

Interactive Maths Teaching in the Primary School



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Starting points for interactive shape and space work

This chapter will ...

- give an introduction to shape and space, briefly considering what might be the key ideas and principles for teaching it;
- provide examples and starting points for interactive teaching of shape and space, again modelling different approaches from Chapter 5.

What is shape and space all about?

Clearly, 'shape and space' is about 'shape' and about 'space', but this trivialisation of the topic hides several important points that, I think, are often lost when I see the topic tackled in schools.

The most important point about shape and space work in schools is that children's focus tends to be on 'shapes', rather than on the notion of 'shape'. Perhaps the best example of this distinction is illustrated by the general confusion over squares, oblongs and rectangles. Every year when I teach a shape and space module to undergraduate student teachers I hear people asking what the difference between these words is. Versions of the question include: 'are oblongs just rectangles?'; 'what's the difference between rectangles and squares?'; 'I was told to say rectangles because oblong was a primary school word'; and so on.

Let me first clear up any confusion. *Rectangle* refers to any quadrilateral (four-sided, two dimensional) shape whose corners are all right angled (note that the German for 'right' is 'recht'). You should now be able to see that this *includes* the squares, since they share this property, but also have the property of four equal length sides. This means that squares *are* rectangles. So what about 'oblong'? Well, note that although 'squares are rectangles', the inverse statement 'rectangles are squares' is not always true, since there are many rectangles that are not square – all those where one pair of sides is longer than the other. What we need is a word to describe these 'non-square' rectangles ... and this is what the word 'oblong' does. Figure 7.1 shows this relationship visually. You will see that, in fact, *square* and *oblong* are best thought of as adjectives, not nouns; they describe the kind of rectangle that we are looking at, but they are used so frequently that they revert to nouns more often than not.

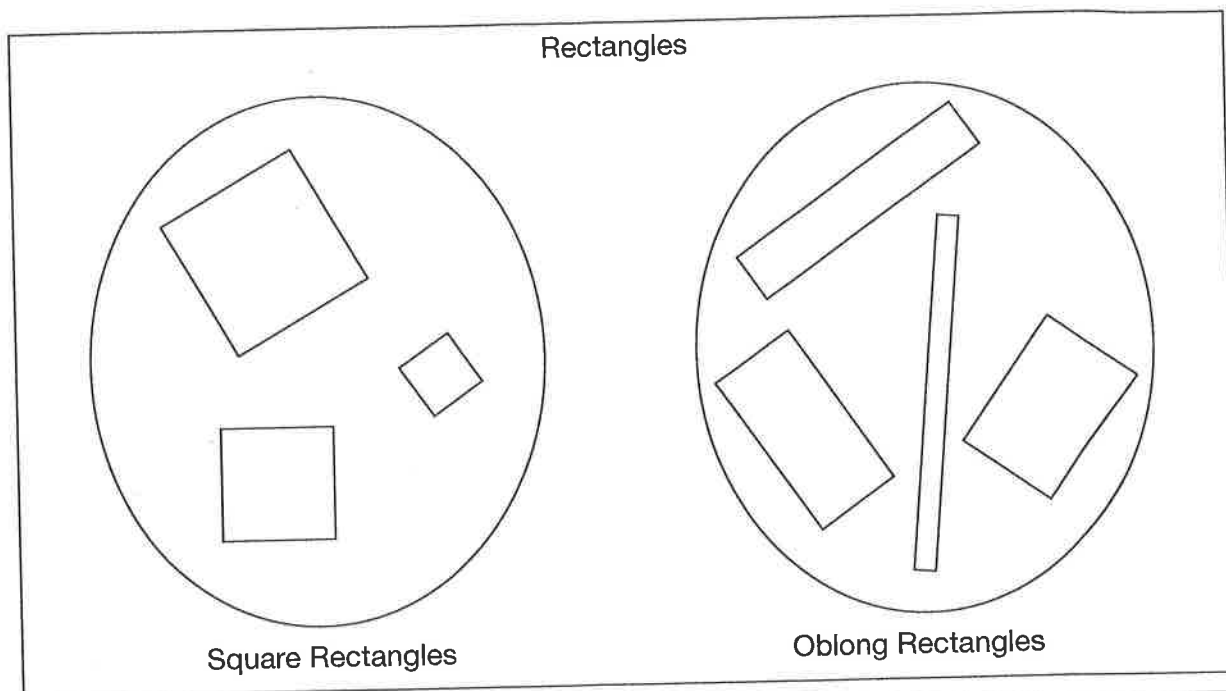


Figure 7.1 Relationships between rectangles

To return now to the distinction between 'shapes' and 'shape', I want to point out that there are lots of 'shapes' which are squares and oblongs, but that each one is named that way because of its 'shape'. Am I teasing you with language? No, the distinction is that each of 'the shapes' is fixed and relates only to that particular one; on the other hand the idea of the 'shape' of squares or oblongs focuses us on the properties that define the whole set of them; spatially what it is about the object that *makes it* a square. This mirrors my claims in Chapter 2 about the nature of maths being essentially to do with ideas. In the case of, say, squares children need to abstract the idea of 'square-ness' from the activity and examples they engage in and then name this idea 'squares'.

Unfortunately, in school, children often do little more than learn the names for 'the shapes' with which they are presented. In other words they tackle things back to front by starting with the names and trying to deduce the properties. It would make better sense to identify the properties, collect all the shapes that fit these properties and then name this set. The name then gets attached to the properties involved – the spatial arrangement that makes the shape what it is – not to isolated examples of them. Of course later, once they know the names and the essential properties that define the set, they will continue to explore further properties.

Where children experience the first of these approaches – naming isolated examples, as they often do – two problems tend to occur. Firstly they cannot understand the *inclusive* nature of shape – that squares can be included *within* rectangles (just as Nick Pratt can be included within 'male' and 'human' etc.). Children who focus on particular shapes, not on the idea of shape, focus on the differences all the time and fail to see the inter-relationships.

Second, and more importantly in the long run, children miss out on all the fun! Simply knowing that a shape is called 'a square' is rather dull; understanding the notion of 'square-ness' and seeing all the relationships and possibilities it holds is much more likely to be of interest. For example, you might like to consider:

- Is a square a parallelogram?
- Is a parallelogram a square?
- Do the diagonals of a square always cross in the middle?
- For rectangles with a fixed perimeter, the square rectangle always has the biggest area (i.e. all the oblong rectangles with the same perimeter have smaller areas) ... how could you demonstrate this?¹

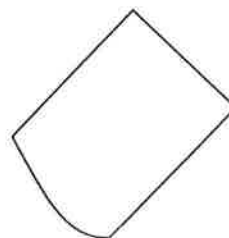
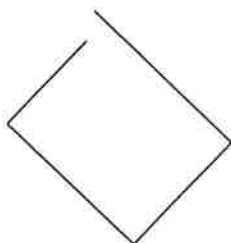
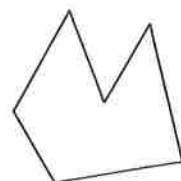
KEY IDEAS



- Teaching of shape and space should focus on the spatial relationships that shapes have.
- We should be helping children to focus on the wider idea of 'shape' and the properties associated with certain sets, not on examples of particular, individual shapes.
- We should also be focusing on the *implications* of shape properties; what each property implies about the shapes and their relationships to other shapes.

IN YOUR CLASSROOM

- Do you show children many examples of each shape name so that they do not come to associate the name only with particular versions of it?
- Do you include highly irregular examples, like this hexagon, as well as the more common ones?
- Do you draw shapes at different angles so that their bases are not always horizontal?
- Do you draw out the connections between 2D and 3D shape, showing children that 2D shapes form the boundaries of 3D shapes?
- Do you use 'near examples' to help children focus on what is and what is not included in each set of shapes? For example, these non-rectangles are all 'near examples':



- Do you sometimes use made-up names for shapes to help children focus on properties (for example, calling quadrilaterals and pentagons 'fourers' and 'fivers' to focus on sides; or rectangles 'squaries' to focus on corners)?

¹ The answers, by the way, are respectively: 'yes, always'; 'only when it's a square one'; 'yes, always'; and 'I'll let you figure it out'.

As I did with number in the previous chapter, I now provide a series of starting points for investigating shape and space across Key Stages 1 and 2, beginning with one example lesson outline. Creating 'purposeful' tasks tends to be easier than for other areas of the mathematics curriculum because of the visual and tactile nature of shape and space. Simply using, holding and manipulating shapes tends to be engaging for children and I have found that they usually need little encouragement to focus intently on what they are doing. On the other hand, this physical manipulation can often be unstructured and children can be too focused on simply 'doing' with little thought given to the implicit shape ideas; the mathematical thinking tends to get lost. My aim, therefore, is to exemplify the shift in emphasis outlined in the first section of this chapter, from looking at shapes towards thinking about properties, patterns and spatial ideas.

Example lesson outline 1: Lines of symmetry (Year 3)

Planning points	Commentary
Learning centre-point: Recognise and sketch more than one line of symmetry. Purposeful task: Create a symmetry puzzle book using cut-out paper shapes.	A typical objective taken from the numeracy curriculum. The lesson assumes that the children already have a working knowledge of line symmetry.
Getting started: Teaching point: Sit children in a horseshoe with individual whiteboards. Remember to use neutral responses to answers to encourage children to keep thinking creatively. Fold a square of paper in half and cut a shape from the folded edge. ► <i>'Sketch what you think will appear when the paper is unfolded.'</i> Fold the paper twice and cut a shape from the folded corner. Again, ask children to sketch what they think will appear. Again using two folds, try cutting shapes from different places on the folds of the paper, asking children to predict the outcomes each time Teaching point: Encourage children to close their eyes and to visualise the pattern in their minds. Model the unfolding carefully to help children to develop a visual picture of the way it works. Teaching point: Resist talking as you unfold the paper, allowing children to concentrate on what is happening visually.	Encourage children to show their predictions and to reason about why the shape will appear that way. Note that you can use an interactive teaching programme and whiteboard for this but, to me, the physical unfolding of the paper is important. Ask children to describe how the pattern works. Focus on where the folds appear and how these act as lines of symmetry. The temptation is to 'tell' the children what is going on yourself. Try, instead, to let them do the thinking. Then ask them to describe back to everyone what they were visualising.

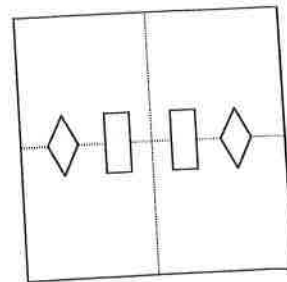
Example lesson outline 1: Lines of symmetry (Year 3) continued

Planning points	Commentary
Follow up questions ... ► What patterns can you describe to help in predicting the shape? ► Can you give a rule(s) for working it out? The main task: Provide paper and scissors for children and pair them up. One child makes his or her own pattern by cutting and unfolding secretly before showing his/her partner; the other child then has to try to replicate the pattern by working out how to cut their own piece of folded paper. Take turns. After a few turns, change to making 'reflection puzzles'. Children create a pattern 'template' – i.e. a folded sheet with bits cut out. They then create several different unfolded patterns, only one of which matches the 'template'. Others have to guess which unfolded pattern originates from the template.	Focus on: Generalisation and Procedures and work as a class to articulate logically the procedures for unfolding the paper, identifying where each 'hole' will end up. Encourage children to think about visualising the pattern mentally and using the language of reflection – flip over, reflect, line of symmetry etc. Encourage children to make 'convincing' non-matching patterns. Discuss shapes that might successfully mislead people doing the puzzle. For example, a square cut on a folded edge opens up into an oblong; but putting squares in the false patterns may help to mislead others.
Preparation for the plenary: Five minutes before the plenary, ask children to stick their 'puzzles' onto a large sheet of paper and to prepare for a 'poster conference'. Each pattern on the paper should be labelled with a number or a letter.	
In the plenary: Place all the posters around the room with an A4 sheet and pencils next to each one. Pairs circulate around the posters and write down the letter or number that they think matches the folded 'template'. They can also give reasons for their choice. After all the posters have been visited pairs return to their own to see if others have guessed correctly.	Encourage children to think logically and to visualise how the template will unfold in each puzzle. Though you might want to round off the lesson with a short comment, the work has been done through the independent discussion in pairs and examination of each poster. I would resist too much discussion after the poster conference as it is likely to kill off the enjoyment of the event!

Interaction in practice – Symmetry (Year 3)

Follow up questions ...

The teacher has folded the paper left to right and then downwards and cut a square and an equilateral triangle from the top edge. The unfolded pattern will be two oblongs and two 'diamonds' (rhombuses) left and right of the vertical fold line.



T: What I want you to do is watch as I unfold the paper slowly and picture it in your mind, like playing a video in your head. Try to make your video go just ahead of mine though, so you see it unfolding just before it actually happens. OK ... here goes.

[Teacher unfolds the paper slowly.]

T: Right, now, I'm going to do it again and this time I want you to do a commentary to yourself as we go. In your head, imagine describing what is happening to someone else, on a phone maybe. Someone who can't see you. Yes? What would you say to them to describe how each part of the pattern was moving? OK, here goes.

[Teacher again unfolds the paper slowly.]

T: OK, now I want you to talk to your partner and describe to each other what you saw. Try to think about how each part of the pattern moved.

[Children spend a minute talking in pairs.]

T: Right. Now I'm going to make it harder. One person in the pair put your hand up. ... OK, you have got to turn round and face the other way. Face the wall over there. Right. Everyone else, you are going to watch me do a new pattern and then you have got to describe it to your partner who is going to try to draw it. Yes, on the whiteboard, that's right.

[Sounds of surprise ... groans of general excitement.]

OK, are you ready? No peeping you lot.

[Teacher cuts a new pattern and slowly unfolds it, then hides it away.]

T: OK, turn round the others. Off you go. The one who wasn't looking has got to draw what their partner describes to them. Think about how it unfolded to help you.

Ch1: Can we draw it for them?

T: No, you can only talk ... and no pointing and drawing with your fingers either!

[Children talk in their pairs, trying to recreate the pattern.]

T: OK, let's see what's happened. Can you sit round the edge of the carpet? [Children shuffle to the edge.] Right. Here is the pattern I cut out [shows the paper pattern]. How have you done? [General chatter which the teacher lets run ... then, interrupting] OK, OK. Listen ... listen. I want you to decide who you think got it closest.

Ch1: Marie-Jane.

Ch2: Mine's useless!

Ch3: Jay. His is good.

Ch4: Yeh, his is really close.

[Children continue to comment freely until a general consensus is reached about the best.]

T: Right Jay, your one has got the vote. Can Luke [partner] describe for us what you said to Jay.

Luke: Um ... it's hard ... I said, like ... no, it's hard.

T: OK, well I'll unfold the paper again and you talk out loud. Everyone else, keep quiet as Luke talks, but be ready to make suggestions for different things to say at the end.

Luke: The square starts in the middle of that side and when you unfold it it goes into an oblong.

Ch5: It's like a ...

T: [Interrupting] No, stop. You are listening now and then you can say it in a minute. Let Luke finish first.

Luke: Then the oblong goes over to the other side ... the same shape ... I mean the same place.

T: Ok, thanks Luke, well done. Now, what about others. Any comments?

Ch5: The oblong is like a double square. It's kinda two squares on top of each other.

[Teacher nods but remains silent. Indicates with a gesture for other comments.]

Ch6: They're the same on each side.

T: Can you say any more about that?

Ch6: The same distance [meaning from the centre-fold; again, teacher nods and gestures].

Ch7: The triangle turns into a diamond.

Ch8: Yeh, it kind of gets long.

Ch6: And it goes on the other side again, like the oblong.

T: What do other people think?

Ch9: It is. It's the same over there as over there [indicating on each side of the vertical fold].

T: Several people have said that now. It's the same over here as over here [indicates the two sides again]. What I want to know now is, if I asked for a rule to describe how it unfolds what would you say? I want a rule that tells me where the shapes end up. Talk together first.

[Children talk in pairs for a minute or so. One or two ask for clarification of the task.]

T: OK, what rules would you have?

Ch4: They're the same size ... the same. Each shape ... is the same ... you know ...

T: No. I don't know yet. Can you explain again.

Ch4: [Pauses to think.] Both the shapes are the same size.

Ch10: Which shapes?

T: Good question. Which shapes?

Ch4: The oblongs. They are the same. And the triangles are.

T: OK, thanks. Any other rules?

Ch9: And they are the same place.

T: How do you mean, same place?

Ch10: He means ...

T: [Interrupting] Hold on. Let her finish.

Ch9: It's the same on this side as that side. The place. This is the same [gets up and indicates distance from central fold to oblong on each side.]

T: Great, OK. Anything else to say?

[The conversation continues for a short while with the teacher continuing to help the children to articulate the ideas behind reflective symmetry.]

It is worth noting how much of the extract above is dominated by the teacher talking, yet it remains interactive in the sense that the children are thinking deeply about how the reflections work. This is a nice example of how interaction can be visual as well as oral – playing up the mental and playing down the oral in 'mental/oral'. It is the visualisation that matters in this task and it is from this that children can then develop the 'rules' for symmetry through discussing these visual images.

Examples of other possible starting points for shape and space

Having presented an example of a lesson outline, the remainder of this chapter provides a series of starting points for investigating shape and space across Key Stages 1 and 2. As in Chapter 6, my aim is to show what is possible – assuming that you will change, adapt, remove and add ideas as you see fit.

Foundation

Learning centre-point	Purposeful task	Interactive opportunities – getting started	Interactive opportunities – plenary
Talk about, recognise and recreate patterns.	Make jewellery for a jeweller's shop and sort it for display.	■ Create repeating patterns (on the board, with beads on threads etc.) and ask children to discuss 'what comes next?'. Make some of the sequences short enough for the pattern not to be fixed so that there are different possibilities open for discussion (e.g. blue-red-blue ... could continue ... RBRB etc. or repeat as BRB BRB etc.) ■ Look together at examples of real necklaces and bracelets with repeating patterns on them. In pairs, try to explain the patterns verbally. ■ 'Record' the patterns verbally by chanting them: 'blue, red, blue, red ...', 'circle, square, square; circle, square, square ...' etc. Then split the class up so that different groups only say their colour or shape and the groups must work together to chant the pattern. ■ Individually or in pairs, begin to make bracelets or necklaces of their own using string and threading beads, pasta etc.	■ Five minutes before the plenary ask children to practise explaining their pattern to a partner. ■ In a circle, lay out the bracelets and choose individuals to show their patterns. Challenge other children to identify and explain the pattern and see if the maker agrees. ■ Ask children how the jewellery might be sorted in the shop window so that similar items were together. Leave the meaning of 'similar' open so that children can give their ideas about how to sort them.

Extensions:

- Set up a jeweller's shop in the classroom and develop the making and sorting through role-play.
- Create 'rules' for the patterns, verbally at first and then recording them on paper to display in the shop using children's own symbols and drawings.
- Make brooches and pendants out of shiny paper in a variety of different shapes. Include different examples of the 'same' shape, sorting them for the display again (i.e. 'five-sided pendants'; 'curved brooches' etc.).
- Create similar patterns in different contexts (open a whole street of shops!) such as printing patterns for a bedroom wallpaper border, T-shirt designs etc. Ask children to explain their patterns and have an adult scribe this for a display of the items made.

Year 1

Learning centre-point	Purposeful task	Interactive opportunities – getting started	Interactive opportunities – plenary
Describe and classify common 3D shapes.	Play 'What am I?' and make puzzle posters for a display.	■ Have two sets of identical 3D shapes, one set hidden and one set visible to the children. Put shapes taken from the hidden set one at a time in a 'feely bag'. Pass round the group and challenge children to identify which of the visible shapes it matches. ■ Choose children to feel a new shape and describe it well enough for others to guess which (visible) shape it is the same as. Have them working in pairs and focus them on appropriate language relating to faces, edges and corners. ■ 'Deconstruct' a cuboid cardboard box by cutting it up so that each of the faces is separated. Ask children to talk about their understanding of the shapes and how they fit together to make the cuboid. ■ Select 3D shapes from the original sets and ask children to discuss in pairs what the faces would look like if 'cut up' like the box. ■ Use thick paint (add some PVA glue to it) to create prints of the sides on sugar paper. ■ Set groups of children printing the faces of different 3D shapes.²	■ Ask children to practise talking with a partner about the shapes on their printing picture. ■ Encourage questions about the names of different shapes not known by children. Discuss the properties of these shapes and link new (possibly made-up) names to them. ■ Choose some children to show their printing pictures and challenge others to decide which 3D shape it came from based on the prints of the faces. Receive different answers and then ask children to justify their decisions.

² Clearly, this will best be done one group at a time whilst the other children work on another task!

Extensions:

- Display the pictures and invite another class into the room to see if they can recognise the original shape. Encourage your children to explain how the pictures work.
- Investigate the shadows that 3D shapes cast when placed on an OHP. Use a cardboard screen round the OHP to hide the shape and see if children can guess what it is as you rotate it onto different surfaces.
- Make a 'shape museum' using packaging from home. Each 'artefact' (shape) must have a label describing some of its key features (properties).

Year 2

Learning centre-point	Purposeful task	Interactive opportunities – getting started	Interactive opportunities – plenary
Make and describe common 2D shapes.	Make shape pictures identifying new shapes within them.	■ Use a large cardboard square and draw round it on the board. Then move the square slightly and draw round it again.³ Discuss the shapes created by the intersection and the remaining regions of the original squares. ■ Repeat this operation creating different intersections each time and discuss the resulting sets of shapes. ■ Challenge children, discussing in small groups, how to create: triangles of different sorts; rectangles of different sorts; as many rectangles/triangles as possible; a five-sided shape; an eight-sided shape etc. Providing two squares of coloured acetate (from a stage lighting shop) or tracing paper will help children to experiment. ■ Let children work individually either on a computer or with plastic shapes to draw around on a sheet of A3 paper. Encourage them to explore the different shapes that can be made.	■ Five minutes before the plenary, introduce the idea of categorising each pattern according to the sides of the shapes created. For example, the two illustrations on the left might be 'two sixers and a fourer' (because a square and two irregular hexagons are created) and 'an eighter, and eight threers' (octagon and eight triangles). ■ Ask children to work out what their examples would be and to bring their thinking to the plenary. ■ During the plenary create a list of different categories. Group them so that, say, all the ones involving 'fourers' are together. ■ Look for 'missing' shapes or new versions together ('has anyone found a fiver ... a different fourer?').

³ Instead of drawing with a marker pen, use Microsoft Word's drawing facility to create two squares. Select the shapes one at a time with a right click on the mouse to get a pop-up menu and choose the 'format autoshape' option. 'Fill' them in two different colours and make them 50% 'transparent'. When placed over each other, the intersection is seen as a mix of the two colours. One or both squares can then be rotated and translated on an interactive whiteboard.

Extensions:

- Use different starting shapes.
- Use more than two shapes to create more complex patterns.
- Set up an 'investigation zone' in the classroom with equipment (a computer, coloured acetates or simply plastic shapes, pencils and paper) for children to experiment with during free time.

Year 3

Learning centre-point	Purposeful task	Interactive opportunities – getting started	Interactive opportunities – plenary
Make and describe 2D shapes and sort them according to their properties.	Make simple shape sets by cutting polygons.	■ Begin by displaying examples of a number of common polygons made from paper. Make the examples different and include some unusual ones (e.g. a very long thin triangle). ■ Ask children to work in pairs and to choose one secretly and plan how they might describe it. ■ Choose pairs to describe their choice out loud for others to try to identify (a harder version is for others to have to ask yes/no questions to identify it). ■ Select a rectangle and cut it (with a straight cut) into two new polygons. Ask children for comments about what they see and think. ■ Cut one of the new polygons again, discussing the two new shapes that are created. Focus on properties, then link a shape name to them. ■ Begin again with a new rectangle. This time cut it differently so that a different set of three shapes is created. ■ Provide lots of paper rectangles and challenge children to make two cuts of their own like this to create some specific sets of three shapes – start by making three rectangles; three triangles; one each of triangle, quadrilateral and pentagon etc.	■ Five minutes before the plenary, ask children to choose one of their sets of polygons to focus on and to get ready to describe it to others in terms of sides and angles. ■ Begin to group the sets of shapes according to children's descriptions of their properties. Focus on number of sides and types of angles. ■ If appropriate, discuss and agree what it means for two shapes to be 'different'. ■ Discuss labels that might be given to each set to describe its contents and build up to a simple display. Include spare rectangles and scissors as part of this and encourage children to return to it in spare moments to find new examples.

Year 3 continued

Learning centre-point	Purposeful task	Interactive opportunities – getting started	Interactive opportunities – plenary
		■ Then challenge them to create as many <i>different</i> sets of three polygons as possible. Leave the meaning of 'different' in this context vague and open to discussion. Are two rectangles of varying size different (because of their size) or the same (because they are both rectangles)? ■ Tip: get children to glue their three shapes onto sugar paper each time so they don't get muddled up.	

Extensions:

- Start with a shape other than a rectangle.
- Allow a third cut to make four polygons and look at the new sets you can make.
- Challenge each other to put the pieces back together again to recreate the original starting shape.
- Use the polygons created as simple *tangram* pieces and investigate new shapes that can be made with them.
- Encourage children to create their own 'challenges' for others and display these (for example: 'What is the shape with the most sides that you can make?'; 'Can you make at least two squares?' etc.).

Year 4

Learning centre-point	Purposeful task	Interactive opportunities – getting started	Interactive opportunities – plenary
Make 2D shapes with increasing accuracy.	Play 'make me draw it' and design a poster explaining the properties of 2D shapes. Hold a poster conference.	■ Challenge the children to <i>force</i> you to draw a chosen 2D shape. Pairs take it in turn to give instructions which you must follow whilst trying to <i>avoid</i> drawing the shape. For example, if children start the instructions for a square by telling you to 'draw a line', you might draw a curved line. ■ Encourage children to talk together about these instructions. (Note how the game demands clarity about the key spatial features of the shape.) Take opportunities, as they arise, to discuss 'what is meant by ...?'. ■ Split the class into two groups and play again with one group giving instructions and the other group trying to avoid drawing the shape. Encourage clarity in what is being said. ■ Set children working in fours with two pairs playing against each other. ■ After ten minutes change the task so that each group must produce a poster explaining the key properties of one or more of the shapes they have been playing with. The poster must explain exactly how to generate the shape.	■ Run a poster conference. Each group places their poster around the edge of the room. Blank A4 sheets are left next to them. Each group circulates around the room looking at the posters and leaving a comment on the A4 sheet. ■ Encourage comments to identify both clear thinking and possible challenges to the instructions for drawing the shape(s). ■ Groups return to their posters to read comments. Then discuss alterations that might be made to their posters in light of any challenges.

Extensions:

- Extend the complexity of the shapes. Circles are particularly interesting. Note that they appear difficult to define, but can easily be forced by giving the following instructions: draw a dot; place the pen anywhere other than on the dot; draw a curve, always remaining an equal distance from the dot and continue until you reach your starting point again.
- Use the posters to write concise definitions of each shape and publish these in a class shape and space 'dictionary'.
- Use LOGO or a floor robot and work out simple programmable instructions to draw the shapes.
- Take long lengths of elastic and stitch the ends together to make a large loop. Challenge groups to stretch the elastic into different shapes by holding it and moving in different ways.

Year 5

Learning centre-point	Purposeful task	Interactive opportunities – getting started	Interactive opportunities – plenary
Constructing 3D models with increasing accuracy.	Work with shape sets to build 3D models. Explore all possible outcomes.	■ Use a 3D building material such as 'Polydron', 'Clix' or 'MATs'.⁴⁴ ■ Provide everyone with just eight 'pieces' of any mix of shapes and ask them to build a 3D solid using some or all of them. Once everyone has made their shape, stand them up and ask them to find similarities between shapes and to make groups accordingly. ■ Start again, rebuilding the shapes. Challenge children to try to build a shape with unusual properties so that they are standing on their own when they form groups. ■ Provide groups with a new 'shape set' and challenge them to find all the possible 3D shapes that can be made from it. Try a limited number of: – triangles and squares only – hexagons and triangles only – hexagons and squares only – octagons and triangles only	■ Prior to the plenary focus children on the faces, edges and vertices of their shape. ■ Use these ideas to talk about the shapes and to classify them in the plenary. ■ Discuss whether or not the children have found them all in each case.

⁴⁴ MATs are cardboard shapes produced by the Association of Teachers of Mathematics. They can be laid out on tables for 2D work or stuck together with glue to make 3D solids.

Extensions:

- Explore all the nets that can be unfolded from each shape.
- Use 'sets' of pipe-cleaners and straws (or a commercially available alternative) to build skeletal models instead of solids. (Note that skeletal models focus children's attention on edges and vertices, rather than faces.)
- Allow children to use just one regular 2D shape, but as many as they like. This will provide an opportunity to find the five regular solids (triangles lead to the tetrahedron, octahedron and icosahedron, squares to the cube and pentagons to the dodecahedron).

Year 6

Learning centre-point	Purposeful task	Interactive opportunities – getting started	Interactive opportunities – plenary
Know properties of common 2D shapes	Hold a debate about controversial shape properties.	■ Draw a set of images like those shown here. Discuss 'what is happening to the kite'. ■ Ask children to discuss in pairs what they think will happen next to it. (Note: it will momentarily become a triangle before becoming a deltoid (arrowhead).) ■ What about stretching it the other way? ■ Is it always a kite, or does it turn into other things? Use this question to discuss how children are defining the idea of 'kite'.⁵ Note how the idea of 'kite' is less certain than it might have been before and needs arguing about, agreeing and defining. ■ Encourage children to deliberately create arguments 'for and against' and try, as a class, to 'attack' the arguments by reasoning and finding counter examples. ■ Split the class into two groups and ask them to discuss one of the following statements (or make up your own). Provide appropriate equipment to help children model the ideas. One group must argue <i>for</i> the statement and the other <i>against</i> it ... – Equilateral triangles are just special cases of isosceles triangles. – Rectangles are special parallelograms.⁶ – Diagonals can never be vertical or horizontal.⁷ – The order of rotational symmetry is always the same as the number of sides for 2D shapes.⁸ – 3D shapes don't have diagonals.⁹ – Four cubes can be glued together in only six different	■ Allow 20–30 minutes for the discussion. ■ Provide 'talking tokens' in two different colours: one for statements; one for questions. ■ Children give up a token of the appropriate colour each time they speak. ■ Take the statements in turn and debate their validity. By controlling the number of tokens you can encourage all children to speak and no one child to dominate.

- ⁵ Note that some dictionaries are happy to include deltoids as kites (inverted kites, if you like) whilst others define kites to have all their angles non-reflex (i.e. $<180^\circ$).
- ⁶ Yes – since parallelograms are defined as being quadrilaterals with two pairs of parallel sides. Rectangles are simply a special case of the parallelogram where all four corners are right angles (and note that squares are special cases of rectangles – and hence of parallelograms too – where all sides are also the same length).
- ⁷ Clearly they can (imagine a square standing on one corner), but this statement encourages children to distinguish between the word 'diagonal' in general and 'the diagonal(s)' of a shape.
- ⁸ True for *regular* shapes, but not for all 2D shapes in general.
- ⁹ Diagonals in 3D shapes are generally only considered to be those that connect vertices *not* in the same face. In other words, diagonals on the faces do not count and all diagonals must pass through points 'inside' the shape. A cube, for example, therefore has four diagonals.
- ¹⁰ There are six different ways to arrange four cubes where 'different' means that rotations and reflections are considered to be 'the same' as each other. If they are considered as 'different' then the number of solutions increases.

Extensions:

- Play 'Call My Bluff' with statements that the children have invented. For each statement children create an accurate answer and two plausible, inaccurate ones and then others must guess which is which.
- Ask children to write down their arguments, focusing on clarity and accuracy. Then email these to children at another school for them to respond to ... and receive similar statements from them to puzzle over.

Summary

- Shape and space work should be about the sense of shape, not just about particular shapes that appear regularly, and their names. Names are important but properties are the key thing and names should be linked to these.
- ▶ How does the claim above relate to your view of shape and space? Do you tend to focus on names or on properties? Are you able to make connections between shapes and understand their properties?
- Encouraging children to sort, construct and take apart shapes and to discuss what they can see will help children to focus on properties.
- ▶ What kinds of tasks have you given children in the past for shape and space? Have these encouraged children to see new properties and to think creatively and logically about the relationships between shapes?



Further reading

Johnston-Wilder, S. and Mason, J. (2005) *Developing Thinking in Geometry*. London: Paul Chapman.

Ebbutt, S. and Mosley, F. (1996) *Exploring Shape and Space*. London: BEAM.