{2} \rho_j (V-U)^2, \quad (1)$$

$$\frac{V-U}{U} = \sqrt{\rho_t / \rho_j}, \quad (2)$$

where ρ_j and ρ_t represent the densities of the penetrator and target, respectively. The density ratio is constant over the entire penetration process. Eq. (1) can be expressed as Eq. (3).

$$U = \frac{V}{1 + \sqrt{\rho_t / \rho_j}}. \quad (3)$$

If V is constant, by integrating Eq. (2) over the total penetration time, then U and $V-U$ can be expressed with penetration depth P and consumed penetrator material length L . Hence, Eq. (4) can be obtained.

$$\frac{P}{L} = \sqrt{\rho_j / \rho_t}. \quad (4)$$

Therefore, by comparing Eqs. (2) and (4), the hydrodynamic theory is expressed as the ratio of density and velocity regardless of strength.

Shaped charge jets (SCJ) have a velocity gradient that decreases from its front end to its rear end. Thus, V , U and L are not constant. Therefore, the penetration of SCJs cannot be explained fully by hydrodynamic theory. The hydrodynamic theory can only be applied to the initial parts of SCJs. Hydrodynamic theory has been complemented by adding terms related to strength [2]. The consistency of this theory has been confirmed to be valid only for jets above a certain velocity [3]. Experiments and numerical analysis have reported that the final penetration depth of a shaped charge warhead decreases as the yield strength of a target increases [4]. Zellner et al. showed the penetration phenomena can be well expressed by only the effect of density when a sufficiently fast jet penetrates

^{*}Corresponding author. Tel.: +82 42 336 0571, Fax.: +82 505 730 1663
E-mail address: yk8063.kang@hanwha.com

[†]Recommended by Associate Editor Ki-Hoon Shin

© KSME & Springer 2018

a ceramic target (higher strength but a lower density than those of steel) [5]. Previous studies [2, 4] have indicated the influence of strength cannot be neglected from the moment of penetration. However, considering the results of References Refs. [3, 5], jet velocities have a range where the influence of target strength on SCJ penetration is negligible.

The limitations of the hydrodynamic theory have been explored in numerous studies. This theory remains valid in the field of the hypervelocity penetration phenomenon. However, the theory is insufficient for predicting the performance of various shaped charge designs. Several studies have measured V or P to assess the penetration performance. However, SCJ penetration should be described in two stages according to hydrodynamic theory. The first is the hypervelocity penetration stage corresponding with hydrodynamic theory. The second is the stage of kinetic energy penetration by the following jet slug. The first and second stages are referred to as hydrodynamic and nonhydrodynamic penetration, respectively. The boundary is considered as the hydrodynamic limit.

The hydrodynamic limit should be identified in advance in studies on SCJ penetration. This recommendation is because selective improvement can be achieved by distinguishing the penetration process before and after the hydrodynamic limit. In this paper, a method of identifying the SCJ hydrodynamic limit using numerical analysis was proposed. The investigation of liner thickness based on the hydrodynamic limit was performed. As a result, the hydrodynamic limit was found to be $V = 4$ km/s. In addition, penetration depths before and after the hydrodynamic limit improved as the apex thickness decreased and the base thickness increased, respectively.

2. Approach

2.1 Hydrodynamic limit identification

Because the velocity of a nonhydrodynamic jet is relatively low, stagnation pressure decreases, and the influence of target strength increases. Therefore, velocity boundary where the density ratio is not established throughout the penetration process and the influence of strength is significant can be regarded as the hydrodynamic limit. To identify this boundary clearly, we considered situations where two targets of the same density but different strengths are penetrated by the same SCJ. Examples of these targets are AISI 1045 and AISI 4340. If similar shaped charges detonate, then SCJs are formed with the same length and velocity distribution until they reach each target. The penetration of the hydrodynamic jet is irrelevant to the target strength. Thus, the penetration depth of each target is almost the same until this jet is consumed. Thereafter, given that the influence of strength is strong in the non-hydrodynamic jet, a considerable difference is observed in the penetration depth.

2.2 BRL 81 mm shaped charge simulation model

A numerical study for the US Army Ballistic Research

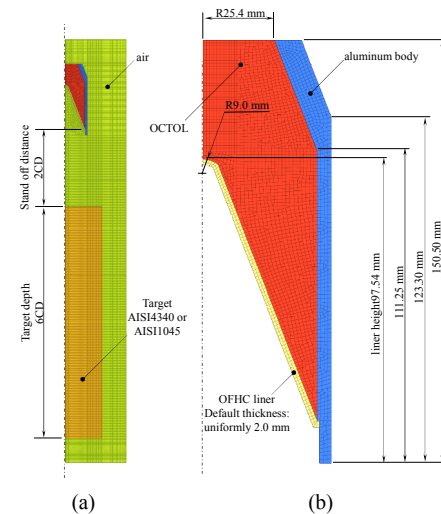


Fig. 1. BRL 81 mm shaped charge warhead simulation model: (a) Two-CD stand-off distance penetration model; (b) detailed specification of Ballistic Research Laboratory (BRL) 81 mm shaped charge warhead.

Laboratory (BRL) was conducted by Bolstad and Mandell using MESA-2D [6]. The jet tip velocity and penetration depth obtained from this numerical analysis were compared with the experimental studies of DiPersio et al. [7]. MESA-2D predicts that the maximum velocity of a jet tip approaches 8.3 km/s. Additionally, the total penetration depth of a target with a Brinell hardness number (BHN) of 320 is approximately 5 charge diameters (CDs) [6], which is slightly higher than the results of DiPersio et al. (4.6–4.9 CD) [7].

This shaped charge warhead contains a liner with a uniform thickness of 2.0 mm comprised of oxygen-free high-thermal-conductivity (OFHC) copper. The density of the OFHC copper (ρ_j) is 8950 kg/m³. The target material is steel with a BHN of 320. The density of RHA (ρ_t) is 7850 kg/m³. Hence, the ratio of densities is $\gamma = (\rho_j / \rho_t)^{1/2} = 1.06$. In this study, BHN 320 RHA was replaced by AISI 4340 steel.

The present authors reproduced the abovementioned studies through a simulation using the LS-DYNA 971 R7 solver. The stand-off distance was set as 2 CDs (1 CD = 81 mm). Target thickness was set as 6 CDs to capture the final penetration state. A two-dimensional axisymmetric model was adopted, because the BRL 81 mm shaped charge has a cylindrical body (Fig. 1).

A detonation was initiated from the back of the BRL 81 mm shaped charge. When the high-pressure wave generated from the detonation point was transmitted to the apex of the liner, the liner was transformed into a jet with an elongated forward shape due to the Munroe (or Neumann) effect. In the finite element analysis using Lagrangian formulation, which describes the behavior of such large deformation phenomena by tracing all nodal points, considerable difficulties in terms of excessive time step reduction because of severe element distortion and the degradation of solution convergence were experienced. Hence, we applied the Eulerian formulation to de-

Table 1. Parameters of the JWL equation of state.

	ρ	D	P_{CJ}	A	B	R_1	R_2	w	E
	(kg/m ³)	(m/s)	(GPa)	(GPa)	(GPa)	-	-	-	J/m ³
OCTOL	1821	8480	34.3	748.6	13.38	4.50	1.00	0.38	5.27E+09

Table 2. Material parameters of the Johnson-Cook constitutive equation and the Mie–Grüneisen equation of state.

	ρ	A_0	B_0	n	C_0	M	T_m	C	S_1	γ_0
	(kg/m ³)	(GPa)	(GPa)	-	-	-	(K)	(m/s)	-	-
OFHC	8960	0.092	0.292	0.31	0.025	1.09	1356	3940	1.489	2.02
Aluminum	2810	0.052	0.477	0.52	0.0025	1.61	893	4596	1.49	2.17
Target (AISI 4340)	7850	0.553	0.6008	0.234	0.013	1.00	1733	4570	1.49	1.954
Target (AISI 1045)	7850	0.792	0.51	0.26	0.014	1.03	1793	3935	1.578	1.69

scribe the behavior of the liner material moving through fixed grids. In addition, boundary conditions were applied to limit the movement of materials transverse to the axis of symmetry. LS-DYNA has a function to calculate an appropriate time step automatically in accordance with the deformation state of the elements. However, for the finite element analysis of materials that experience severe deformation, such as a shaped charge liner, a time step smaller than the automatically calculated time step is required. Therefore, we set the time step calculated by LS-DYNA to be interpreted as artificially reduced by 0.2 times. Generally, an extremely fine mesh distribution is required to describe an accurate behavior. However, the reduction in mesh size involves a large increase in execution time for analysis. Therefore, the multimaterial arbitrary-Lagrange option, which enables a variety of materials to exist within one Eulerian grid, was added to reduce the execution time for analysis.

The components of the shaped charge warhead sustain a large deformation under a high strain rate load because of explosive pressure. The hardening effects from such conditions were considered by applying the Johnson-Cook constitutive relationship.

$$\sigma_y = (A_0 + B_0 \varepsilon_p^n) \left(1 + C_0 \ln \frac{\dot{\varepsilon}_p}{\dot{\varepsilon}_0} \right) \left\{ 1 - \left(\frac{T - T_r}{T_m - T_r} \right)^m \right\}, \quad (5)$$

where σ_y is the flow stress, ε_p is the equivalent plastic strain, $\dot{\varepsilon}_p$ is the plastic strain rate, $\dot{\varepsilon}_0$ is the effective plastic strain rate, T_m is the melting point, and T_r is the ambient temperature. A_0 , B_0 , C_0 , m and n are the parameters of the Johnson-Cook constitutive equation.

The relationship between the pressure and volume of the components due to hypervelocity impact was considered with the Mie–Grüneisen equation of state.

$$p = \frac{\rho_0 C^2 \mu \left\{ 1 + \left(1 - \frac{\gamma_0}{2} \right) \mu \right\}}{\left\{ 1 - (S_1 - 1) \mu \right\}^2}, \quad (6)$$

where p is the pressure, C is the speed of sound, S_1 is the coefficient of the Hugoniot curve, and $\mu = \rho / \rho_0 - 1$ refers to the ratio of initial and current densities.

The relationship between the pressure and volume of the OCTOL high explosive was considered with Jones–Wilkins–Lee (JWL) equation of state.

$$p = A \left(1 - \frac{\omega}{R_1 v} \right) e^{-R_1 v} + B \left(1 - \frac{\omega}{R_2 v} \right) e^{-R_2 v} + \frac{\omega E}{v}, \quad (7)$$

where $v = \rho_0 / \rho$ is the ratio of the densities of explosive (ρ_0 , solid part) and detonation products (ρ), E is the internal energy, D is the Chapman–Jouguet (CJ) detonation velocity, P_{CJ} is the CJ pressure, and A , B , R_1 , R_2 and ω are the JWL parameters.

The parameters of the JWL and Mie–Grüneisen equations of state and the Johnson-Cook constitutive relationship [8] are listed in Tables 1 and 2.

The accuracy of the finite element analysis depends strongly on mesh size. The most important area in the analysis of the shaped charge warhead is the area around the axis of symmetry, where the jet is formed and the target is penetrated. In this study, the BRL 81 mm shaped charge penetration analysis model was set to have the minimum element size (representative element size: h) in this area. In particular, while maintaining h uniformly in the penetration direction, the size of the element was gradually increased by h as the coordinate increased in the radial direction to prevent an excessive increase in the degree of freedom (Fig. 2(a)). Based on these mesh generation criteria, the maximum jet velocity and maximum penetration depth for h ranging from 0.25 mm to 1.0 mm were compared with MESA-2D [6] and DiPersio et al. [7]. Fig. 3 and Table 3 show the comparison of the maximum jet velocity and target penetration depth in accordance with h . The comparison indicates that the analysis result clearly converges as h decreases. However, the analysis result for $h = 0.50$ mm is in a good agreement with Refs. [6, 7]; whereas that for $h = 0.25$ mm is slightly higher. Although the analysis results for $h = 0.25$ mm and $h = 0.50$ mm do not show substantial differences, the representative element size of this

Table 3. Analysis case setting based on liner apex and base thickness.

h	N	P_h	V_h
(mm)	-	(CD)	(km/s)
0.25	1,944	5.23	8.419
0.50	972	4.84	8.270
0.75	648	4.17	8.071
1.00	486	1.65	7.862
MESA-2D [6]	-	5.40	8.300
DiPersio et al. [7]	-	4.60 ~ 4.90	-

h : Representative mesh size of BRL 81 mm shaped charge model

N : Element layer number throughout target depth (486 mm / h)

P_h , V_h : penetration depth and maximum jet tip velocity for h

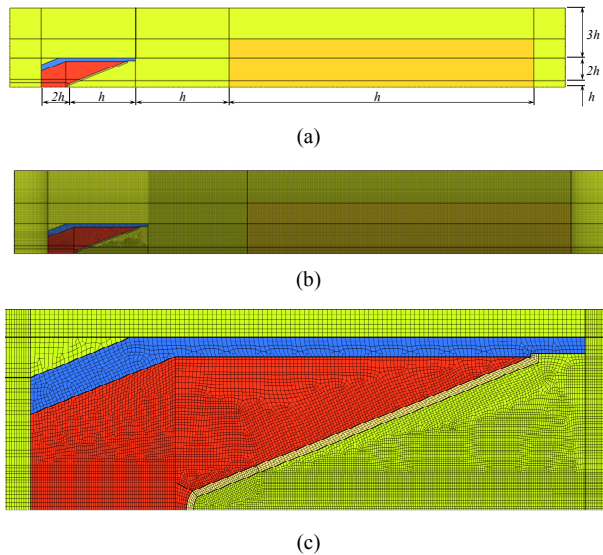


Fig. 2. Mesh generation for the BRL 81 mm shaped charge warhead: (a) Area segmentation for setting the element size per area; (b) mesh distribution of the entire analysis area ($h = 0.50$ mm); (c) detailed element distribution of BRL 81 mm shaped charge warhead.

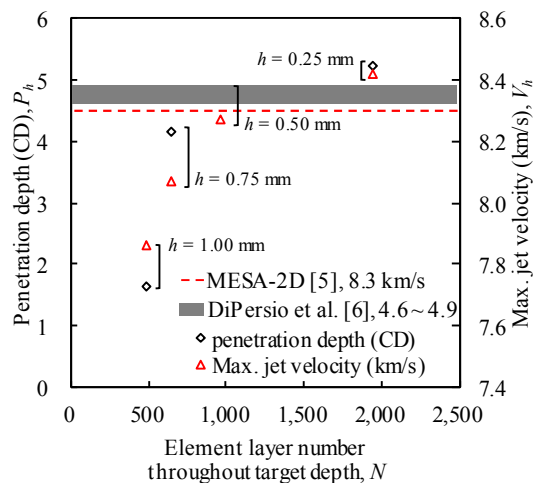


Fig. 3. Maximum jet velocity and maximum penetration depth change in accordance with the representative element size. As the number of element layers in the target height of 6 CDs (486 mm) increases, the tendency to converge to a specific value is confirmed.

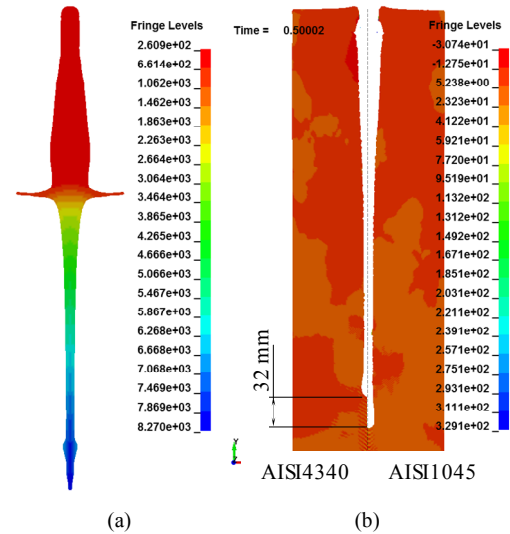


Fig. 4. Simulation results of BRL 81 mm shaped charge: (a) Jet velocity distribution immediately before the collision. The maximum velocity = 8.27 km/s; (b) comparison of total penetration between AISI 4340 and AISI 1045 at the termination time ($t = 500$ μs).

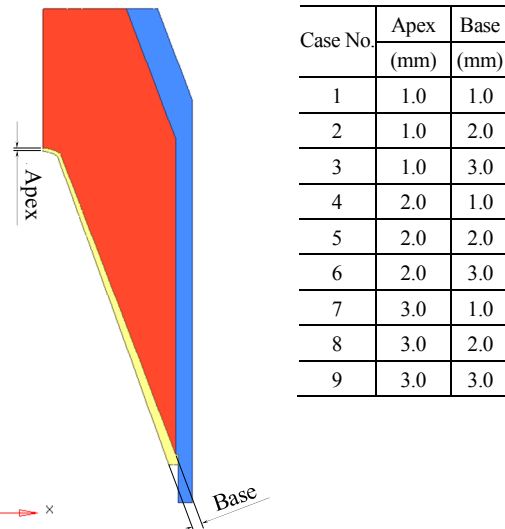


Fig. 5. Thickness change locations. Thickness was changed only on the outer surface of liner, and only the effect of the liner design on penetration characteristics is considered.

study was specified as $h = 0.50$ mm in consideration of the execution time of analysis.

The results of the penetration analysis of the AISI 1045 and AISI 4340 targets are shown in Fig. 4. In the case of the two-CD stand-off distance, the total penetration depths of the AISI 4340 and AISI 1045 targets obtained through simulation are 4.84 (392) and 5.23 CDs (424 mm), respectively.

2.3 Liner design parameter study setup

A case study was conducted on the variation in the thickness of the liner to investigate the penetration capability before and after the hydrodynamic limit. The abovementioned analysis is

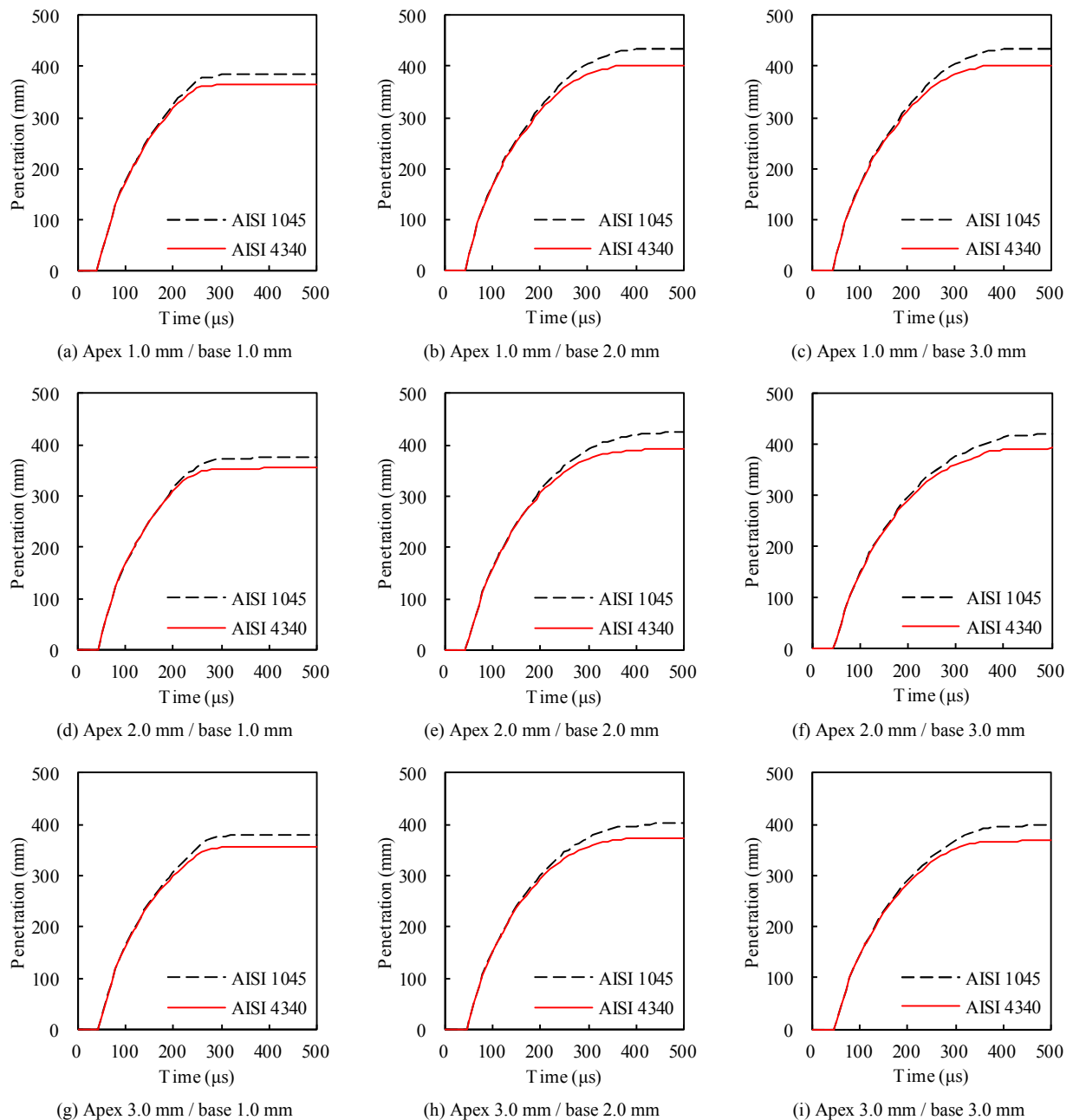


Fig. 6. Penetration depth by analysis case over time. In the AISI 1045 target, which has less strength than that of AISI 4340, the deeper penetration is shown. From the start of the penetration, the penetration depth is almost the same, until 70–80 μs has elapsed. Afterwards, however, the penetration depth differences gradually increase. In the case of a thin base thickness, the penetration is terminated relatively early in time.

for one case of the uniform thickness liner. Nine analysis cases where the thickness of the apex and base was changed by ± 1.0 mm were compared (Fig. 5). The thickness of the outer surface was changed to consider only the influence of the liner thickness. All cases were set to have the same body design.

3. Results

3.1 Hydrodynamic limit

The changes in penetration with time for each simulation

case are shown in Fig. 6. The initial collision time of the jet tip and target is similar in all the cases, which is approximately 43 μs . The penetration amount of the AISI 1045 and AISI 4340 targets is almost the same during 70–80 μs . However, a measurable difference is observable at the end of penetration depth. The penetration of the AISI 1045 target is deeper than that of the AISI 4340 target, thereby revealing the influence of the target strength. To ascertain the difference between the penetration amounts of the two targets, the accumulated penetration depth difference in the entire penetration process ($\Delta P(t_i)$)

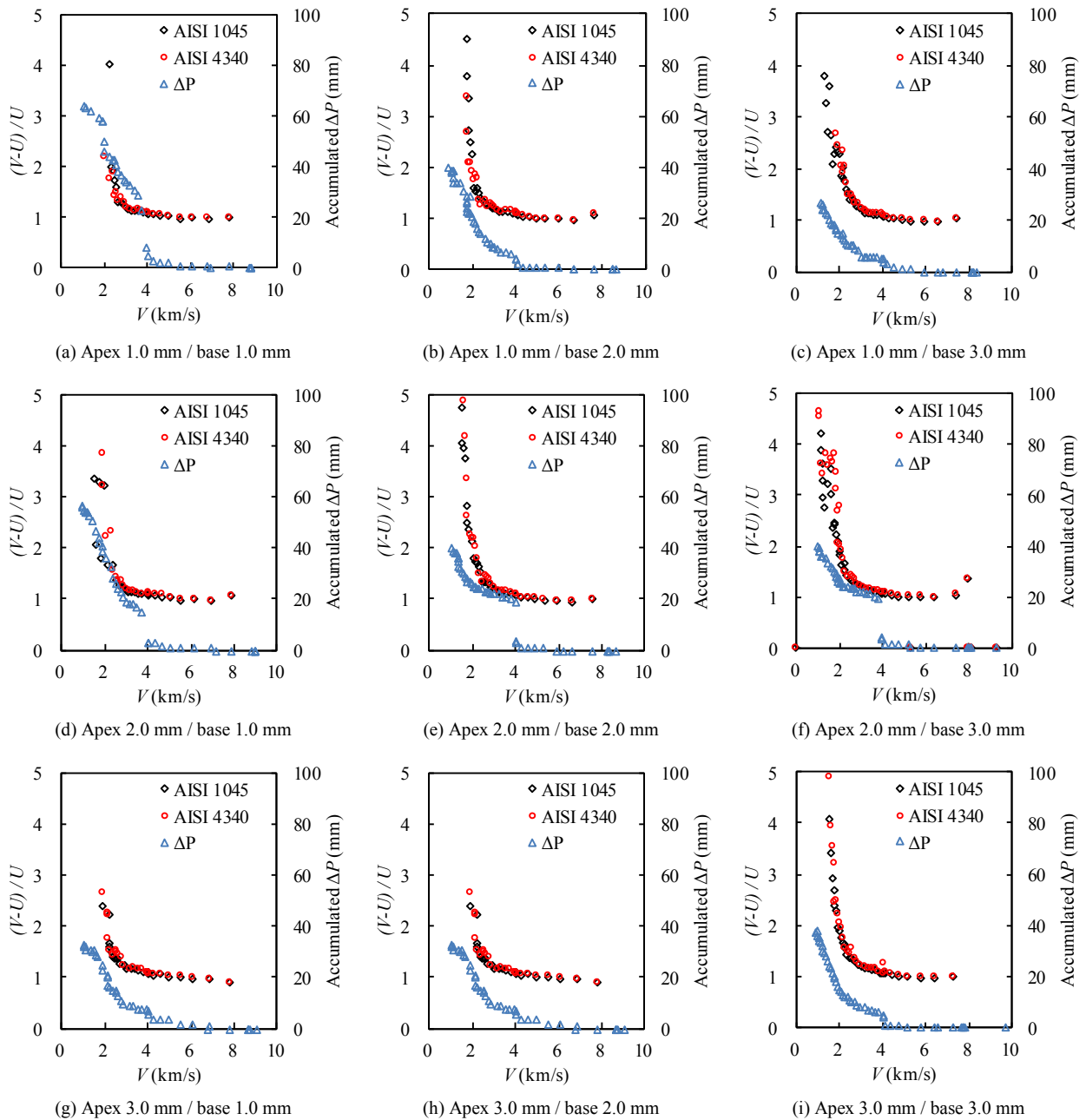


Fig. 7. Comparison of ratios of velocities, $(V-U)/U$ ($\diamond$ AISI 1045, $\circ$ AISI 4340) and accumulated penetration depth differences between AISI 1045 and AISI 4340, ΔP ($\triangle$). In all analysis cases, the ratios of velocities were approximated to the ratio between the densities of OFHC and steel: $1/\gamma = (\rho_i / \rho_j)^{1/2} = (V-U)/U = 0.94$, until V was larger than 4 km/s. However, as V reduces to be less than 4 km/s, the ratios of velocities do not correspond with the ratios of densities. Moreover, the accumulated penetration depth difference rapidly increases.

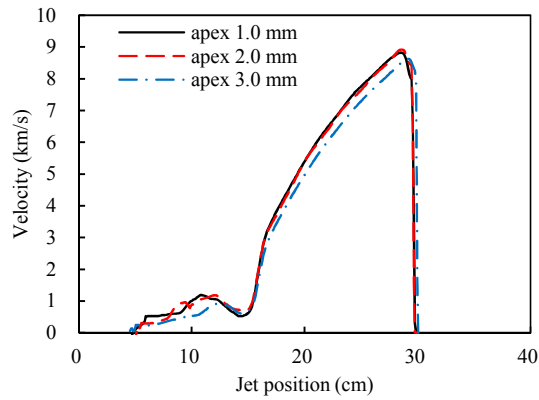
was defined as follows:

$$\Delta P(t_i) = \sum_{t=0}^{t_i} |dP_{A4340} - dP_{A1045}|_{t=t_i}, \quad (8)$$

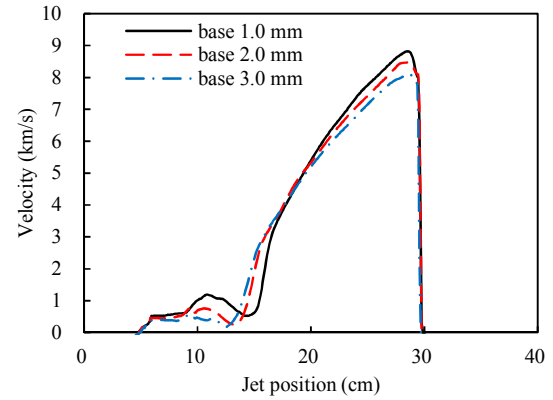
where dP_{A4340} and dP_{A1045} denote the penetration increment in AISI 4340 and AISI 1045, respectively.

Fig. 7 shows the ratio of the velocities of the two targets de-

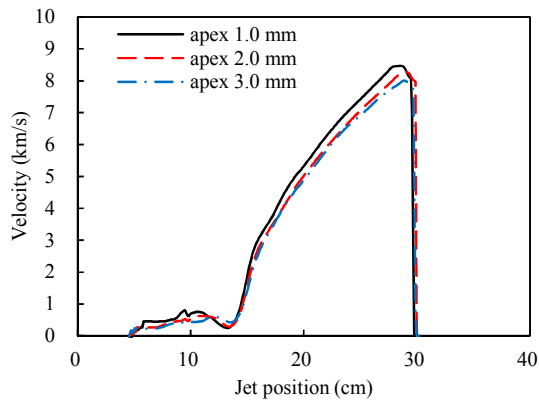
fined by Eq. (2). This ratio was evaluated using the V and U obtained from the numerical analysis. The ratio of the velocities agrees well with the ratio of densities, $1/\gamma = (\rho_i / \rho_j)^{1/2} = 0.94$, in the jet part at a velocity exceeding 4 km/s. Therefore, the hydrodynamic theory is satisfied in jets at a velocity above 4 km/s. However, when V is less than 4 km/s, the ratio of the densities of both targets and ΔP rapidly increase. This increase appears at approximately $V = 4$ km/s. Therefore, the hydrodynamic limit is identified as 4 km/s.



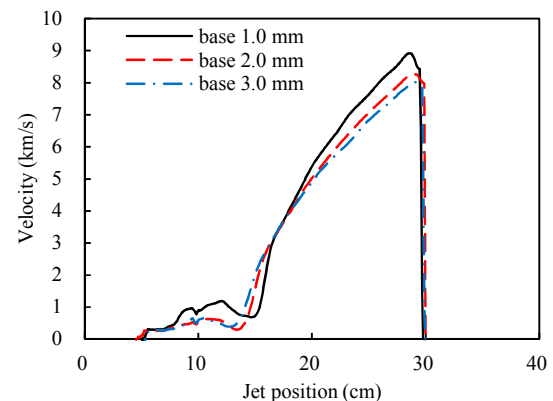
(a) Base thickness = 1.0 mm



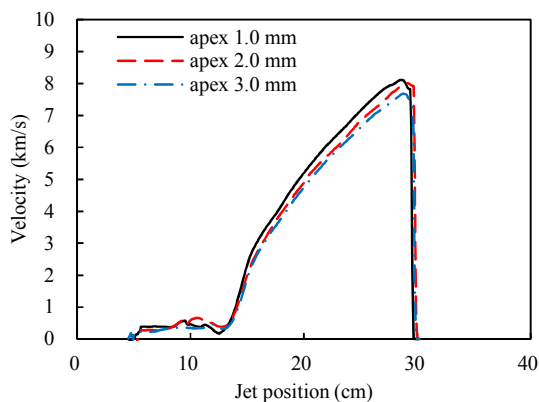
(a) Apex thickness = 1.0 mm



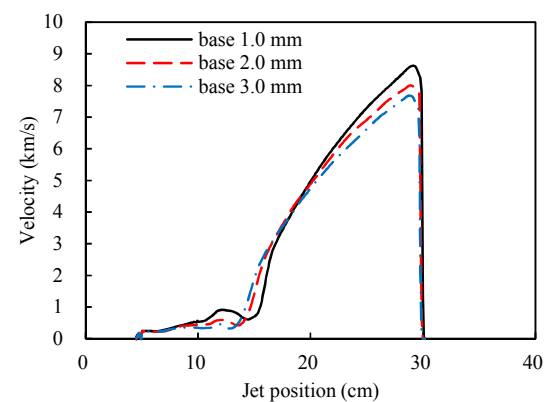
(b) Base thickness = 2.0 mm



(b) Apex thickness = 2.0 mm



(c) Base thickness = 3.0 mm



(c) Apex thickness = 3.0 mm

Fig. 8. Jet velocity distributions immediately before colliding with the target. For the same base thickness, the distributions of the high velocity region increase as the apex thickness decreases. However, the distribution of the low velocity region doesn't show a strong relevance.

3.2 Influence of changes in liner thickness on the penetration of hydrodynamic and nonhydrodynamic jets

The jet velocity distributions immediately before the collision with the target are shown in Figs. 8 and 9. For the same base thickness (Fig. 8), as the apex thickness increases, the velocity of the hydrodynamic jet ($V \geq 4$ km/s) decreases; whereas, that of the nonhydrodynamic jet ($V < 4$ km/s) does

Fig. 9. Jet velocity distributions immediately before colliding with the target. For the same apex thickness, the distributions of the high velocity region increase as the base thickness decreases. However, the distribution of the low velocity region decreases with base thickness.

not change remarkably. For the same apex thickness (Fig. 9), as the base thickness increases, the velocity of the hydrodynamic jet ($V \geq 4$ km/s) decreases; whereas, that of the nonhydrodynamic jet ($V < 4$ km/s) increases.

The comparisons between the hydrodynamic and nonhydrodynamic penetration depths are shown in Fig. 10. For the same base thickness, the hydrodynamic penetration depth decreases as the apex thickness increases. On the contrary, for

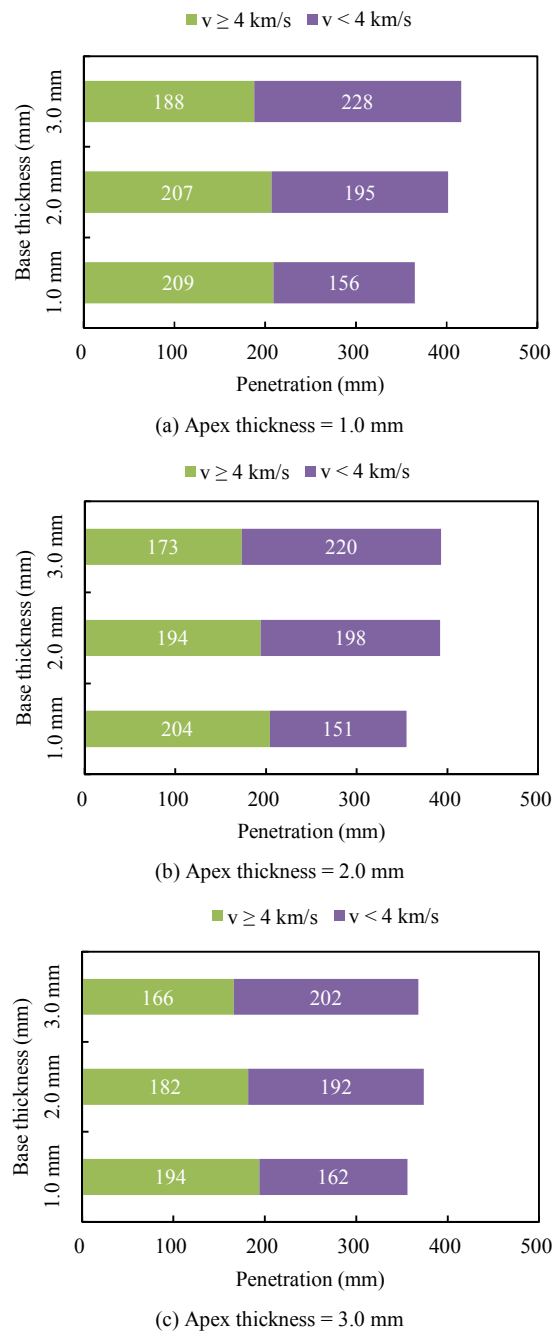


Fig. 10. Comparisons of penetration depth before and after the hydrodynamic limit. At the same apex thickness, nonhydrodynamic penetration depth increases with base thickness. At the same base thickness, hydrodynamic penetration depth decreases as apex thickness increases. In all cases, the total penetration depth decreases with base thickness.

the same apex thickness, as the base thickness increases, the hydrodynamic penetration depth decreases, and the nonhydrodynamic penetration depth increases.

The difference between the hydrodynamic and nonhydrodynamic penetration amounts affects the entire penetration depth. In several cases of deep hydrodynamic penetration, the final penetration depth is less than that in the other cases due

to less nonhydrodynamic penetration. Such cases were identified by comparing the base cases with low thickness, particularly when the base thickness was 1.0 mm.

4. Discussion

In this work, the hydrodynamic limit, specifically, the applicability of the hydrodynamic theory, was identified using a numerical analysis of the penetration of targets. AISI 1045 and AISI 4340 were adopted as target materials; these materials have the same density but different strengths. In the jet part, the evaluated ratios of density and velocity did not match. Furthermore, the accumulated differences in the penetration depth also increased, when the velocity was less than 4 km/s. Therefore, the hydrodynamic limit was identified to be $V = 4$ km/s. This identification method can be regarded as a fully quantified identification method based on previous studies conducted over several decades.

A case study considering the variation in the apex and base thicknesses of the liner was conducted based on the hydrodynamic limit. As the apex thickness decreased, the hydrodynamic jet velocity ($V \geq 4$ km/s) distribution and the hydrodynamic penetration depth increased. The result that the faster jet formed from the thin apex was in agreement with the study of Fedorov et al. [9]. Abrahamson and Goodier also suggested that the penetration depth can be evaluated using Eq. (9) when the jet velocity distribution before penetration is known [10].

$$P = \left\{ \left(\gamma + \frac{1}{v_L^2} \right) \int_0^L \{v(\xi)\}^\gamma d\xi - L \right\}, \quad (9)$$

where $v(\xi)$ is the jet velocity distribution and ξ is a coordinate with an origin located at the jet tip. The penetration depth of nonuniform velocity jets is proportional to the integration of jet velocity $v(\xi)$ through position ξ . Refs. [4, 8] reported that the SCJ with a thin apex thickness liner could form a fast hydrodynamic jet and deepen further its hydrodynamic penetration.

The decrease in the hydrodynamic penetration in accordance with the increase in the base thickness was due to the assumption of the variation in liner thickness, which was set to linearly change from the apex to the base. When the base thickness increased, the thickness around the apex point increased. Therefore, it could be caused by the decrease in the hydrodynamic jet velocity and the hydrodynamic penetration depth.

The nonhydrodynamic penetration depth, which was influenced by the target strength, increased as the base thickness increased. This result was because of a large number of liner materials supplied to the nonhydrodynamic jet. Therefore, the penetration capability to overcome the target strength was increased by a jet having a substantial kinetic energy.

As such, the authors recommend that the identification of the hydrodynamic limit should be prioritized when designing a shaped charge warhead. When the required penetration depth is within the hydrodynamic penetration depth, a liner

with an appropriately thin apex thickness should be selected. A liner with sufficient base thickness should also be selected when a deep penetration depth is required.

This work covered only the effect of liner design and a hydrodynamic limit on the penetration depth by using a numerical analysis. Nonetheless, various parameters that affect the penetration capability of the shaped charge warhead, such as the stand-off distance, explosive type, and ratio of the warhead length to diameter, exist. Thus, the authors will conduct further studies that focus on the effect of these parameters with consideration of the hydrodynamic limit, as introduced in this study. Through further research, an easier performance evaluation of shaped charge warheads is expected to be possible when several design parameters are decided prior to executing detail designs. The addition of hydrodynamic limit considerations to the ability of SCJs to penetrate various target configurations, such as Arnold and Rottenkolber [11], is also expected to contribute toward improving its performance.

5. Conclusions

In this study, the hydrodynamic limit of the shaped charge warhead identification method was proposed. Using this method, the penetration characteristics of the SCJ before and after the hydrodynamic limit were investigated. Results showed the hydrodynamic penetration capability improved as the apex thickness decreased and the nonhydrodynamic penetration capability improved as the base thickness increased. Various design parameters of the shaped charge warhead are expected to be optimized selectively and systematically using the proposed method.

Nomenclature

V	: Jet velocity
U	: Penetration ratio
ρ_p, ρ_t	: Density of jet and target
γ	: Density ratio, $(\rho_p/\rho_t)^{1/2}$
P	: Penetration depth
h	: Representative element size
L	: Penetrator of jet length
σ_y	: Flow stress
A_0	: Johnson-Cook yield stress
B_0	: Johnson-Cook hardening coefficient
n	: Johnson-Cook hardening exponent
C_0	: Johnson-Cook hardening coefficient
ε_p	: Equivalent plastic strain
$\varepsilon_p, \varepsilon_0$	: Plastic strain and plastic strain rate
m	: Thermal softening exponent
p	: Pressure
C	: Speed of sound
μ	: The ratio of initial and current densities
γ_0	: Grüneisen gamma
S_I	: Coefficient of the Hugoniot curve
A	: JWL EOS coefficient

B	: JWL EOS coefficient
ω	: JWL EOS coefficient
R_1	: JWL EOS coefficient
R_2	: JWL EOS coefficient
E	: JWL EOS coefficient

References

- [1] G. Birkhoff, D. P. MacDougall, E. M. Pugh and S. G. Taylor, Explosives with lined cavities, *Journal of Applied Physics*, 19 (6) (1948) 563-582.
- [2] R. J. Eichelberger, Experimental test of the theory of penetration by metallic jets, *Journal of Applied Physics*, 27 (1) (1956) 63-68.
- [3] W. A. Gooch, M. S. Burkins, W. P. Walters, A. A. Kozhushko and A. B. Sinani, Target strength effect on penetration by shaped charge jets, *International Journal of Impact Engineering*, 26 (1-10) (2001) 243-248.
- [4] T. Elshenawy, A. Elbeih and Q. M. Li, Influence of target strength on the penetration depth of shaped charge jets into RHA targets, *International Journal of Mechanical Sciences*, 136 (2018) 234-242.
- [5] M. B. Zellner et al., Shaped charge jet penetration of Alon® ceramic assessed by proton radiography and computational simulations, *Procedia Engineering*, 103 (2015) 663-670.
- [6] J. Bolstad and D. Mandell, *Calculation of a shaped charge jet (king MESA-2D and MESA-3D hydrodynamic computer codes)*, Los Alamos National Laboratory, University of California for the United States Department of Energy (1992).
- [7] R. DiPersio, J. Simon and A. Merendino, *Penetration of shaped-charge jets into metallic targets*, US Army Ballistic Research Laboratory, BRL (1965).
- [8] G. R. Johnson and W. H. Cook, A constitutive model and data for metals subjected to large strains, high strain rates and high temperatures, *Proceedings of the 7th International Symposium on Ballistics*, The Hague, Netherlands (1983).
- [9] S. V. Fedorov, Y. M. Bayanova and S. V. Ladov, Numerical analysis of the effect of the geometric parameters of a combined shaped-charge liner on the mass and velocity of explosively formed compact elements, *Combustion, Explosion, and Shock Waves*, 51 (1) (2015) 130-142.
- [10] G. R. Abrahamson and J. N. Goodier, Penetration by shaped charge jets of nonuniform velocity, *Journal of Applied Physics*, 34 (1) (1963) 195-199.
- [11] W. Arnold and E. Rottenkolber, High explosive initiation behavior by shaped charge jet impacts, *Procedia Engineering*, 58 (2013) 184-193.



Youngku Kang has been working as a CAE engineer at the Defense R&D Center of Hanwha Corporation for the past seven years. His major areas of specialization include modeling and simulation of explosives and shock phenomena and structural analysis of ammunitions.